SOA and CAS: Exam P, Probability[1]
Chapter 9: Percentiles

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(1) *Defintion:* Medium measures the center of a distribution, besides mean and mode

Medium	$X_P^{0.5}$: which makes $F_X(x) = 0.5 <=> \int_{-\infty}^{X_P^{0.5}} f_X(x)\, dx = 0.5$ ("50th percentile of X" is at $X_P^{0.5}$)
25th Percentile	$X_P^{0.25}$: which makes $F_X(x) = 0.25 <=> \int_{-\infty}^{X_P^{0.25}} f_X(x)\, dx = 0.25$ ("25th percentile of X" is at $X_P^{0.25}$)
75th Percentile	$X_P^{0.75}$: which makes $F_X(x) = 0.75 <=> \int_{-\infty}^{X_P^{0.75}} f_X(x)\, dx = 0.75$ ("75th percentile of X" is at $X_P^{0.75}$)
Mid-range	$mid - range = a + \frac{(b-a)}{2}$ *For example:* $f_X(x)=x^2$ $(1<x\leq 5)$ $<=>$ $mid-range = 1 + \frac{(5-1)}{2} = 3$

For example: Question gives $f_X(x) = \frac{x^2}{21}$ $(1 \leq x \leq 4)$, then

(i) What is medium $X_P^{0.5}$? Solve: $\int_1^{\mathbf{X}_P^{0.5}} \frac{x^2}{21}\, dx = 0.5 => X_P^{0.5} = 3.19125$

(ii) What is mid-range? Solve: $X = \underbrace{a}_{1} + \dfrac{(\overbrace{b}^{4} - \overbrace{a}^{1})}{2} = 2.5$

(2) *Property:*

    (2.1) If g(X) is a *monotonically strictly increasing* function of X

        Then, "g(X)'s a^{th} percentile" is g of "X's a^{th} percentile"

        For example: 75th percentile of X is 2. Then, 75th percentile of $(X-1)^3 = (\underbrace{X_P^{0.75}}_{2} - 1)^3 = 1$

    (2.2) If g(X) is a *monotonically strictly decreasing* function of X

        Then, "g(X)'s $(100 - a)^{th}$ percentile" is g of "X's a^{th} percentile"

        For example: 75th percentile of X is 2. Then, (100-75)=25th percentile of $(1-X)^3 = (1 - \underbrace{X_P^{0.75}}_{2})^3 = -1$