

# SOA and CAS: Exam P, Probability<sup>1</sup>

## Chapter 9: Percentiles

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January 13, 2024

(1) *Defintion:* Medium measures the center of a distribution, besides mean and mode

Medium	$X_P^{0.5}$ : which makes $F_X(x) = 0.5 \Leftrightarrow \int_{-\infty}^{X_P^{0.5}} f_X(x) dx = 0.5$ ("50th percentile of $X$ " is at $X_P^{0.5}$ )
25 <sup>th</sup> Percentile	$X_P^{0.25}$ : which makes $F_X(x) = 0.25 \Leftrightarrow \int_{-\infty}^{X_P^{0.25}} f_X(x) dx = 0.25$ ("25th percentile of $X$ " is at $X_P^{0.25}$ )
75 <sup>th</sup> Percentile	$X_P^{0.75}$ : which makes $F_X(x) = 0.75 \Leftrightarrow \int_{-\infty}^{X_P^{0.75}} f_X(x) dx = 0.75$ ("75th percentile of $X$ " is at $X_P^{0.75}$ )
Mid-range	$mid - range = a + \frac{(b-a)}{2}$ <i>For example:</i> $f_X(x)=x^2$ ( $1 < x \leq 5$ ) $\Leftrightarrow mid-range = 1 + \frac{(5-1)}{2} = 3$

For example: Question gives  $f_X(x) = \frac{x^2}{21}$  ( $1 \leq x \leq 4$ ), then

(i) What is medium  $X_P^{0.5}$ ? Solve:  $\int_1^{X_P^{0.5}} \frac{x^2}{21} dx = 0.5 \Rightarrow X_P^{0.5} = 3.19125$

(ii) What is mid-range? Solve:  $X = \underbrace{a}_1 + \frac{\underbrace{(b-a)}_4}{2} = 2.5$

(2) *Property:*

(2.1) If  $g(X)$  is a *monotonically strictly increasing* function of  $X$

Then, " $g(X)$ 's  $a^{th}$  percentile" is  $g$  of " $X$ 's  $a^{th}$  percentile"

For example: 75<sup>th</sup> percentile of  $X$  is 2. Then, 75<sup>th</sup> percentile of  $(X - 1)^3 = \underbrace{(X_P^{0.75} - 1)}_2^3 = 1$

(2.2) If  $g(X)$  is a *monotonically strictly decreasing* function of  $X$

Then, " $g(X)$ 's  $(100 - a)^{th}$  percentile" is  $g$  of " $X$ 's  $a^{th}$  percentile"

For example: 75<sup>th</sup> percentile of  $X$  is 2. Then,  $(100-75)=25^{th}$  percentile of  $(1-X)^3 = \underbrace{(1 - X_P^{0.75})}_2^3 = -1$

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