# SOA and CAS: Exam P, Probability[1]
## Chapter 8: Variance and Other Moments

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(1) *Definition:*

Variance measures the "dispersion", which is the $2^{nd}$ central moment

$$Var(X) = E[(X-\mu)^2] = \begin{cases} Discrete: \sum_x (x-\mu)^2 * P(X=x) \\ Continuous: \int_{-\infty}^{+\infty} (x-\mu)^2 * f_X(x)\, dx \end{cases}$$

$$= \underbrace{E[(X)^2]}_{2^{nd}\ raw\ moment} - [E(X)]^2$$

Recall: "Mean" is the "expected value"

| | |
|---|---|
| *Discrete* | $E(X) = \sum_x x * P(X=x)$ |
| *Continuous* | $E(X) = \int_{-\infty}^{+\infty} x * f_X(x)\, dx \ (-\infty < X < +\infty)$<br>$E(X) = \int_{0}^{+\infty} [1 - F(x)]\, dx \ (X\ can\ only\ take\ nonnegative\ value)$ |

Moments: skewness is the $3^{rd}$ central moment. Kurtosis is the $4^{th}$ central moment

$$Skewness(X) = \frac{E[(X-\mu)^3]}{\sigma^3}$$

$$Kurtosis(X) = \frac{E[(X-\mu)^4]}{\sigma^4}$$

(2) *Property:*

(2.1) *Single Random Variable:*

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|---|
| $E(c) = c$ |
| $E[a + (a+b)X] = a + (a+b)E(X)$ |
| $Var(c) = 0$ |
| $Var(\underbrace{a}_{(variance=0)} + (a+b)X) = (a+b)^2 Var(X)$ |

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(2.2) *Multiple Random Variable:*

| |
|---|
| $E(X_1 + X_2) = E(X_1) + E(X_2)$ |
| $E[g(X_1) + g(X_2)] = E[g(X_1)] + E[g(X_2)]$ |
| $E(aX_1 + bX_2) = aE(X_1) + bE(X_2)$ |
| $If\ X_1\ and\ X_2\ are\ independent <=> \ \rho = 0 <=> \ Cov(X_1, X_2) = 0$ |
| $Var(X_1 + X_2) = Var(X_1) + Var(X_2) + \ 2 * \underbrace{Cov(X_1, X_2)}_{\rho\sigma_{x_1}\sigma_{x_2}}$<br>$= Var(X_1) + Var(X_2) + 2 * \underbrace{\rho}_{X_1,\ X_2\ independent <=> \rho=0} * \sigma_{x_1}\sigma_{x_2}$<br>$= Var(X_1) + Var(X_2)\ (if\ X_1\ and\ X_2\ are\ independent)$ |
| $Var(aX_1 + bX_2) = a^2Var(X_1) + b^2Var(X_2) + 2ab * Cov(X_1, X_2)$<br>$= a^2Var(X_1) + b^2Var(X_2) + 2ab * \rho * \sigma_{x_1}\sigma_{x_2}$<br>$= a^2Var(X_1) + b^2Var(X_2)\ (if\ X_1\ and\ X_2\ are\ independent)$ |
| $Var[\underbrace{a}_{variance=0} + (-b)X + cY] = (-b)^2Var(X) + (c)^2Var(Y)$ |

For example: Give $Z = 3X - Y - 5$, $Var(X) = 1$, $Var(Y) = 2$

What is $Var(Z)$?

Slove: $Var(Z) = Var(3X - Y - \underbrace{5}_{variance=0}) = 3^2Var(X) + (-\mathbf{1})^2Var(Y) = 11$