SOA and CAS: Exam P, Probability[1]
Chapter 7: Mean

Yi Li [2]

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(1) *Definition:* Mean is the "Expected Value"

Discrete	$E(X_{r.v.}) = \sum_i x_i * P(X = x_i)$
Continuous	$E(X_{r.v.}) = \int_{-\infty}^{+\infty} x * f_X(x)\ dx\ (-\infty < X_{r.v.} < +\infty)$ $If\ X_{r.v.}\ is\ nonnegative,\ then\ E(X_{r.v.}) = \int_0^{+\infty} [1 - F_X(x)]\ dx$

(2) *General Formula*

Discrete	$E[g(X)] = \sum_x g(x) * P(X = x)$
Continuous	$E[g(X)] = \int_{-\infty}^{+\infty} g(x) * f_X(x)\ dx$

(3) "Benefit Limit" *versus* "Deduction"

Benefit Limit	Discrete	$E[g(X)] = \sum_x g(x) * P(X = x)$ $where\ g(x) = \begin{cases} x & (X < u) \\ u & (X \geq u) \end{cases}$ $E[g(X)] = \sum_{-\infty}^{x=u} x * p(x) + \sum_{x=u}^{+\infty} u * p(x)$
	Continuous	$E[g(X)] = \int_{-\infty}^{+\infty} g(x) * f_X(x) dx$ $where\ g(x) = \begin{cases} x & (X < u) \\ u & (X \geq u) \end{cases}$ $E[g(X)] = \int_{-\infty}^{u} x * f_X(x)\ dx + \int_u^{+\infty} u * f_X(x)\ dx$
Deduction	Discrete	$E[g(X)] = \sum_x g(x) * P(X = x)$ $where\ g(x) = \begin{cases} 0 & (X < d) \\ x & (X \geq d) \end{cases}$ $E[g(X)] = \sum_{x=d}^{+\infty} x * p(x) = \sum_{-\infty}^{x=d} 0 * p(x) + \sum_{x=d}^{+\infty} x * p(x)$
	Continuous	$E[g(X)] = \int_{-\infty}^{+\infty} g(x) * f_X(x)$ $where\ g(x) = \begin{cases} 0 & (X < d) \\ x & (X \geq d) \end{cases}$ $E[g(X)] = \int_d^{+\infty} x * f_X(x)\ dx = \int_{-\infty}^{d} 0 * f_X(x)\ dx + \int_d^{+\infty} x * f_X(x)\ dx$

(4) *Property:*

(4.1) $E(c) = c$

(4.2) $E[cg(X)] = cE[g(X)]$

(4.3) $E[g_1(X) + g_2(X) ... + g_n(X)] = E[g_1(X)] + E[g_2(X)] + ... + E[g_n(X)]$

(4.4) $f_X(x) = F_X'(x) = tan(\Theta)$

[2] Email: liyifinhub@outlook.com This note was drafted when I was preparing for the exam. Please email me if you find any errors.My personal website http://www.yilifinhub.com