SOA and CAS: Exam P, Probability¹ Chapter 7: Mean

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(1) Definition: Mean is the "Expected Value"

Discrete	$E(X_{r.v.}) = \sum_{i} x_i * P(X = x_i)$
Continuous	$E(X_{r.v.}) = \int_{-\infty}^{+\infty} x * f_X(x) dx (-\infty < X_{r.v.} < +\infty)$
	If $X_{r.v.}$ is nonnegative, then $E(X_{r.v.}) = \int_0^{+\infty} [1 - F_X(x)] dx$

(2) General Formula

Discrete	$E[g(X)] = \sum_{x} g(x) * P(X = x)$
Continuous	$E[g(X)] = \int_{-\infty}^{+\infty} g(x) * f_X(x) dx$

(3) "Benefit Limit" versus "Deduction"

	Discrete	$E[g(X)] = \sum g(x) * P(X = x)$
Benefit Limit		where $g(x) = \begin{cases} x & (X < u) \\ u & (X \ge u) \end{cases}$
		$E[g(X)] = \sum_{-\infty}^{x=u} x * p(x) + \sum_{x=u}^{+\infty} u * p(x)$
		$E[g(X)] = \int_{-\infty}^{+\infty} g(x) * f_X(x) dx$
	Continuous	where $g(x) = \begin{cases} x & (X < u) \\ u & (X \ge u) \end{cases}$
		$E[g(X)] = \int_{-\infty}^{u} x * f_X(x) dx + \int_{u}^{+\infty} u * f_X(x) dx$
Deduction		$E[g(X)] = \sum g(x) * P(X = x)$
	Discrete	where $g(x) = \begin{cases} 0 & (X < d) \\ x & (X \ge d) \end{cases}$
		$E[g(X)] = \sum_{x=d}^{+\infty} x * p(x) = \sum_{-\infty}^{x=d} 0 * p(x) + \sum_{x=d}^{+\infty} x * p(x)$
		$E[g(X)] = \int_{-\infty}^{+\infty} g(x) * f_X(x)$
	Continuous	where $g(x) = \begin{cases} 0 & (X < d) \\ x & (X \ge d) \end{cases}$
		$E[g(X)] = \int_d^{+\infty} x * f_X(x) dx = \int_{-\infty}^d 0 * f_X(x) dx + \int_d^{+\infty} x * f_X(x) dx$

(4) Property:

- (4.1) E(c) = c
- (4.2) E[cg(X)] = cE[g(X)]
- $\begin{array}{l} (4.3) \ E[g_1(X) + g_2(X) ... + g_n(X)] = E[g_1(X)] + E[g_2(X)] + ... + E[g_n(X)] \\ (4.4) \ f_X(x) = F_X'(x) = tan(\Theta) \end{array}$

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