SOA and CAS: Exam P, Probability[1]
Chapter 6: Conditional Probability for Random Variable

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January 13, 2024

(1) *Property:*

$$P(X > a | X < b) = \frac{P(X > a \ \cap \ X < b)}{P(X < b)} = \frac{P(a < X < b)}{P(X < b)}$$

(1.a) if question gives $f_X(x)$, then $P(X > a | X < b)$ becomes

$$P(X > a | X < b) = \frac{P(a < X < b)}{P(X < b)} = \frac{\int_a^b f_X(x) \ dx}{\int_{-\infty}^b f_X(x) \ dx}$$

(1.b) if question gives F(x), then $P(X > a | X < b)$ becomes

$$P(X > a | X < b) = \frac{P(a < X < b)}{P(X < b)} = \frac{F(b) \ - \ F(a)}{F(b)}$$

where $F(x) = \int_{-\infty}^x f_X(x) dx$

(1.c) After applying "Bayes' Theorem":

$$P(X > a | X < b) = \frac{P(a < X < b)}{P(X < b)} = \frac{P(X < b | X > a) * P(X > a)}{P(X < b | X > a) * P(X > a) \ + \ P(X < b | X \le a) * P(X \le a)}$$

(2) *Deduction Case (Example):*

For example: Question gives (i) deduction of 100 ; (ii) density function $f_X(x) = \frac{1000-x}{500,000}$ $(0 < x < 1000)$

What is P (payment of 500 or less)?

Slove: P (payment of 500 or less)
$\iff$ P (a positive payment is made): $P(X \le 600 | X > 100)$
$\iff$

$$P(X \le 600 | X > 100) = \frac{\int_{100}^{600} f_X(x) \ dx}{\int_{100}^{1000} f_X(x) \ dx}$$

(3) *Conditional CDF (Example):*

For example: Question gives (i) $F_{x|x>0} = \underbrace{1 - (\frac{1500}{1500 + x})(x > 0)}_{F_{x<x|x>0}}$; (ii) $P(X > 0) = 0.2$

What is $P(X > 500)$?

Solve:

$$P(X > 500) = P(X > 500 | X > 0) * P(X > 0)$$
$$= (\frac{1500}{1500 + \underbrace{x}_{500}}) * 0.2$$