

SOA and CAS: Exam P, Probability¹

Chapter 6: Conditional Probability for Random Variable

Yi Li²
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(1) *Property:*

$$P(X > a | X < b) = \frac{P(X > a \cap X < b)}{P(X < b)} = \frac{P(a < X < b)}{P(X < b)}$$

(1.a) if question gives $f_X(x)$, then $P(X > a | X < b)$ becomes

$$P(X > a | X < b) = \frac{P(a < X < b)}{P(X < b)} = \frac{\int_a^b f_X(x) dx}{\int_{-\infty}^b f_X(x) dx}$$

(1.b) if question gives $F(x)$, then $P(X > a | X < b)$ becomes

$$P(X > a | X < b) = \frac{P(a < X < b)}{P(X < b)} = \frac{F(b) - F(a)}{F(b)}$$

$$\text{where } F(x) = \int_{-\infty}^x f_X(x) dx$$

(1.c) After applying “Bayes’ Theorem”:

$$P(X > a | X < b) = \frac{P(a < X < b)}{P(X < b)} = \frac{P(X < b | X > a) * P(X > a)}{P(X < b | X > a) * P(X > a) + P(X < b | X \leq a) * P(X \leq a)}$$

(2) *Deduction Case (Example):*

For example: Question gives (i) deduction of 100 ; (ii) density function $f_X(x) = \frac{1000-x}{500,000}$ ($0 < x < 1000$)

What is P (payment of 500 or less)?

Solve: P (payment of 500 or less)

$$\iff P(\text{a positive payment is made}) : P(X \leq 600 | X > 100)$$

\iff

$$P(X \leq 600 | X > 100) = \frac{\int_{100}^{600} f_X(x) dx}{\int_{100}^{1000} f_X(x) dx}$$

(3) *Conditional CDF (Example):*

For example: Question gives (i) $F_{x|x>0} = 1 - \underbrace{\left(\frac{1500}{1500+x}\right)}_{F_{x<x|>0}} (x > 0)$; (ii) $P(X > 0) = 0.2$

What is $P(X > 500)$?

Solve:

$$\begin{aligned} P(X > 500) &= P(X > 500 | X > 0) * P(X > 0) \\ &= \left(\frac{1500}{1500 + \underbrace{x}_{500}}\right) * 0.2 \end{aligned}$$

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²Email: liyifinhub@outlook.com. This note was drafted when I was preparing for the exam. Please email me if you find any errors. My personal website <http://www.yilifinhub.com>