SOA and CAS: Exam P, Probability[1]
Chapter 3: Conditional Probability (Sets)

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(1) *Defintion:* Porbability of "one event" depends on "another event"

For example: if a person smokes, then the dying probability of this person is higer than a person who does not smoke. That is the porbability of "dying" depends on "smoking".

This gives you the conditional probability $P(B|A) = \frac{P(B \cap A)}{P(A)}$

If B set is known to be *mutually exclusive and exhausitve*

Then, Bayes' Theorem gives

$$
\begin{aligned}
P(B|A) &= \frac{P(B \cap A)}{P(A)} \\
&= \frac{P(B \cap A)}{P(A \cap B_1) + P(A \cap B_2) + ... + P(A \cap B_n)} \\
&= \frac{P(A|B) * P(B)}{P(A|B_1) * P(B_1) + P(A|B_2) * P(B_2) + ... + P(A|B_n) * P(B_n)}
\end{aligned}
$$

Now think, what is $P(B|A^C)$?

$$
\begin{aligned}
P(B|A^C) &= \frac{P(B \cap A^C)}{P(A^C)} \\
&= \frac{P(A) - P(B \cap A)}{1 - P(A)}
\end{aligned}
$$

(2) *Mutually Exclusive* $\Longleftrightarrow$

$$
\begin{cases}
P(A \cup B) = P(A) + P(B), \text{ (Normally you have } P(A \cup B) = P(A) + P(B) - P(A \cap B), \text{ here } P(A \cap B) = 0) \\
P(A \cup B|H) = P(A|H) + P(B|H) \\
P(A \cap B) = 0
\end{cases}
$$

(3) *Mutually Independent* $\Longleftrightarrow$

$$
\begin{cases}
P(A \cap B) = P(A) * P(B) \\
P(A^C \cap B^C) = P(A^C) * P(B^C), \text{ where } P(A^C) = 1 - P(A) \text{ and } P(B^C) = 1 - P(B) \\
P(A \cap B|H) = P(A|H) * P(B|H)
\end{cases}
$$

(4) *Identify "Conditional Probability":*

(4.1) Question asks "Probability of A, given B" $\Longleftrightarrow$ Question asks you to calculate P(A | B)

(4.2) Question gives: "*of those* who insure A, 15% insures B", or "*of the* policyholders with A, 15% have B". Both indicate the conditional probability P(B | A)=15%

(4.3) A conditional probability means that the probability of A depends on B. For example: the dying probablity of a smoker is definitely different from that of a non-smoker. Therefore, smoking is a condition that affects the probability of death. Note that sometimes the question might be: a random selected person has A, what is the probability of B. This also refers to a conditional probability.

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