SOA and CAS: Exam P, Probability[1]
Chapter 23: Central Limit Theorem

Yi Li [2]

January 13, 2024

(1) *Definition*: If (i) $X_i \sim$ some distribution with mean=μ, variance=σ^2
(ii) X_i and X_j are independent

Then, $sum(X_i) = X_1 + X_2 ... + X_n \sim N(n\mu, n\sigma^2)$

(2) *Probability*:

Example (2.1): 10 X_i and $X_i \sim Uniform(0, 12)$
Then, $sum(X_i) = X_1 + X_2 + ... + X_{10} \sim N(10 * \frac{0+12}{2}, 10 * \frac{(12-0)^2}{12})$

Example (2.2): 5 X_i and $X_i \sim Exponential(\theta = mean = 2)$
Then, $sum(X_i) = X_1 + X_2 ... + X_5 \sim N(5 * \theta, 5 * \theta^2)$

Example (2.3): Let X_i *stands for* "the number of car" in "one day" $\sim Poisson(\lambda = 5)$
Then, Y "the number of car" in "100 day" follows

$$Y = sum(X_i) = X_1 + X_2 ... + X_{100} \sim N(100 * 5, 100 * 5)$$

Thus, $Probability(Y \geq 1050) = 1 - P(Y \leq 1050)$ where

$$P(Y \leq 1050) = P(\underbrace{\frac{Y - 100 * 5}{\sqrt{100 * 5}}}_{Z} \leq \frac{1050-100*5}{\sqrt{100*5}}) = \Phi(\frac{1050-100*5}{\sqrt{100*5}}) = 0.92$$

Example (2.4): (i) A line, 50 person in front of you
(ii) each person needs X minutes: $X_i \sim Exponential(\theta = 1)$ $(x > 0)$
Question: What is P(waiting time more than 60 minutes)?

Solve: P(waiting time more than 60 minutes) $\iff P(X_1 + X_2 ... + X_{50} > 60)$
If the total waiting time is Y, then we have
$Y = sum(X_i) = X_1 + X_2 ... + X_{50} \sim N(50 * \theta,\ 50 * \theta^2)$, which implies

$$P(Y > 60) = 1 - P(Y \leq 60)$$
$$where\ P(Y \leq 60) = P(\underbrace{\frac{Y - 50 * 1}{\sqrt{50 * 1^2}}}_{Z} \leq \frac{60-50*1}{\sqrt{50*1^2}}) = \Phi(\frac{60-50*1}{\sqrt{50*1^2}}) = 0.92$$

(3) *Percentile:* $X_p = \mu + \sigma Z_p$

For example: (i) $X_i \sim Uniform(10^5, 10^6)$
(ii) Y is the sum of $X_i : Y = X_1 + X_2 ... + X_{50},$ *that is*

$$Y \sim N(50 * \frac{10^5 + 10^6}{2},\ 50 * \frac{(10^6 - 10^5)^2}{12})$$

Question: What is 75^{th} of Y?

Solve:

$$75^{th} \text{ of Y} = \mu + \sigma * \underbrace{Z_p^{75^{th}}}_{0.68}$$
$$where\ \mu = 50 * \frac{10^5 + 10^6}{12}$$
$$\sigma = \sqrt{50 * \frac{(10^6 - 10^5)^2}{12}}$$

---

[2] Email: liyifinhub@outlook.com. This note was drafted when I was preparing for the exam. Please email me if you find any errors. My personal website http://www.yilifinhub.com