SOA and CAS: Exam P, Probability[1]
Chapter 12 and 27 Marginal Distribution

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Chapter 12: Marginal Distribution
Chapter 27: Marginal Distribution for Continuous Random Variables

(1) *Give "joint" distribution function, calculate the "marginal" distribution function*

(1.a) *Discrete:*

Joint Distribution Function	Marginal Distribution Function
$f(x,y)$	$f_X(x) = \underbrace{\sum_y f(x,y)}_{sum\ up\ all\ Ys}$
	$f_Y(y) = \underbrace{\sum_x f(x,y)}_{sum\ up\ all\ Xs}$

For example: give the following joint distribution function

		Y		
		1	2	3
X	1	$\underbrace{0.4}_{P(X=1,Y=1)}$	$\underbrace{0.12}_{P(X=1,Y=2)}$	$\underbrace{0.08}_{P(X=1,Y=3)}$
	2	$\underbrace{0.3}_{P(X=2,Y=1)}$	$\underbrace{0.06}_{P(X=2,Y=2)}$	$\underbrace{0.04}_{P(X=2,Y=3)}$

Then, $P_X(1) = P(X=1,Y=1) + P(X=1,Y=2) + P(X=1,Y=3) = 0.4 + 0.12 + 0.08 = 0.6$

$P_Y(1) = P(X=1,Y=1) + P(X=2,Y=1) = 0.4 + 0.3 = 0.7$

$P(1,1) - P_X(1) = 0.4 - 0.6$

(1.b) *Continue:*

Joint Distribution Function	Marginal Distribution Function
$f(x,y)$	$f_X(x) = \underbrace{\int_{-\infty}^{+\infty} f(x,y)\ dy}_{integrated\ over\ all\ Ys}$
	$f_Y(y) = \underbrace{\int_{-\infty}^{+\infty} f(x,y)\ dx}_{integrated\ over\ all\ Xs}$

For example: give $f(x,y) = 2 \ \ (0 \le x \le y \le 1)$
Then, we have $f_X(x) = \int_x^1 f(x,y)\ dy \ \ (0 \le x \le 1)$ (*draw* a line parallel to the Y-axis, $0 \le \boxed{x \le y \le 1}$)
$f_Y(y) = \int_0^y f(x,y)\ dx \ (0 \le y \le 1)$ (*draw* a line parallel to the X-axis, $\boxed{0 \le x \le y} \le 1$)
$E(XY^2) \overset{indepedent}{=} E(X) * E(Y^2)$, *because* $f(x,y) = 2$ (*constant*)

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