SOA and CAS: Exam P, Probability[1]
Chapter 11 and 15: Joint Distribution

Yi Li [2]

January 13, 2024

Chapter 11 Joint Distribution
Chapter 25 Joint Distribution for Continuous Random Variables

(1) *Multiple random variables: If X, Y, Z are mutually independent, then we have*

(i) $\rho_{X,Y} = 0, \quad \rho_{X,Z} = 0, \quad \rho_{Y,Z} = 0$
(ii) $Cov(X,Y) = 0, \quad Cov(X,Z) = 0, \quad Cov(Y,Z) = 0$
(iii) $Var(X+Y+Z) = Var(X) + Var(Y) + Var(Z)$
(iv) $E(XY) = E(X) * E(Y)$
(v) $p(x,\, y,\, z) = p(x) * p(y) * p(z)$
(vi) $f(x,\, y,\, z) = f(x) * f(y) * f(z)$
(vii) if $F(x,y)$ is the product of "function x" and "function y"
then we know, (a) X and Y are independent
(b) $X's$ df is the "function x", $Y's$ df is the "function y"
(viii) if $f(x,y) = a$ (constant), for example: $f(x,y) = 3$
then, X and Y are independent $\iff$ $f(x,y)$=$f(x) * f(y)$

(2) *Definition:*

(i) $F(x,y) = (X \leq x \; and \; Y \leq y)$
(ii) $P(a < x \leq b,\, c < y \leq d) = F(b,d) - F(a,d) - F(b,c) + F(a,c)$

(3) *Give "joint pdf $f(x,y)$", calculate cdf:*

Type I: $f(x,y) = 1.2(x^2 + y)$ $(0 \leq x \leq 1,\, 0 \leq y \leq 1)$

Then, $Pr(X \leq 0.5, Y \leq 0.4) = \int_0^{0.5} \int_0^{0.4} 1.2(x^2 + y)\; dydx$

Type II: $Pr(X + Y \leq 0.8)$

Then, $Pr(X + Y \leq 0.8) = \int_0^{0.8} \int_0^{0.8-x} 1.2(x^2 + y)\; dydx$

(that is: x full range, y into x by drawing a line parallel to y)

Type III: $Pr(X + Y > 0.8) = 1 - Pr(X + Y \leq 0.8)$, where $Pr(X + Y \leq 0.8)$ is shown in Type II

(4) *Independent:*

If $F(x,y)$ is the product of "function x" and "function y"

Then (4.1) X and Y are independent

(4.2) $X's$ df is the "function x", $Y's$ df is the "function y"

For example: $F(x,y) = [1 - (0.5)^{x+1}][1 - (0.3)^{y+1}]$

Then (1) X and Y are independent

(2) $F_X(x) = [1 - (0.5)^{x+1}]$

$F_Y(y) = [1 - (0.3)^{y+1}]$

[1]The purpose of the use is non-commercial research and/or private study. Please do not copy or distribute without permission of the author.

[2]Email: liyifinhub@outlook.com. This note was drafted when I was preparing for the exam. Please email me if you find any errors. My personal website http://www.yilifinhub.com