SOA and CAS: Exam P, Probability[1]
Chapter 10: Mode

Yi Li [2]

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(1) *Defintion:* Mode measures the "center of a distribution", besides mean and medium[3]

(2) *Discrete and Continuous Cases:*

(2.2) *Discrete:*

    (2.2.a) Mode: the observation that appears most frequently
    For example: $\{1, 2, 2.5, 2.5, \mathbf{3, 3, 3}, 4, 2\} \Longrightarrow$ "mode" is 3

    (2.2.b) Mode: the event has the highest probability
    For example: $P(N = 0) = 0.1$
                  $P(N = 1) = 0.2$
                  $P(N = 2) = 0.7 \Longrightarrow$ "mode" is 2

(2.3) *Continue:* two steps to calculate the mode
    Step 1: $\frac{df_X(x)}{dx} = 0 => x = \{x_1, x_2\}$
    Step 2: $\frac{d^2 f_X(x)}{dx^2} < 0 => x = x_1$ (delete x_2)

    For example: Question gives $f_X(x) = x^2(1 - x)$
                  What is the mode?

    Step 1: $\frac{df_X(x)}{dx} = \frac{d[x^2(1-x)]}{dx} = x(2 - 3x) = 0 => x_1 = 0 \ or \ x_2 = \frac{2}{3}$
    Step 2: $\frac{d^2 f_X(x)}{dx^2} = \frac{d^2[x^2(1-x)]}{dx^2} = 2 - 6x < 0 => mode = x_2 = \frac{2}{3}$

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[3]Mean is the expected value and medium is the 50th percentile.