

SOA and CAS: Exam P, Probability¹

Chapter 10: Mode

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January 13, 2024

(1) *Defintion*: Mode measures the “center of a distribution”, besides mean and medium³

(2) *Discrete and Continuous Cases*:

(2.2) *Discrete*:

(2.2.a) Mode: the observation that appears most frequently
For example: $\{1, 2, 2.5, 2.5, \mathbf{3}, \mathbf{3}, \mathbf{3}, 4, 2\} \implies$ “mode” is 3

(2.2.b) Mode: the event has the highest probability
For example: $P(N = 0) = 0.1$
 $P(N = 1) = 0.2$
 $P(N = 2) = 0.7 \implies$ “mode” is 2

(2.3) *Continue*: two steps to calculate the mode

Step 1: $\frac{df_X(x)}{dx} = 0 \implies x = \{x_1, x_2\}$

Step 2: $\frac{d^2f_X(x)}{dx^2} < 0 \implies x = x_1$ (delete x_2)

For example: Question gives $f_X(x) = x^2(1-x)$
What is the mode?

Step 1: $\frac{df_X(x)}{dx} = \frac{d[x^2(1-x)]}{dx} = x(2-3x) = 0 \implies x_1 = 0$ or $x_2 = \frac{2}{3}$

Step 2: $\frac{d^2f_X(x)}{dx^2} = \frac{d^2[x^2(1-x)]}{dx^2} = 2-6x < 0 \implies mode = x_2 = \frac{2}{3}$

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³Mean is the expected value and medium is the 50th percentile.