SOA and CAS: Exam FM[1]
Written Solutions: 86-117

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This document only provides written solutions to official example problems 86-117. For official sample questions, check out the official websites of Society of Actuaries and the Casualty Actuarial Society.

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[2]Email: liyifinhub@outlook.com The written solutions were drafted when I was preparing for the exam. Please email me if you find any errors. My personal website: `https://yilifinhub.com/`

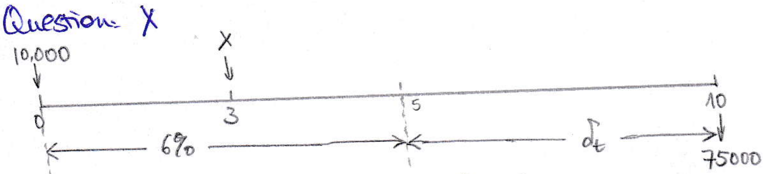
Exam FM Question 86

Give: ① A bank agrees to: lend 10,000 now, and, X 3-year later, in exchange for a single payment of 75,000 at the end of 10-year

② For the first 5 years: annual rate 6%

For $t \geq 5$, force of interest: $\delta_t = \frac{1}{1+t}$

Question: X



Solve: $\underset{\uparrow AV}{75000} = 10{,}000\,(1+6\%)^5 \cdot \underbrace{e^{\int_5^{10} \delta_t \, dt}}_{e^{\int_5^{10} d\ln(1+t)}} + \underline{X} \cdot (1+6\%)^2 \cdot \underbrace{e^{\int_5^{10} \delta_t \, dt}}_{\frac{11}{6}}$

$= e^{\ln 11 - \ln 6}$

$= \frac{11}{6}$

$\Rightarrow X = 24498.78$ Ⓒ

Exam FM Question 87:

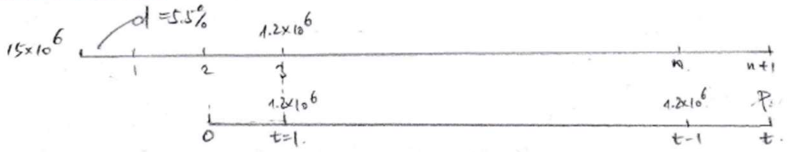
Give: Loan of 15,000,000 at annual discount rate of 5.5%.

(i) Loan repaid to be repaid with n annual payments of 1,200,000 plus a drop payment one year after the nth payment

(ii) The fitsy payment is due 3 years after the loan is taken out

Question: what is the amount of the drop payment?

Exam FM Question 87



Solve: First: get t: $15\times10^6 = v^2_{5.5\%}\left(1.2\times10^6\, a_{\overline{t}|i}\right)$ where $(1-d)^{-1} = (1+i) \Rightarrow i \cong 5.82\%$

$d = 5.5\%$, $i = 5.82\%$

$\Rightarrow a_{\overline{t}|5.82\%} \cong 14 \Rightarrow t \cong 29.79$ since the final payment is a drop payment

$\Rightarrow t = 30$ { 29 level payments of 1.2×10^6; 30^{th} Final drop payment

Second: $15\times10^6 = \underbrace{v^2_{5.82\%}}_{0.893}\left[1.2\times10^6 \cdot a_{\overline{29}|5.82\%} + P v^{30}_{5.82\%}\right] = 14842731.6 + 0.1636133P$

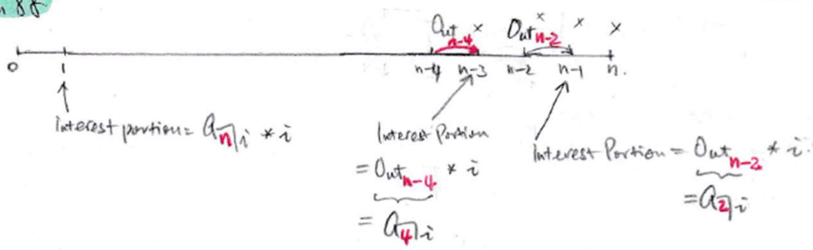
$\Rightarrow P \cong 961220.14$ (D)

Exam FM Question 88:

Give: n-year loan with equal annual payments at the end of each year. The interest portion of the payment at (n-1) is equal to 0.5250 of the interest portion of the payment at (n-3) and is equal to 0.1427 of the interest portion of the first payment

Question: what is n?

Exam FM Question 88



Solve: $X\cdot\frac{1-v^2}{i}\times i = 0.5250\cdot X\cdot\frac{1-v^4}{i}\times i = 0.1427\cdot X\cdot\frac{1-v^n}{i}\times i$

$\boxed{1-v^2 = 0.5250(1-v^4)} = 0.1427(1-v^n)$

$\Rightarrow$ let $v^4 = y^2 \Rightarrow 0.5250y^2 - y + 0.475 = 0 \Rightarrow y = \frac{1\pm\sqrt{1-4\times0.5250\times0.475}}{2\times0.5250} = 0.904762 \Rightarrow i = 5.1315\%$

$y = v^2$

Then: Use $1-v^2 = 0.1427(1-v^n)$

$i = 5.1315\%$

$\Rightarrow 1-v^n = 0.6674 \Rightarrow v^n = 0.3326 \Rightarrow n = \frac{\ln(0.3326)}{\ln v} \cong 22$ (C)

-1.1008714

$v = \frac{1}{1.051315}$

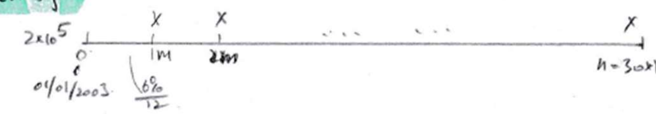
-0.05

Exam FM Question 89:

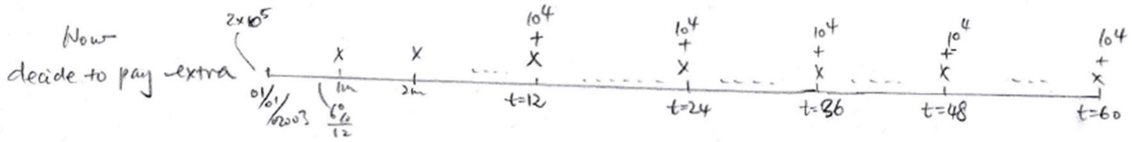
Mike took out a 30-year mortgage loan on January 1, 2003 in the amount of 200,000 at an annual interest rate 6% compounded monthly. The loan repaid by end-of-month level payments with the first payments on Jan 31, 2003. Mike repaid an extra 10,000 in addition to the regular monthly payment on each Dec 31 in the years 2003 through 2007

Question: what is the date on which Mike will make his last drop payment?

Exam FM Question 89



$\Rightarrow X\, a_{\overline{30\times12}|\frac{6\%}{12}} = 2\times10^5 \Rightarrow X = 1199.1$



$$2\times10^5 = \Big(\underbrace{10^4 v^{12}_{\frac{6\%}{12}}}_{9419.05} + \underbrace{10^4 v^{24}_{\frac{6\%}{12}}}_{8871.86} + \underbrace{10^4\cdot v^{36}_{\frac{6\%}{12}}}_{8356.45} + \underbrace{10^4\cdot v^{48}_{\frac{6\%}{12}}}_{7871} + \underbrace{10^4\cdot v^{60}_{\frac{6\%}{12}}}_{7413.722}\Big) + 1199.1\, a_{\overline{n}|\frac{6\%}{12}}$$

$$\Rightarrow 1199.1\, a_{\overline{n}|\frac{6\%}{12}} = 158067.92 \Rightarrow n = 215.77 \Big\} \Rightarrow n = 216 \quad \because \frac{n=216}{12} = 18 \Rightarrow \begin{cases} \text{integer} \Rightarrow \text{Dec} \\ 2003+18 = 2021 \end{cases}$$

Final drop payment

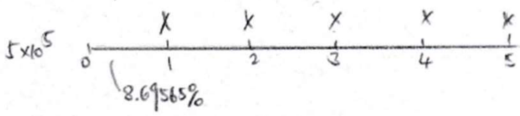
(D)

Exam FM Question 90:

5-year loan of 500,000 with an annual discount rate of 8% repaid by level end-of-year payments. If the first 4 payments had been aroundedup to the next multiple of 1,000, the final payment would be X

Question: what is X?

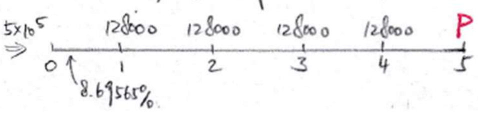
Exam FM Question 90



$d = 8\% \Leftrightarrow (1-d)^{-1} = (1+i) \Rightarrow i \approx 8.69565\%$

Solve: $5\times10^5 = X\cdot a_{\overline{5}|8.69565\%} \Rightarrow X \approx 127532.71$ round up 1000 $\Rightarrow X' = 128000$

Since $X\uparrow$ goes up. so $\Rightarrow$ the first 4 payments are 128000, the last 5^{th} payment is a drop one



$\Rightarrow 5\times10^5 = 128000\, a_{\overline{4}|8.69565\%} + P\cdot v^5_{8.69565\%} \Rightarrow P \approx 125220.79$

(D)

Exam FM Question 91:

A plan: invest X at the beginning of each month in a zero-coupon bond in order to accumulate 100,000 at the end of 6 months. The price of each bond as a percentage of redemption value is given as

Maturity(months)	1	2	3	4	5	6
Price	0.99	0.98	0.97	0.96	0.95	0.94

Question: what is X given that the bond prices will not change during the 6-month period?

Exam FM Question 91

Maturity (months)	1	2	3	4	5	6
Price	0.99	0.98	0.97	0.96	0.95	0.94

Solve: invest X in a 6-month bond now, can receive Y in 6-months. $\frac{X}{0.94}=\frac{Y}{1} \Rightarrow Y=\frac{X}{0.94}$

invest 0.94 now, get 1 in 6 month

1 month after, invest X in a 5-month bond, can receive $\frac{X}{0.95}$ in 5 months

2 months after, invest X in a 4-month bond, can receive $\frac{X}{0.96}$ in 4 months

3 months after, invest X in a 3-month bond, can receive $\frac{X}{0.97}$ in 3 months

4 month after, invest X in a 2-month bond, can receive $\frac{X}{0.98}$ in 2 months.

5 month after, invest X in a 1-month bond, can receive $\frac{X}{0.99}$ in 1 month

$$\Rightarrow \frac{X}{0.94}+\frac{X}{0.95}+\frac{X}{0.96}+\frac{X}{0.97}+\frac{X}{0.98}+\frac{X}{0.99}=10^5 \Rightarrow 6.22X=10^5 \Rightarrow X\cong 16078.30 \text{ (B)}$$

Exam FM Question 92:

Loan of X repaid with annual level payments at the end of year for 10 years. Moreover:

(i) The interest paid in the first year is 3600

(ii) The principal repaid in the 6th year is 4871

Question: what is X?

Exam FM Question 92.

$$\begin{array}{cccccccc} X & P & P & P & P & P & \cdots & P \\ 0 & 1 & 2 & 3 & 4 & 5 & \cdots & 10 \end{array}$$

i

Solve. i) Interest paid in 1^{st} year $\Rightarrow P[a_{\overline{10}|i} * i]=3600 \Rightarrow P(1-v^{10})=3600$ (A)

ii) Principal repaid in the 6^{th} year $\Rightarrow P[v^5]=4871$ (B)

Recall		level payment	interest	Pricipal repaid
	$t=1$	1	$1-v^n$	v^n.
$\Rightarrow$	$t=6$	1	$1-v^5$	v^5

$$\Rightarrow \frac{(A)}{(B)}=\frac{1-v^{10}}{v^5}=\frac{3600}{4871} \Rightarrow v^5=0.69656 \Rightarrow i\cong 7.5\%$$

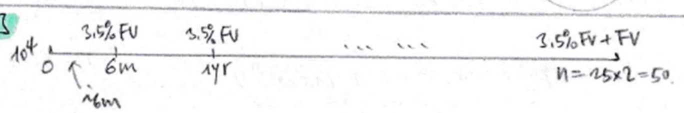
$$\left.\begin{array}{l} \\ \text{(A)}\ P[a_{\overline{10}|i} * i]=3600 \end{array}\right\} \Rightarrow \underbrace{Pa_{\overline{10}|i}}_{X}=48000 \text{ (D)}$$

Exam FM Question 93:

25-year bond with semi-annual coupons, redeemable at par, price of 10,000. Annual effective rate is 7.05% with annual coupon rate 7%

Question: what is the redemption value of the bond?

Exam FM Question 93



$(1+i_{6m})^2 = 1.0705 \Rightarrow i_{6m} = 3.465\%$

Solve: $\underbrace{(3.5\% \, FV) \cdot \underbrace{a_{\overline{50}|3.465\%}}_{\frac{1-v^{50}_{3.465\%}}{i_{3.465\%}}}}_{0.82615 \, FV} + \underbrace{FV \cdot v^{50}_{3.465\%}}_{0.18211 \, FV} = 10^4$

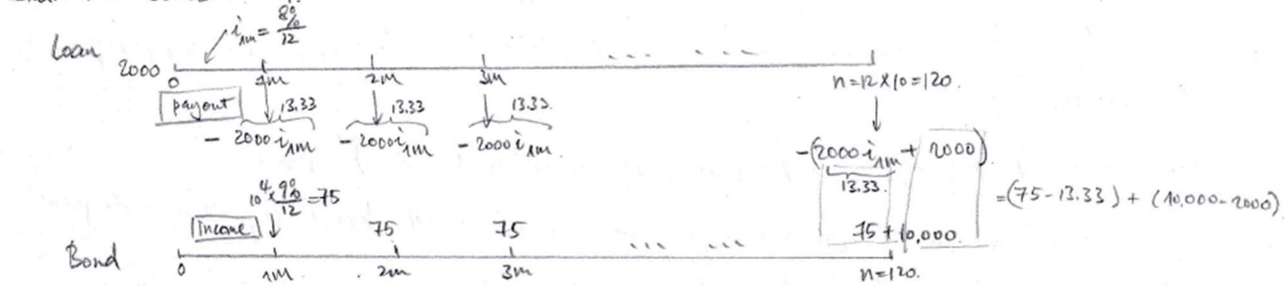
$\Rightarrow 1.0082615 \, FV = 10^4 \Rightarrow FV \cong 9918.06$ (A)

Exam FM Question 94:

In order to be able to use 8,000 to purchase a 10,000 bond. Jeff takes out a 10-year loan of 2,000 from a bank and will make interest-only payments at the end of each month at a rate of 8% convertible monthly. He immediately pays 10,000 for a 10-year bond with a par value of 10,000 and 9% coupons paid monthly

Question: what is the annual rate that Jeff will realize on his 8,000 over the 10-year period?

Exam FM Question 94



Solve: $\underbrace{8000}_{-PV} = \underbrace{(75 - 13.33)}_{PMT} \, \underbrace{a_{\overline{120}|i_{1m}}}_{\frac{1-v^{120}_{1m}}{i_{1m}}} + \underbrace{8000}_{FV} \, v^{120}_{1m}$

$\Rightarrow i_{1m} = 0.770875\% \Rightarrow \boxed{(1+i_m)^{12} = 1 + i_{annual}} \Rightarrow i_{annual} \doteq 9.653\%$ (B)

Exam FM Question 95:

A bank issues 3 annual coupon bonds redeemable at par. All 3 bonds have the same term, price, and annual yield rate. The first bond has face value 1,000 and annual coupon rate 2.58%. The second bond has face value 1,100 and annual coupon rate 4.40%. The third bond has face value 1,320 and annual coupon rate r.
Question: what is r?

Exam FM Question 95

First Bond: P

0	1	2	3	… …	n
P	52.8	52.8	52.8	… …	52.8 + 1000

Second Bond: P

0	1	2	3	… …	n
P	48.4	48.4	48.4	… …	48.4 + 1100

Third Bond: P

0	1	2	3	… …	n
P	1320r	1320r	1320r	… …	1320r + 1320

Solve:

(eq1) $52.8\, a_{\overline{n}|i} + 1000 v^n = P$

(eq2) $48.4\, a_{\overline{n}|i} + 1100 v^n = P$

(eq3) $(1320r)\, a_{\overline{n}|i} + 1320 v^n = P$

$\Rightarrow$ eq(1) − eq(2) $\Rightarrow \boxed{a_{\overline{n}|i} = \frac{100}{4.4} v^n}$ plug into eq(1) $\Rightarrow \boxed{v^n = \frac{P}{2200}}$

plug into eq(3)

plug into eq(3)

$$\Rightarrow (1320r) \times \frac{100}{4.4} \times \frac{P}{2200} + 1320 \frac{P}{2200} = P \Rightarrow r = \frac{0.4}{13.63636} \cong 2.9333\% \text{ (B)}$$

Exam FM Question 96:

A bond that is redeemable for 250 in 6 years from now. The investor has just received a coupon of c and each subsequent semiannual coupon will be 2% larger than the preceding coupon. The PV of this bond immediately after the payment of the coupon is 582.53 assuming an annual yield rate of 4%
Question: what is the c?

Exam FM Question 96

0 ($n=0$)	6m ($n=1$)	1yr ($n=2$)	… …	$n = 6\times2 = 12$
582.53	$1.02C$	1.02^2C	… …	$1.02^{12}C + 250$

i_{6m}

$(1+i_{6m})^2 = 1.04 \Rightarrow i_{6m} = 0.0198$

Solve:

$$582.53 = \underbrace{C\times1.02v_{6m} + C\times1.02^2v_{6m}^2 + \dots + C\times1.02^{12}v_{6m}^{12}}_{C\times1.02v_{6m}\underbrace{(1+1.02v+\dots+(1.02v)^{11})}_{\frac{1-(1.02v_{6m})^{12}}{1-1.02v_{6m}}}} + \underbrace{250v_{6m}^{12}}_{197.5877}$$

$$= 12.01531C + 197.5877$$

$$\Rightarrow C \cong 32.0376 \text{ (C)}$$

Exam FM Question 97:

An n-year bond with annual coupons:

(i) Redemption value at maturity 1890

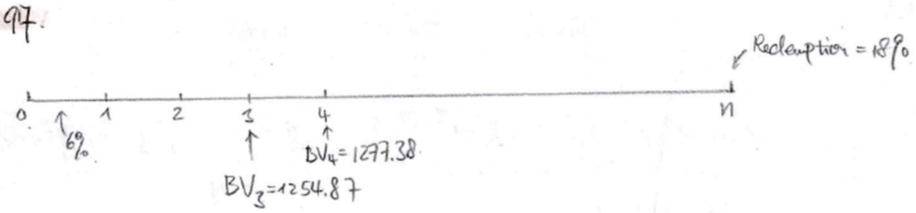
(ii) The annual effective rate is 6%

(iii) The book value immediately after the 3rd coupon is 1254.87

(iv) The book value immediately after the 4th coupon is 1277.38

Question: what is the n?

Exam FM Question 97.



Solve: Use the relationship formula: $\underbrace{BV_3}_{1254.87} \times (1+\overset{6\%}{i}) = \underbrace{BV_4}_{1277.38} + Coupon_{t=4} \Rightarrow Coupon_{t=4} = 52.7822$

Then: $BV_3 = 1254.87 = 52.7822\, a_{\overline{n-3}|6\%} + 1890\, v^{n-3}$

total n payments, less 3 payments

Calculator: $\underset{FV}{-1254.87}$, $\underset{PMT}{52.7822}$, $\underset{I/Y}{6}$, $\underset{FV}{1890}$ $\Rightarrow n-3=17 \Rightarrow n=20$ (E)

Exam FM Question 98:

An n-year bond with semi-annual coupons:

(v) Par value and redemption value are 2,500

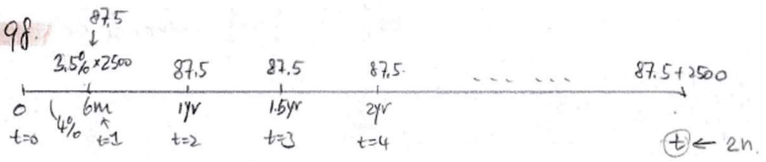
(vi) The annual coupon rate is 7% payable semi-annually

(vii) The annual nominal yield to maturity is 8% convertible semi-annually

(viii) The book value immediately after the 4th coupon is 8.44 greater than the book value immediately after the 3rd coupon

Question: what is the n?

Exam FM Question 98.



Solve: Use the relationship formula: $BV_3 \times (1+4\%) = BV_4 + Coupon_{t=4}$

From question, we know: $BV_3 + 8.44 = BV_4$) plug in $\Big\} \Rightarrow BV_3 = 2398.5$

We also know: $BV_3 = 2398.5 = 87.5\, a_{\overline{t-3}|4\%} + 2500\, v^{t-3}$

less 3 past payments

Calculator: $\underset{PV}{-2398.5}$, $\underset{PMT}{87.5}$, $\underset{I/Y}{4}$, $\underset{FV}{2500}$ $\Rightarrow t-3=10 \Rightarrow \boxed{t=13} \Rightarrow n=\frac{13}{2}$ year $= 6.5$yr (A)

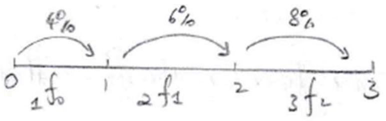
Exam FM Question 99:

The one-year forward rates, deferred t years, are estimated to be

Year (t)	0	1	2	3	4
Forward Rate	4%	6%	8%	10%	12%

Question: what is the spot rate for a zero-coupon bond maturing 3 years from now?

Exam FM Question 99



Solve $(1+r)^3 = 1.04 \times 1.06 \times 1.08 \Rightarrow r \cong 0.05987$ (C)

Exam FM Question 100:

(i) Annuity A pays 1 at the beginning of each year for 3 years

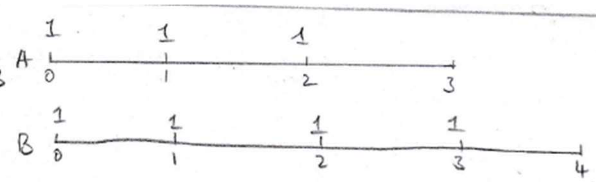
(ii) Annuity B pays 1 at the beginning of each year for 4 years

(iii) The Macaulay duration of Annuity A at the time of purchase is 0.93. Both annuities offer the same yield rate

Question: what is Macaulay duration of Annuity B at the time of purchase?

Exam FM Question 100

$D_A^{mac} = 0.93$



Solve. ∵ (A) $0.93 = \dfrac{v + v^2 \times 2}{1 + v + v^2} \Rightarrow 1.07v^2 + 0.07v - 0.93 = 0 \Rightarrow v = \dfrac{1.9263}{2 \times 1.07} \cong 0.9$

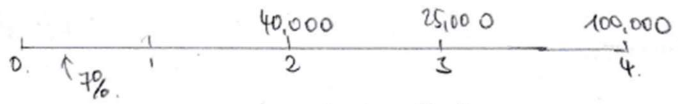
$\therefore D_B^{mac} = \dfrac{0.9 + 0.9^2 \times 2 + 0.9^3 \times 3}{1 + 0.9 + 0.9^2 + 0.9^3} \cong 1.3682$ (B)

Exam FM Question 101:

Cash flows are 40,000 at time 2 (in years), 25,000 at time 3, and 100,000 at time 4. The annual effective rate is 7%

Question: what is the Macaulay duration?

Exam FM Question 101



Solve. $D^{mac} = \dfrac{40000v^2 \times 2 + 25000v^3 \times 3 + 10^5 v^4 \times 4}{40000v^2 + 25000v^3 + 10^5 v^4}$ where $v = \dfrac{1}{1.07} \cong 0.93458$

$= \dfrac{436252.15}{131633.45} = 0.93458$

Exam FM Question 102:

Rhonda purchases a perpetuity providing a payment of 1 at the beginning of each year. The Macaulay duration of this perpetuity is 30 years

Question: what is the modified duration of this perpetuity?

Exam FM Question 102

Perpetuity-due $= \frac{1}{d}$

increasing-perpetuity-immediate $= \frac{1}{i} + \frac{1}{i^2}$

increasing perpetuity due $= \frac{1}{d^2}$

$D^{mac} = (1+i) \times D^{MOD}$

Solve: $D^{mac} = \dfrac{(1\times v)\times 1 + (1\times v^2)\times 2 + (1\times v^3)\times 3 + \cdots}{\underbrace{1 + v + v^2 + v^3 + \cdots}_{\frac{1}{1-v}}}$ $\Leftrightarrow$ $I\ddot{a}_{\overline{\infty}|} = \frac{1}{i} + \frac{1}{i^2}$; $\frac{1}{d}$ or $\frac{1}{1-v}$

$= \dfrac{\frac{1}{i} + \frac{1}{i^2}}{\frac{1}{1-v}}$, $v = \frac{1}{1+i}$ $= \boxed{\dfrac{1+\frac{1}{i}}{1+i} = 30} \Rightarrow i = \frac{1}{30}$

Thus: $D^{mac} = (1+i)D^{MOD} \Rightarrow D^{MOD} = \dfrac{1}{1+i} D^{mac} \approx 29.03225$ (C)

where $i = \frac{1}{30}$ and $D^{mac} = 30$

Exam FM Question 103:

Question: what is the following statements regarding immunization are true?

Exam FM Question 103

Which of the following statements regarding immunization are true?

I. If long-term interest rates are lower than short-term rates, the need for immunization is reduced.

Wrong: Immunization is NOT related to "long/short" term rate (term structure)

II. Either Macaulay or modified duration can be used to develop an immunization strategy.

Correct. Strategy needs more than Mac & Mod duration

III. Both processes of matching the PV of the cash flows or the flows themselves will produce exact matching.

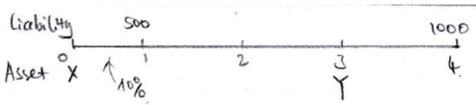
wrong. PV cannot produce exact matching.

Exam FM Question 104:

500 and 1,000 to be paid at the end of year 1 and year 4. The company sets up an investment program to match the duration and the PV of the above obligation using an annual rate of 10%. This program produces asset cash flows of X and Y in 3 years.

Question: what is X and determine whether the investment program satisfies the Redington immunization?

Exam FM Question 104.



Solve: Check "Redington Immunization": (in terms of i)

Check → (1) PV: $X + Y(1+i)^{-3} = 500(1+i)^{-1} + 1000(1+i)^{-4} \Rightarrow \overbrace{X + Y(1+i)^{-3}}^{PV\ Asset} - \overbrace{500\times(1+i)^{-1} - 1000\times(1+i)^{-4}}^{PV\ Liability} = 0$

check → (2) Duration. (1st derivative w.r.t i) $-3\times Y\times(1+i)^{-4} + 500(1+i)^{-2} + 4000(1+i)^{-5} = 0$ where $i=10\%$

Check → (3) Convexity. (2nd derivative w.r.t i) $\underbrace{12\times Y(1+i)^{-5}}_{Convexity\ Asset} - \underbrace{1000(1+i)^{-3} - 20000(1+i)^{-6}}_{Liability} > 0$

From (2), we have: $2.049Y = 2483.6853 + 413.22314 \Rightarrow Y \cong 1413.82$. plug into (1).

Using $Y \cong 1413.82$ & (1), we have: $X \cong 75.36$ (A)

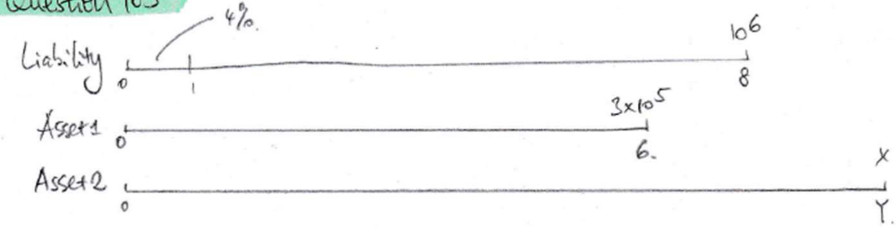
Using $X \cong 75.36$ & $Y \cong 1413.83$ to check (3): $\underbrace{12\times 1413.82\times 1.1^5}_{Asset} - \underbrace{1000(1.1)^3 - 20000(1.1)^6}_{Liability} = -1506.34 < 0$ (NOT hold)

Exam FM Question 105:

A liability of 1,000,000 due in 8 years from now. Full immunization. Asset I give a cash flow of 300, 000 6 years from now. Asset II provides a cash flow of X, exactly y years from now, where y>8. The annual rate is 4%

Question: what is X?

Exam FM Question 105



Solve: Fix at $t=8$.

(1) PV: $3\times10^5(1+i)^2 + X(1+i)^{-(Y-8)} = 10^6$ eq(1) where $i=4\%$

(2) Duration: $2\times3\times10^5(1+i) + (8-Y)\cdot X\cdot(1+i)^{7-Y} = 0$ eq(2)

From eq(1), we have: $X\cdot(1+i)^{8-Y} = 6.7552\times10^5$ plug into eq(2), we have: $Y-8 = 0.960682$

Use eq(1) & $Y-8 = 0.960682$, we have: $0.963X = 6.7552\times10^5 \Rightarrow X = 701408.2608$ (D)

Exam FM Question 106:

Liabilities of 573 due at the end of year 2 and 701 due at the end of year 5. A portfolio comprises 2 zero-coupon bonds, Bond A and Bond B. Question: which portfolio produces a Redington immunization using a rate 7%?

Exam FM Question 106

$\begin{cases} \text{PV} & \dfrac{PV_1 \times t_1 + PV_2 \times t_2}{PV_1 + PV_2} \\ \text{Convexity} & \dfrac{PV_1 \times t_1^2 + PV_2 \times t_2^2}{PV_1 + PV_2} \end{cases}$ (this question, directly give PV_1 & PV_2 of Asse (→ check: Convexity formula).

Solve, Redington Immunization: ($i=7\%$)

① Check PV (at $t=0$): Liability $= 573v^2 + 701v^5 \doteq 1000$

check (A), $PV = 500 + 500 = 1000$ (✓)

(B): $PV = 572 + 428 = 1000$ (✓)

(C): $PV = 182 + 1092 > 1000$ (X)

(D): $PV = 637 + 637 > 1000$ (X)

(E): $PV = 1000$ (✓)

② Check Duration: $D^{\text{Liability}} = \dfrac{(573v^2) \times 2 + (701v^5) \times 5}{1000} = 3.5$

check (A): $D = \dfrac{500 \times 1 + 500 \times 6}{1000} = 3.5$ (✓)

Check (B): $D = \dfrac{572 \times 1 + 428 \times 6}{1000} = 3.14$ (X)

check (E): $D = 3.5$ (✓)

③ check Convexity: check (A): $C_A^{\text{Asset}} = \dfrac{500 \times \boxed{1^2} + 500 \times \boxed{6^2}}{1000} = 18.2$

(ok✓) $\vee$ $C_A^{\text{Liability}} = \dfrac{(573v^2) \times \boxed{2^2} + (701v^5) \times \boxed{5^2}}{1000} = \dfrac{2001.9215 + 12495.08}{1000} = 14.497$

check (E): $C_E^{\text{Asset}} = \dfrac{1000 \times \boxed{3.5^2}}{1000} = 12.25$

(X) $\wedge$ $C_A^{\text{Liability}} = 14.497$

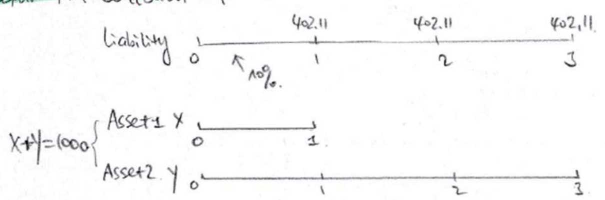
Thus: choose (A).

Exam FM Question 107:

Liabilities of 402.11 due at the end of each of next 3 years. The company will invest 1000 today to fund these payouts, Investments available are 1-year and 3-year zero-coupon bonds, yield at 10% annual rate. The company wishes to match the duration of asset to the duration of its liability

Question: Determine how much the company should invest in each bond?

Exam FM Question 107



Solve: To match duration: $D^{mac}_{\text{Liability}} = \dfrac{1 \times (402.11v) + (402.11v^2) \times 2 + (402.11v^3) \times 3}{1000}$ where $v = \frac{1}{1.1}$

$= \dfrac{365.554545 + 664.644628 + 906.333584}{1000} = \dfrac{1936.532757}{1000} = 1.936533.$

Then: $D^{mac}_{\text{Asset}} = \boxed{\dfrac{X \times 1 + Y \times 3}{1000} = 1.936533} = D^{mac}_{\text{Liability}}$

We also know: $X + Y = 1000$

$\Rightarrow 2Y = 936.533 \Rightarrow Y = 468.2665$ (D)

Exam FM Question 108:

Given the following information about a company's liabilities:

- PV 9697
- Macaulay duration 15.24
- Macaulay convexity 242.47

Create an investment portfolio by making investments into 2 of the following 3 zero-coupon bonds: 5-year, 15-year, and 20-year. Redington immunized. The annual effective yield rate is 7.5%.

Question: Determine which of the following portfolios the company should create?

Exam FM Question 108

Give: Liability: (1) PV=9697 ; (2) $15.24 = D^{mac}$; (3) $C^{liability} = 242.47$

Solve: (1) check PV

(A) 3077 + 6620 = 9697 (V)

(B) 6620 + 3077 = 9697 (V)

(C) 465 + 9232 = 9697 (V)

(D) 4156 + 5541 = 9697 (V)

(E) 9232 + 465 = 9697 (V)

(2) check D^{mac}

(A) $D_A^{mac} = \dfrac{3077 \times 5 + 6620 \times 20}{9697} = 15.24$ (V)

(B) $D_B^{mac} = \dfrac{6620 \times 5 + 3077 \times 20}{9697} = 9.7597$ (X)

(C) $D_C^{mac} = \dfrac{465 \times 15 + 9232 \times 20}{9697} = 19.76$ (X)

(D) $D_D^{mac} = \dfrac{4156 \times 15 + 5541 \times 20}{9697} = 17.887$ (X)

(E) $D_D^{mac} = \dfrac{9232 \times 15 + 465 \times 20}{9697} = 15.24$ (V)

(3) Check $C^{Asset} > C^{liability}$

(A) $C_A^{Asset} = \dfrac{3077 \times \boxed{5^2} + 6620 \times \boxed{20^2}}{9697} = \boxed{281.007 > 242.47} = C^{liability}$ ✓

(E) $C_E^{Asset} = \dfrac{9293 \times \boxed{15^2} + 465 \times \boxed{20^2}}{9697} = \boxed{234.807 < 242.47} = C^{liability}$ ✗

⇒ A

Exam FM Question 109:

20,000 deposit on which it guarantees to pay an annual rate of 10% for 2 years. A customer needs to withdraw half of the accumulated value at the end of the 1st year. The remaining will be taken at the end of the 2nd year. There are 3 investments options available: (1) Bond H: 1-year zero-coupon bond yielding 10% annually (2) Bond I: 2-year zero-coupon bond yielding 11% annually (3) Bond J: 2-year zero-coupon bond yielding 12% annually.

Question: Determine which strategies produces the highest profits?

Exam FM Question 109

Solve: notice (1) highest profit ≡ lowest cost portfolio

(2) Bond I and J are replaceable

Liability: time 1: $\frac{2\times10^4\times(1.1)}{2} = \boxed{1.1\times10^4}$; time 2: $\frac{2\times10^4\times1.1}{2}\times1.1 = \boxed{1.21\times10^4}$

Bond H: 10%, 0 to 1: FV_H (Face Value$_H$)

Bond H: 11%, 0 to 2: FV_I

Bond J: 0: Price = FV_J; time 1: $0.12FV_J$; time 2: $1.12FV_J$

time 1: $1.1\times10^4 = FV_H + 0.12FV_J$

time 2: $1.21\times10^4 = FV_I + 1.12FV_J$ eq(2)

$$\rightarrow \text{Cost of portfolio} = \underbrace{\frac{FV_H}{1.1}}_{price_H} + \underbrace{\frac{FV_I}{1.11^2}}_{price_I} + \boxed{\underbrace{FV_J}_{price_J}} = 10^4 - 0.10909FV_J + 0.982\times10^4 - 0.90901FV_J + FV_J = 19820 - 0.0181FV_J$$

$\rightarrow$ To achieve: $\text{Min}(19820 - 0.0181FV_J) \Leftrightarrow \text{Max}(FV_J) \Leftrightarrow$ No "Bond H", $\Leftrightarrow$ eq(2) $FV_I = 0 \Leftrightarrow FV_J = \frac{1.21\times10^4}{1.12}$

$\Rightarrow FV_J = 10803.571$ (E)

Exam FM Question 110:

Need to match liabilities of 25,000 payable in 1 year and 20,000 payable in 2 years with specific assets. The following assets are available. (1) 1-year bond with annual coupon of 6.75% (2) 2-year bond with annual coupon of 4.5% at par (3) 2-yearzero-coupon bond yielding 5.00% annual effective

Question: What is the smallest amount needed to match these liabilities?

Exam FM Question 110

lowest cost = using highest yield (iii has a higher yield than ii, thus choose iii).

Liabilities: time 1: 25,000; time 2: 20,000

0 to 1: yield = coupon rate = 6.75%

0 to 2: yield = 5%

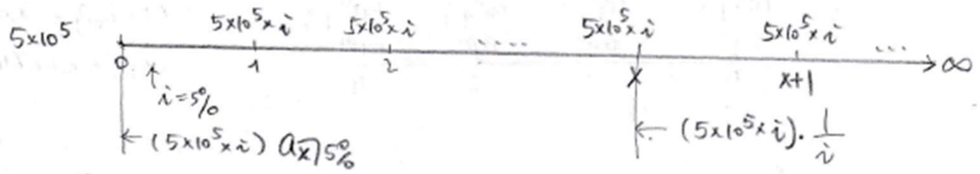
$$\text{Solve: Lowest cost} = \underbrace{\frac{25{,}000}{1+6.75\%}}_{23419.2037} + \underbrace{\frac{20{,}000}{(1+5\%)^2}}_{18140.589} \cong 41559.79275 \text{ (C)}$$

Exam FM Question 111:

An estate of 500,000. Interest on the state is paid to John for the first X years at the end of each year. Karen receives annual interest rate from the end of year X+1 forever. An annual rate of 5%, The PV of Karen's interest payment is 1.59 times the PV of the John's

Question: What is X?

Exam FM Question 111



Solve: $1.59\times(5\times10^5\times i)\, a_{\overline{x}|5\%} = v^x\cdot(5\times10^5\times \cancel{i})\cdot\frac{1}{\cancel{i}}$

$$1.59\times(\cancel{5\times10^5\times i})\cdot\frac{1-v^x}{\cancel{i}} = v^x\cdot(\cancel{5\times10^5})$$

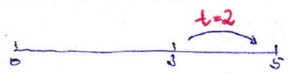
$$1.59(1-v^x)=v^x$$

$$1.59=2.59v^x$$

$$v^x=0.6139$$

$$x=\frac{\ln(0.6139)}{\ln(v)}\Big|_{v=\frac{1}{1.05}}=\frac{-0.4879238}{-0.04879}\cong 10 \quad (E)$$

Exam FM Question 112



Sherry:

From "annual rate" to "force of interest": $(1+7\%) = e^{\delta\times 1} \Rightarrow \delta = \ln(1.07) = 0.06766.$ eq(1)

Gomer:

not 2, use original t

From "simple rate" to "force of interest": step1: $\boxed{\frac{\partial[\ln(1+y\%\cdot t)]}{\partial t} = \delta} \Rightarrow \delta = \frac{y\%}{1+y\%\times t}$ eq(2).

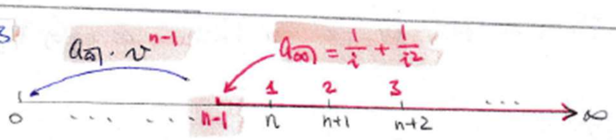
At t=2: (step 2)

eq(1) = eq(2) $\Rightarrow \boxed{0.06766 = \frac{y\%}{1+y\%\times \underset{2}{t}}} \Rightarrow y = 7.825\% \Rightarrow$ Gomer's account $= 1000(1+7.825\%\times 2) = 1156.5$ (A)

Exam FM Question 113:

Which of the following is an expression for the PV of a perpetuity with annual payments of 1, 2, 3,⋯., where the first payment will be made at the end of n years, using an annual rate of i?

Exam FM Question 113



* $a_{\overline{\infty}|}=\frac{1}{i}+\frac{1}{i^2}$
Be careful about the t=0
* $d=vi$

Solve: $PV = v^{n-1}\cdot \underbrace{a_{\overline{\infty}|}}_{\frac{1}{i}+\frac{1}{i^2}}$

$= v^{n-1}\times\left(\frac{1}{i}+\frac{1}{i^2}\right)$

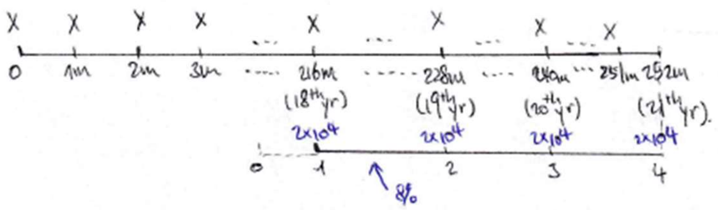
$= v^{n-1}\,\frac{1+i}{i^2} = \frac{v^{n-2}}{i^2} = \frac{v^{n-2}\times v^2}{\underbrace{i^2\times v^2}_{d=iv}} = \frac{v^n}{d^2}$ (D)

Exam FM Question 114:

A college plan. Jennifer plans to invest X at the beginning of each month for the next 21 years. Beginning at the 18th year, she will withdraw 20,000 annually. The first withdrawal at the end of the 21st year will exhaust the account. Annual rate of 8%

Question: What is X?

Exam FM Question 114



$(1+i_m)^{12}=1.08$

$\Rightarrow i_m = 0.006436$

Solve: $(2\times10^4)\,\underbrace{s_{\overline{4}|8\%}}_{\frac{(1+i)^4-1}{i_{=8\%}}} = X\cdot \underbrace{\ddot{s}_{\overline{252}|0.006434}}_{\frac{(1+i)^{252}-1}{d_{=\frac{0.006436}{1.006436}}}}$, with $i = 0.006436$

$\Rightarrow 630.98965X = 90122.24 \Rightarrow X = 142.8268$ (B)

Exam FM Question 115.

Give: ① $i > 0$

② PV of 20-year "continuous annuity", of \$1 per year, equal to

1.5 times the PV of a 10-year "continuous annuity", of \$1 per year

Formular:

$$\boxed{\bar{a}_{\overline{n}|} = \frac{1-v^n}{\delta} = \frac{i}{\delta} a_{\overline{n}|}}$$

$$\frac{a_{\overline{2n}|}}{a_{\overline{n}|}} = 1 + v^n$$

$$\bar{s}_{\overline{n}|} = \frac{(1+i)^n - 1}{\delta}$$

$$\Leftrightarrow \bar{a}_{\overline{20}|} = 1.5\, \bar{a}_{\overline{10}|}$$

$$\Leftrightarrow \frac{i}{\delta} a_{\overline{20}|} = 1.5 \frac{i}{\delta} a_{\overline{10}|}$$

$$\Leftrightarrow \frac{a_{\overline{20}|}}{a_{\overline{10}|}} = 1.5$$

$$\Leftrightarrow 1 + v^{10} = 1.5$$

$$\Leftrightarrow v^{10} = 0.5 \Leftrightarrow \underbrace{v}_{\frac{1}{1+i}} = 0.5^{\frac{1}{10}} = 0.933 \Rightarrow 1+i \cong 1.07177 \Rightarrow i \cong 0.07177.$$

Question: AV of "7-year" "continuous annuity" of \$1 per year

Solve: $\bar{s}_{\overline{7}|} = \frac{(1+i)^7 - 1}{\delta} = \frac{(1+i)^7 - 1}{\ln(1+i)} = \frac{0.6245}{0.06931} \cong 9.0102$ (D)

where $\delta = \ln(1+i)$

Alternative: Give: $\bar{a}_{\overline{20}|} = 1.5\, \bar{a}_{\overline{10}|}$

$$\Leftrightarrow \frac{1 - e^{-\delta \cdot 20}}{i} = 1.5 \frac{1 - e^{-\delta 10}}{i}$$

$$\Leftrightarrow e^{-20\delta} - 1.5 e^{-10\delta} + 0.5 = 0$$

$$\Leftrightarrow (e^{-10\delta} - 0.5)\underbrace{(e^{-10\delta} - 1)}_{\text{delete: since } \delta \neq 0} = 0$$

$$\Leftrightarrow e^{-10\delta} = 0.5$$

$$\Leftrightarrow -10\delta = \ln(0.5) \Rightarrow \delta = -\frac{\ln(0.5)}{10} \cong 0.069315$$

Then: $\bar{s}_{\overline{7}|} = \frac{e^{n\delta} - 1}{\delta} = \frac{e^{7\delta} - 1}{\delta} = \frac{e^{7 \times 0.069315} - 1}{0.069315} \cong 9.01$ (D)

Recall $\bar{s}_{\overline{7}|} = \frac{(1+i)^7 - 1}{\delta}$, where $(1+i) = e^{\delta t}$, $t = 1$

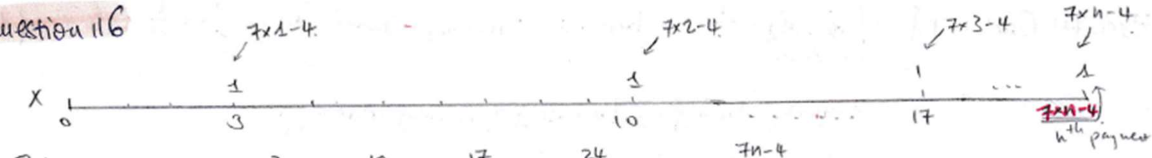
$$= \frac{e^{\delta \cdot 7} - 1}{\delta}$$

Exam FM Question 116:

An annuity having n payments of 1 has a PV of X. The first payment is made at the end of 3 years and the remaining payments are made at 7-year intervals thereafter

Question: What is X?

Exam FM Question 116



Solve: $PV = X = 1\cdot v^3 + 1\cdot v^{10} + 1\cdot v^{17} + 1\cdot v^{24} + \cdots + 1\cdot v^{7n-4}$ $\quad q = v^7$

$$= \frac{v^3 - v^{7n-4}\times v^7}{1-v^7} \quad \leftarrow \frac{a_1 - a_n q}{1-q}$$

$$= \frac{v^3 - v^{7n+3}}{1-v^7}$$

$$= -\frac{(1-v^3)-(1-v^{7n+3})}{1-v^7}$$

$$= -\frac{(1-v^3)/i - (1-v^{7n+3})/i}{(1-v^7)/i} = -\frac{a_{\overline{3}|} - a_{\overline{7n+3}|}}{a_{\overline{7}|}} = \frac{a_{\overline{7n+3}|} - a_{\overline{3}|}}{a_{\overline{7}|}} \quad (B)$$

Exam FM Question 117:

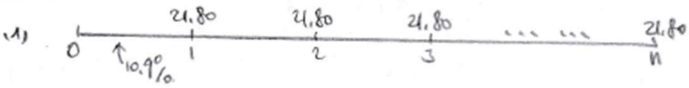
An annual rate of 10.9%, each of the following are equal to X:

- The accumulated value at the end of n years of an n-year annuity-immediate paying 21.80 per year
- The PV of a perpetuity-immediate paying 19,208 at the end of each n-year period

Question: What is X?

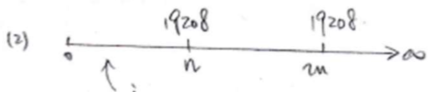
Exam FM Question 117

(1)



$\Rightarrow AV_1 = X = 21.8\, s_{\overline{n}|10.9\%}$

(2)



$\Rightarrow AV_2 = X = \dfrac{19208}{j}$, $j = 1.109^n - 1$ $\quad$ eq(2)

$j: (1+j) = (1+10.9\%)^n \Rightarrow j = (1.109)^n - 1$

Solve: $\because AV_1 = AV_2 = X$

$$\therefore 21.8\, \underbrace{s_{\overline{n}|10.9\%}}_{\frac{(1+10.9\%)^n - 1}{i}} = \frac{19208}{1.109^n - 1}$$

$$\Rightarrow 21.8\,\frac{1.109^n - 1}{0.109} = \frac{19208}{1.109^n - 1} \Rightarrow (1.109^n - 1)^2 = 96.04 \Rightarrow 1.109^n = 10.8 \quad \text{plug into eq(2)}$$

we have: $\underset{AV_2}{X} = \dfrac{19208}{10.8 - 1} = 1960$ (C)