SOA and CAS: Exam FM[1]
Written Solutions: 327-385

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This document only provides written solutions to official example problems 327-385. For official sample questions, check out the official websites of Society of Actuaries and the Casualty Actuarial Society.

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[2] Email: liyifinhub@outlook.com The written solutions were drafted when I was preparing for the exam. Please email me if you find any errors. My personal website: `https://yilifinhub.com/`

Exam FM Question 327

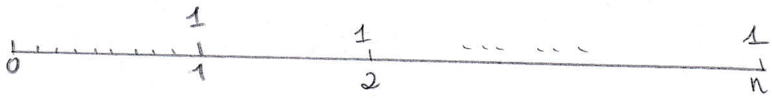
Give: ① n-year annuity, with a payment of 1 at the "end of each year", has a present value of 5.5554. at a annual nominal rate, convertible m times per year.

② i) j is the effective rate of interest per interest conversion peiod

ii) $(1+j)^m = 1.0614$

Question: What is n?

Solve:



$$\left[\left(1+\underbrace{\frac{i_{annual}}{m}}_{j}\right)^m\right]^1 = 1+i_{annual}$$

$$\Rightarrow \boxed{5.5545 = 1\times a_{\overline{n}|i_{annual}}} = 1\times\frac{1-v_{annual}^n}{i_{annual}} \quad \text{where: } i_{annual} = \underbrace{(1+j)^m}_{1.0614} - 1 = 0.0614$$

$$\boxed{= 1\times\frac{1-\left(\frac{1}{1.0614}\right)^n}{0.0614}}$$

$\Rightarrow n \cong 7$

Exam FM Question 328

Give: ①. Option 1: buy a car: pay 15,000, at the end of 3 year, car worth 8,000

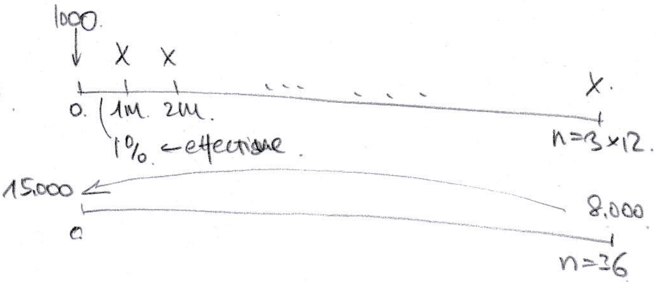
② Option 2: lease: 1000 down payment, X at each monthy for 3 year then return the car. 租辆车 预付1000, 之后每月付X

③ Customer: indifferent to both opinions. $i_{annual} = \overset{\overset{normal}{\downarrow}}{12\%}$ convertible monthly.

Question: What is X?

Solve. PV (drop in Value) = PV (lease this car)

$$15000 - 8000\left(\frac{1}{1+0.01}\right)^{36} = 1000 + X\underbrace{a_{\overline{36}|1\%}}_{30.11}$$

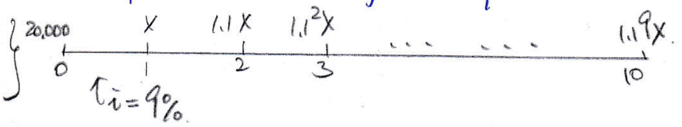


$X \cong 279.26$ Ⓒ//

Exam FM Question 329

Give: ① Loan of 20,000, repaid with 10 increasing installment paid at the end of each year. Each installment will be 10% greater than the preceding installment.

② Annual rate = 9%



Question: "Amount of principal" in the 2nd installment ⇔ $\underbrace{1.1X}_{Amt} - Outstanding_{t=1} * i$.

Solve: First. $20{,}000 = Xv + (1.1X)v^2 + (1.1^2X)v^3 + \cdots + (1.1^9X)v^{10}$

$$= Xv\left(1+1.1v+1.1^2v^2+\cdots+(1.1v)^9\right) = Xv\frac{1-(1.1v)^{10}}{1-1.1v} \quad \text{where } i=9\%$$

$\Rightarrow X \cong 2091.435$

Second: $Question = 1.1\overset{2091.435}{X} - \underbrace{Outstanding_{t=1}}_{20{,}000(1+9\%)-X_{\nwarrow 2091.435}} \times 9\% \cong 526.80$ Ⓑ//

Exam FM Question 330.

Give: ① At time $t=0$, 3 is deposited into an investment fund earning interest at a force of interest $\delta_t = \frac{1}{t+1}$

② At the same time, X is as deposited into a bank account paying interest at an annual effective rate of 5%

③ In the third year, the interest earned on the first account, and the interest credited on the second account are exactly the same.

Question: What is X?

Solve: Force of Interest

First: "Interest earned in "year 3"" : $\boxed{AV_{t=3} - AV_{t=2}} = 3 \times e^{\int_0^3 \frac{1}{t+1} dt} - 3 \times e^{\int_0^2 \frac{1}{t+1} dt} = 3e^{\ln(3+1)} - 3e^{\ln(2+1)}$

$= 3(4-3) = 3.$

Second: "Interest earned in "year 3"" of 2nd account : $\cancel{AV_{t=3} - AV_{t=2} = X(1.05)^3 - X(1.05)^2}$

$$AV_{t=2} \times i = [X(1.05)^2] \times \underset{5\%}{i}$$

$\Leftrightarrow \quad 3 = [(1.05)^2 X] \times 5\% \Rightarrow X \cong 54.42$ Ⓔ

Exam FM Question 331.

Give: ① Current loan rates are based on the following term structure of interest rate

Investment Length (years)	Spot rate
3	5.75%
5	7.25%

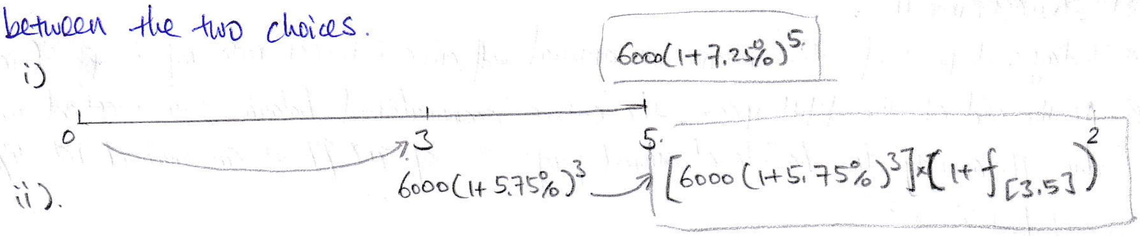
② Loan of 6000 to be repaid with single payment of principal + interest, at the end of 5-year.

③ The borrower has two choices.

i) a 5-year loan

ii) a 3-year loan to be repaid with a single payment of principal interest by taking out a 2-year loan at the beginning of year 4.

Question: What is "annual effective interest rate" on the 2-year loan, such that borrower is indifferent between the two choices.



Solve: $\cancel{6000} \cdot (1+7.25\%)^5 = [\cancel{6000}(1+5.75\%)^3] \times (1+f_{[3.5]})^2$

$\Rightarrow f_{[3.5]} \cong 9.54\%$ Ⓓ

Exam FM Question 332:

Give:

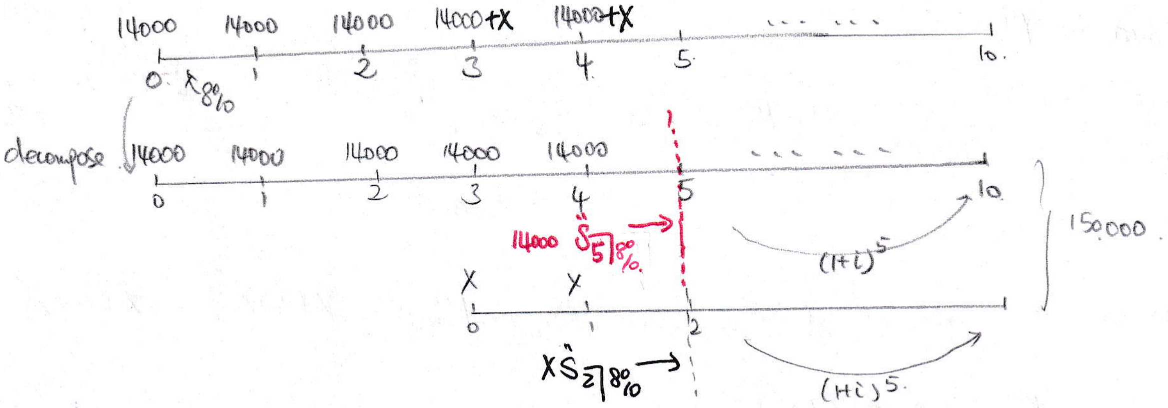
① An investor wants to accumulate 150,000 to purchase a business in 10-year

② The investor deposits 14,000 into an account at the beginning of each of years one through five.

③ The investor deposits an additional amount X at the beginning of years four and five to meet this goal.

④ the account earns interest at an rate 8%.

Question: What is X?



Solve: $\left(14000\,\underbrace{\ddot{s}_{\overline{5}|8\%}}_{\frac{(1+i_{8\%})^5-1}{d}} + X\underbrace{\ddot{s}_{\overline{2}|8\%}}_{\frac{(1+8\%)^2-1}{d}}\right) \times (1+8\%)^5 = 150{,}000 \Rightarrow X \cong 5878$ ⓒ //

where $d = \frac{8\%}{1+8\%}$

Exam FM Question 333.

Give:

→ Determin which of the following statments about "immunization strategies" are true.

(I ✗) "Redington Immunization" protects against any change (small change) in interest rates.

(II ✗). "Full immunization" occurs only when the duration of the assets is greater than (equal to) the duration of the liabilities.

(III ✗) Immunization techniques strive to arrange the asset portfolio, such that the convexity of the assets is equal to "greater than" the convexity of the liabilities.

✓(A) None (B) I and II only (C) I and III only (D) II and III only (E) The correct answer is not given by (A). (B). (C) or (D).

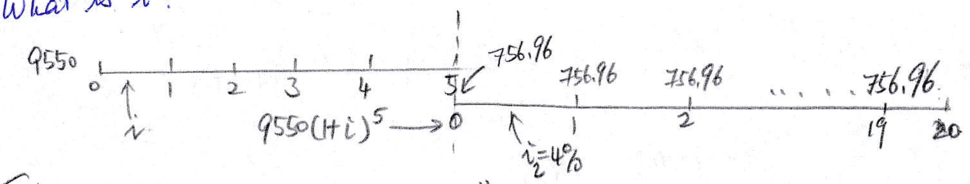
Exam FM Question 334

Give:

① Today's deposit of 9550 earns an annual effective interest rate of i for five-year.

② At the end of the fifth year, the entire accumulated balance is reinvested into a 20-year annuity-due. The annuity due has level annual payments of 756.97 at an annuat rate 4%.

Question: What is i?



Solve: $9550(1+i)^5 = 756.96 \times \underbrace{\ddot{a}_{\overline{20}|4\%}}_{\underbrace{\frac{1-v^{20}}{d}}_{\cong 14.1345}} \Rightarrow i \cong 2.23\%$ Ⓑ //

where $v = \frac{1}{1.04}$, $d = \frac{0.04}{1.04}$

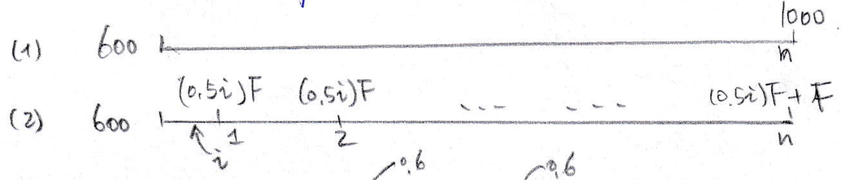
Exam FM Question 335

Give: ① Zero-Coupon bond, with Face Value 1000, price 600, mature in "n-year", yield rate i

② Second bond, same time to maturity, same yield, coupon rate = $0.5i$, yield rate i

Question: What is the Face Value of the 2nd bond?

Solve:



(1) $\Leftrightarrow 1000v^n = 600 \Rightarrow v^n = 0.6$

(2) $\Leftrightarrow [(0.5i)F]\,a_{\overline{n}|i} + F\cdot v^n = 600$

$$[(0.5\cancel{i})F]\times\frac{1-\overset{0.6}{v^n}}{\cancel{i}} + F\cdot \overset{0.6}{v^n} = 600 \Rightarrow F = 750 \text{ (B)}$$

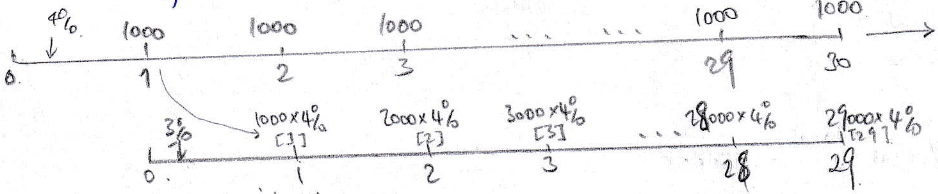
Exam FM Question 336

$\boxed{IS_{\overline{n}|} = \frac{\ddot{S}-n}{i}}$ or $\boxed{IS_{\overline{n}|} = \underset{\sim}{Ia_{\overline{n}|}}\times(1+i)^n = \underset{\sim}{\frac{\ddot{a}-nv^n}{i}}\times(1+i)^n}$.

Give: ① An actuary invests 1000 at the end of each year for 30 years. The investment will earn interest at 4%.

② At the end of each year, the interest will be re-invested at 3% rate.

Question. Accumulated value of the investment at the end of 30-year period.



principle

$\longrightarrow AV_1 = 1000\times 30$

$$\Leftrightarrow AV_2 \overset{\text{interest}}{=} \left[1000\times4\%\times\left(\underbrace{Ia_{\overline{29}|3\%}}\right)\right](1+3\%)^{29} \quad \text{or} \quad AV_2 = 40\,\boxed{IS_{\overline{29}|3\%}}$$

$$\frac{\ddot{S}_{\overline{29}|3\%}-\overset{29}{n}}{3\%}$$

$$\frac{\ddot{a}-nv^{\overset{n-29}{}}_{\frac{1}{1.03}}}{\underset{3\%}{i}}\ (\text{where } i=3\%).\qquad \frac{1-v^{29}}{d}\ \text{where } d=\frac{3\%}{1.03}$$

$$\text{Thus: Question} = AV_1 + AV_2 = 1000\times30 + \underbrace{40\times\frac{\frac{1-v^{29}}{d}-29v^{29}}{3\%}\times(1.03)^{29}}_{23433.9252} \cong 53433.91 \text{ (B)}$$

Exam FM Question 337

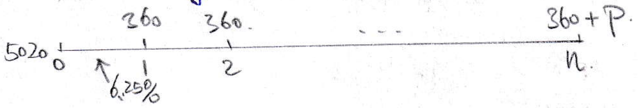
Definition: $\boxed{\text{Ballon Payment}}$: when get $n=16.7$, then 16 payments → 16 regular level payments; → 16th (last payment) = $\overbrace{P + \text{regular payment}}^{\text{balloon payment}}$

$\boxed{\text{Prop Payment}}$: when get $n=16.7$ then total 17 payments → 16 regular level payments; → 17th (last payment) = $\overbrace{P}^{\text{drop payment}} <$ regular one

Give: (1) Purchase of 5020. paid of with a loan at annual effective rate of 6.25%.

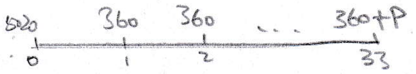
(2) Payments made at end of each year until the loan is paid off. Each payment is 360 except for a final balloon payment, which is less than 720.

Question: What is the "Ballon Payment"?



Solve: First: $\underset{FV=-5020}{5020} = 360\,a_{\overline{n}|6.25\%} \Rightarrow n \cong 33.96$ } $\Rightarrow N = 33 \Rightarrow$ Timeline

∵ Ballon Payment

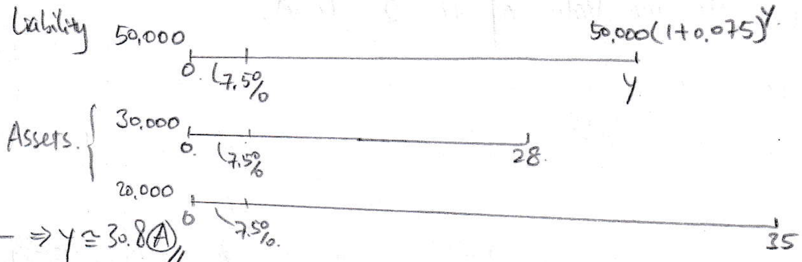


$$\text{Second: } 5020 = \underbrace{360\,a_{\overline{33}|6.25\%}}_{4980.946} + P\cdot\underbrace{v^{33}_{6.25\%}}_{0.13525} \Rightarrow P \cong 288.754 \Rightarrow \text{Balloon Payment} = 360 + P \cong 648.754 \text{ (C)}$$

Exam FM Question 338

Give:
(1) A company is required to pay $50{,}000(1.075)^y$ in y years, the company invests 30,000 in a 28-year zero-coupon bond and 20,000 in a 35-year zero-coupon bond to Redington immunize its position against small changes in interest rates
(2) annual rate 7.5%

Question: What is y?



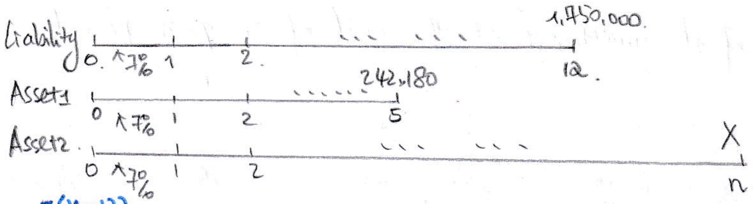
Solve: $D^{mac}_{Liability} = D^{mac}_{Asset}$

$$y = \frac{30{,}000 \times 28 + 20{,}000 \times 35}{50{,}000} \Rightarrow y \cong 30.8 \text{ (A)}$$

Exam FM Question 339

Give:
(1) It has a single liability of 1,750,000 to be paid 12-year from now.
(2) Its asset portfolio consists of a zero-coupon bond maturing in 5-year for 242,180 and a zero-coupon bond maturing in n years.
(3) At an annual rate 7%, Company J's position is fully immunized.

Question: what is n?



Fully Immunized:

Solve.

$$\begin{cases} (1)\ 242180(1+i)^7 + X(1+i)^{-(n-12)} = 1750000 \\ (2)\ 7 \cdot \underbrace{242180(1+i)^6 \cdot (1+i)}_{242180(1+i)^7} - (n-12) \cdot X \underbrace{(1+i)^{-(n-12)-1} \cdot (1+i)}_{(1+i)^{-(n-12)}} = \underbrace{0 \cdot (1+i)}_{0} \end{cases}$$

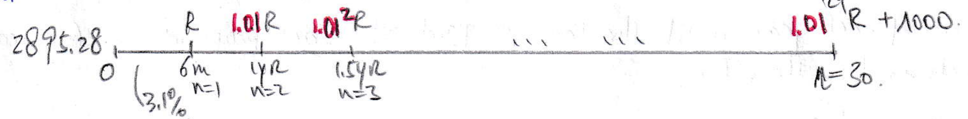
$$\Rightarrow \begin{cases} (1)\ \underset{7\%}{242180(1+i)^7} + \boxed{X(1+i)^{-(n-12)}} = 1750000 \Rightarrow X(1+i)^{-(n-12)} = 1361111.842 \\ (2)\ 7\left(\underset{7\%}{242180(1+i)^7}\right) - (n-12)\boxed{\left(X(1+i)^{-(n-12)}\right)} = 0 \end{cases}$$

$$\Rightarrow (n-12) \cong 2 \Rightarrow n \cong 14 \text{ (B)}$$

Exam FM Question 340.

Give:
(1) A 15-year bond with semi-annual coupons is purchased for 2895.28. The bond is redeemable for 1000.
(2) The first coupon payment is equal to R. and each subsequent coupon is 1% larger than the previous coupon payments.
(3) The annual nominal rate 6.2% convertible semi-annually.

Question: What is R?



$$\text{Solve: } 2895.28 = \underbrace{Rv + 1.01Rv^2 + 1.01^2Rv^3 + \cdots + 1.01^{29}Rv^{30}}_{Rv(\underbrace{1 + 1.01v + 1.01^2v^2 + \cdots + 1.01^{29}v^{29}}_{\frac{1-(1.01v)^{30}}{1-1.01v}})} + 1000v^{30} \quad \text{where } i_{6m} = 3.1\%$$

$$\leftarrow \frac{1-aq^n}{1-q}$$

$$\Leftrightarrow 2895.28 = Rv \cdot \frac{1-(1.1v)^{30}}{1-1.1v} + 1000v^{30}$$

where $i_{6m} = 3.1\%$

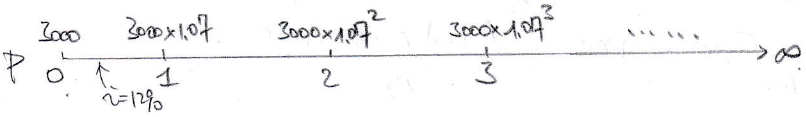
$$\Rightarrow R \cong 113.75 \text{ (B)}$$

Exam FM Question 341

Give: (1) An annuity writter sells an annual perpetuity-due, with a first payment of 3000. The payments increase by 7% annually.

(2) The purchase, P, of the perpetuity is based on a 12% annual force of interest

Question: what is P?



Solve:

$$P = 3000 + (3000\times 1.07)v + (3000\times 1.07^2)v^2 + \cdots\cdots$$

$$= 3000\,\underbrace{(1 + 1.07v + (1.07v)^2 + \cdots)}_{\frac{1}{1-1.07v}} \text{ where } i = 12\%$$

$$\cong 58826.087 \text{ Ⓒ}$$

Exam FM Question 342.

constant force of interest $\delta_{1year} = \delta_{6m}\times 2$; $\delta_{n\text{-}year} = \delta_{1\text{-}year}\times n$

Give: (1) An investor opens a savings account with no initial deposit that pays a constant 4.5% annual force of interest. The investor deposits 700 at the end of every 6-month period for 20 years.

(2) The account balance immediately after the last deposit is X

Question: Determine which of the following is an equation of value that can be used to solve for X?

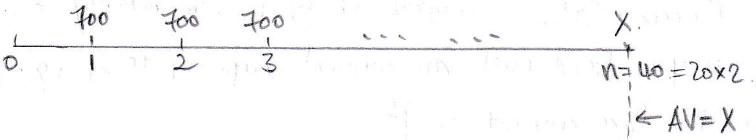
(A) $\sum_{n=0}^{39} 700e^{0.0225n} = Xe^{-0.9}$

(B) $\sum_{n=0}^{39} 700e^{-0.0225n} = Xe^{0.9}$

(C) $\sum_{n=0}^{39} 700e^{-0.0225n} = Xe^{-0.9}$

(D) $\sum_{n=0}^{39} 700e^{0.0225n} = Xe^{0.9}$

✓(E) $\sum_{n=1}^{40} 700e^{-0.0225n} = Xe^{-0.9}$



Solve: First. $AV_{t=40} = X$, thus cannot forward. only can backward.

$\Rightarrow$ delete B & D.

Second: $\because e^{-0.9} = e^{-\underset{annual}{0.045}\times\underset{20\text{-}year}{20}} \Rightarrow$ back to t=0

$\therefore$ left side: the first 700 must be $700e^{-0.0225\times(year=1)}$

Thus, n starts from 1, not 0 $\Rightarrow$ choose Ⓔ

Exam FM Question 343.

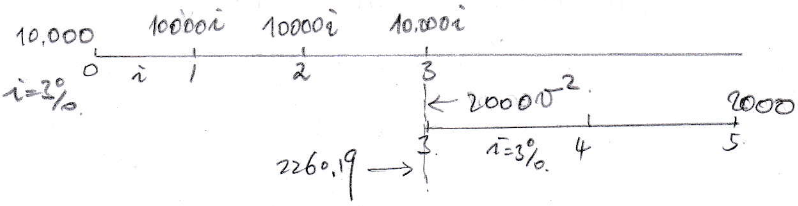
Give: (1) A five-year interest-only loan in the amount of 10,000 has annual payments, and an annual effective rate i. At the end of year 5, the borrower pays off the principal along with the last interest payment. ∀time t, repay interest, no principal.

(2) To finance the principal payment, the borrower buys the following two zero-coupon bonds that both mature at the end of year 5

	Time of Purchase	Par Value	Annual yield rate
Bond 1	End of year 3	2000	3.0%
Bond 2	End of year 4	8000	2.5%

(3) It costs 2260.19 at the end of year 3 to pay the interest due & to buy Bond 1

Question: What is i?



Solve: $2260.19 = 10.000i + 2000v^2$ where $i=3\%$.

$10.000i = 374.9982$

$i \cong 3.75\%$. Ⓓ

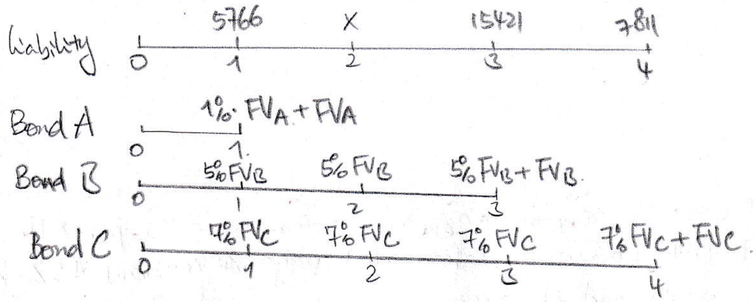
FM Exam Question 344.

Give: (1) A company faces the following liabilities at the end of the corresponding years:

Year	1	2	3	4
Liability	5766	X	15421	7811

(2) The company can invest in the following 3 annual coupon bonds redeemable at par:

	Term (in years)	Annual coupon rate
Bond A	1	1%
Bond B	3	5%
Bond C	4	7%.



Solve: At $t=4$: $7811 = 1.07 FV_C \Rightarrow FV_C = 7300$.

At $t=3$. $15421 = 1.05 FV_B + 7300 \times 7\% \Rightarrow FV_B = 14200$

At $t=2$. $X = 5\% \times 14200 + 7\% \times 7300 = 1221$ Ⓐ //

FM Exam Question 345.

coupons are paid semi-annually = coupon rate compounded semi-annually

An n-year bond with an annual coupon rate of r% has the following characteristics:

Give.

(i) The face amount is 980

(ii) Coupons of 49 are paid semi-annually.

(iii) The annual nominal yield rate convertible semi-annually is (r+1.8%)

(iv) The purchase price is 915.70

(v) the redemption value is 1000

Question: What is n?

Solve: First: coupons of 49 paid semi-annually $\Leftrightarrow 980 \times \frac{r\%}{2} = 49 \Rightarrow r\% = 10\%$

Second: annual nominal yield rate $= r + 1.8\% = 11.8\%$. $\Rightarrow r_{6m} = \frac{11.8\%}{2} = 5.9\%$

Third:

$$\underbrace{915.7}_{PV=-915.7} = \underbrace{49}_{PMT}\, a_{\overline{2n}|\underbrace{5.9\%}_{I/Y}} + \underbrace{1000}_{FV} v^{2n}_{5.9\%} \Rightarrow 2n = 12 \Rightarrow n = 6 \text{ years. Ⓔ} //$$

Exam FM Question 346

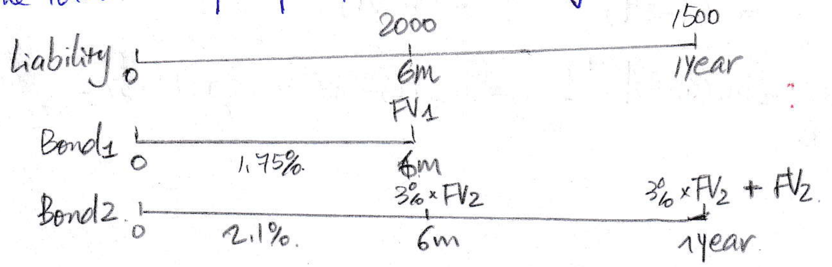
Give: (1) A liability consists of a payment of 2000 to be paid 6 months from now and a payment of 1500 to be paid one year frome now.

(2) The only investments avaliable are:

(i) Six-month "zero-coupon" bonds with an annual normal yield rate of 3.5% convertible semi-annually, and

(ii) One-year par value bonds with an annual coupon rate of 6% payable semi-annually and an annual nominal yield rate of 4.2% convertible semi-annually.

Question: the total cost of a portfolio that exactly matches the liability cash flows?



Solve: "Cash Flows Merch": $t = 1$year: $1.03FV_2 = 1500 \Rightarrow FV_2 = 1456.31$

$t = 0.6$year: $FV_1 + 0.03FV_2 = 2000 \Rightarrow FV_1 = 1956.31$

Thus: cost of portfolio $= \underbrace{\frac{FV_1}{1+1.75\%}}_{1922.66} + \underbrace{\frac{0.03FV_2}{1+2.1\%}}_{42.79} + \underbrace{\frac{(0.03+1)FV_2}{(1+2.1\%)^2}}_{1438.93} \cong 3403.38$

Exam FM Question 347

Give: ① A store purchased couch #1 for X two months ago and plans to sell it for 1500 six months from today

② The same store purchases couch #2 for X today and plans to sell it for 1500 four mouths from today

③ The annual force of interest is a constant 10%.

④ The current value of the store's cash flows from the pur

Question: Calulate the current value of the store's cash flows from the purchase and sale of couch #1.

Solve. First: $260 = 1500 \cdot e^{-\frac{4}{12}\times 10\%} - X \Rightarrow X \cong 1190.82$ ← hold because force of interest is constant

Second: $-Xe^{\frac{2}{12}\times 10\%} + 1500e^{-\frac{6}{12}\times 10\%} \cong 216.01$ Ⓐ

Exam FM Question 348

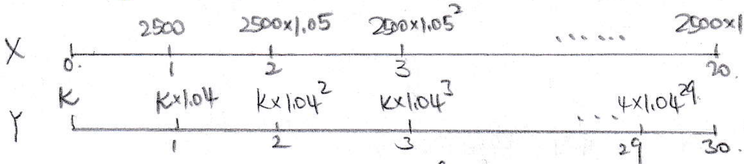
You are given the following information regarding 2 annuities with annual payments:

Give: (i) Annuity X is a 20-payment annuity-immediate which provides an initial payment of 2500 and each subsequent payment is 5% larger than the preceding payment.

(ii) Annuity Y is a 30-payment annuity-due which provides an initial payment of K and each subsequent payment is 4% larger than the preceding payment.

(iii) Annual rate 4%. Annuity X and Y have the same present value.

Question: What is K?



$\Leftrightarrow PV = K + K\times\underbrace{1.04v}_{1} + \cdots + K\underbrace{(1.04v)}_{1}^{29} = 30K$

Solve: $30K = 2500v + 2500\times 1.05v^2 + \cdots + 2500\times 1.05^{19}v^{20}$

$= 2500v(1 + 1.05v + \cdots + (1.05v)^{19}) = 2500v\frac{1-(1.05v)^{20}}{1-1.05v}$ where $i = 4\%$

$\Rightarrow K \cong 1758.33$ Ⓐ

Exam FM Question349

$\rightarrow \left[\underbrace{(1-2d)^{-1\times0.5}}_{\text{Annual}}\right]^{0.4/1\text{ yearly}}$ and $(a^m)^n = a^{m\times n}$ and $\ln a - \ln b = \ln\frac{a}{b}$

Give: The annual force of Interest is $\delta_t = \frac{2}{10-t}$ (for $0 \le t < 10$). in which t is measured in years.

Question: Calculate the equivalent annual nominal discount rate compounded every two years. for the period $(2 \le t \le 2.4)$.

Solve: → discount rate, compound annually (Forward) $\overset{\boxed{\text{Annually}}}{(1-d)^{-1}}$

→ discount rate, compound semi-anually (Forward) $(1-\frac{d}{2})^{-1\times2}$

→ discount rate, compound every 2-year (Forward) $(1-2d)^{-1\times0.5}$

Left: $\left[\underbrace{(1-2d)^{-0.5}}_{1\text{-year}}\right]^{\overbrace{\text{Year}}^{2.4-2}} = (1-2d)^{-0.5\times0.4} = (1-2d)^{-0.2}$

Right: $e^{\int_{2}^{2.4}\frac{2}{10-t}dt} = e^{[-2\ln(10-t)|_2^{2.4}]} = e^{-2[\ln(7.6)-\ln(8)]} = e^{+2[\ln\frac{8}{7.6}]} = e^{\ln[\underbrace{(\frac{8}{7.6})^2}_{t}]} = (\frac{8}{7.6})^2$

→ $e^{2\ln x} = e^{\ln(x^2)} = x^2$

Thus: $(1-2d)^{-0.2} = (\frac{8}{7.6})^2 \Rightarrow d \cong 0.20068$ Ⓐ//

Exam FM Question 350

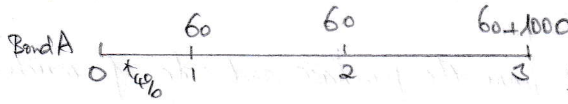
Give: (1) A portfolio consists of two bonds. Bond A is a 3-year 1000 Face amount bond with an annual coupon rate of 6% paid annually.

(2) Bond B is a 1-year zero-coupon bond.

(3) Both bonds yield an annual effective rate of 4%

Question: What is the percentage of the portfolio to invest in Bond A to obtain a Macaulay duration of 2-year.

Solve:



First: $\text{Price}_{\text{Bond A}} = 60a_{\overline{3}|4\%} + 1000v^3 = 1055.50$

Second: $2 = \frac{(60v)\times1 + (60v^2)\times2 + (1060v^3)\times3 + X\cdot1}{1055.50 + X}$ where $i=4\%$

$\Rightarrow X \cong 885.65$

Thus: Question $= \frac{1055.50}{1055.50 + 885.65} \cong 54.4\%$ Ⓓ//

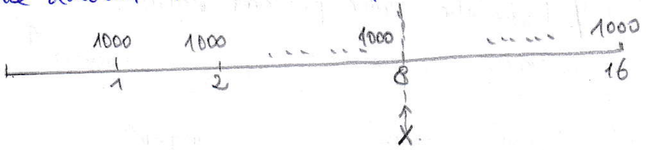
Exam FM Question 351

Give: ① Bank A: lends a certain sum of money at an annual effective rate of 7%. The loan is to be repaid by 16 annual payments of 1000. with the first payment due after one year.

② Immediately after receiving the 8th payment, Bank A sells the right to receive the remaining 8 payments to Bank B. Bank A's yield on the entire transaction is an annual effective interest rate of 6%.

Question: What is the amount that Bank B paid to assum the loan?

Solve:



$$\underbrace{\text{PV(Investment)}} = \underbrace{\text{PV(Revenue)}}$$

$$1000a_{\overline{16}|7\%} = 1000a_{\overline{8}|6\%} + X\cdot v_{6\%}^{8} \Rightarrow X \cong 5159.05$$ Ⓐ//

Exam FM Question 352

Give: (1) A zero-coupon bond with a face amount of 1000 sells for a price of 640 and matures in n years, where n is a whole number

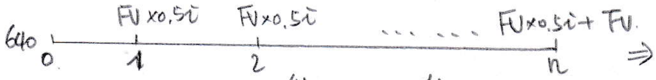
(2) A second bond has the same price, same time until maturity, and same annual effective yield. It pays annual coupons at an annual rate equal to 50% of the annual effective rate.

Question: What is the face value of the second bond?

Solve. First bond

$\Rightarrow 1000v^n \doteq 640 \Rightarrow v^n = 0.64$

Second bond



$\Rightarrow (FV \times 0.5i) \cdot a_{\overline{n}|i} + FV \cdot v^n = 640$ eq(1).

eq(1): $(FV \times 0.5\cancel{i}) \frac{1-\overset{0.64}{v^n}}{\cancel{i}} + FV \times \overset{0.64}{v^n} = 640 \Rightarrow 0.82FV = 640 \Rightarrow FV = 780.48$ Ⓐ//

Exam FM Question 353.

$D^{mac} = (1+i)D^{mod}$; $D^{mod} = \frac{-\partial P/\partial i}{P}$

Give: A bond is price at 950, giving an annual effective yield to maturity of 9%. At 9%, the derivative of the price of the bond w.r.t the yield rate is -4750.

Question: What is the Macaulay duration of the bond in years.

Solve. $D^{mac} = (1+i) D^{mod}$ where $D^{mod} = -\frac{\frac{\partial P}{\partial i}}{P} = -\frac{-4750}{950} = 5$.

$\Rightarrow D^{mac} = (1+9\%) \times 5 = 5.45$ Ⓔ//

Exam FM Question 354

(1) Bond Price = redemption price, coupon rate = yield rate (2) Portfolio $D^{mac} = \frac{PV_1 \cdot D_1^{mac} + \cdots + PV_n \cdot D_n^{mac}}{PV_1 + \cdots + PV_n}$

Give: (1) An investor purchases a portfolio consisting of 3 bonds.

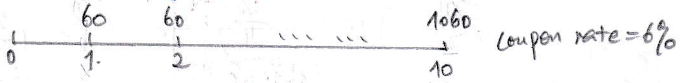
(2) Bond A has annual coupons of 6% and matures for its face amount of 1000 in 10-year. It is purchased for 1000.

(3) Bond B and C are zero-coupon bonds, maturing for 1000 each in 5 and 10 years, respectively.

(4) All 3 bonds have the same yield rate.

Question: What is the Macaulay duration in years at the time of purchase of the portfolio w.r.t the yield rate?

Price 1000, Bond A



coupon rate = 6%

Bond B.

Bond C.

Solve. First: Bond A: Price = Redemption Value $\Rightarrow$ yield rate = coupon rate = 6%

Second: $$D^{mac}_{Bond A} = \frac{\underbrace{(60v) \times 1}_{56.6} + \underbrace{(60v^2) \times 2}_{106.8} + \underbrace{(60v^3) \times 3}_{151.13} + \underbrace{(60v^4) \times 4}_{190.10} + \underbrace{(60v^5) \times 5}_{224.177} + \underbrace{(60v^6) \times 6}_{253.785} + \underbrace{(60v^7) \times 7}_{279.32} + \underbrace{(60v^8) \times 8}_{301.158} + \underbrace{(60v^9) \times 9}_{319.625} + \underbrace{(1060v^{10}) \times 10}_{5918.98}}{1000}$$ where i=6%

$\cong 7.8016.$

$D^{mac}_{Bond B} = 5$

$D^{mac}_{Bond C} = 10$

Thus: $$D^{mac}_{portfolio} = \frac{1000 \times 7.8016 + \overbrace{(1000v^5) \times 5}^{747.56} + \overbrace{(1000v^{10}) \times 10}^{558.39}}{1000 + 1000v^5 + 1000v^{10}}$$ where i = 6%

$\cong 7.426$ Ⓑ//

Exam FM Question 355 (i) Perpetuity $= \frac{1}{i}$ (ii) Increasing Perpetuity $= \frac{1}{i} + \frac{1}{i^2}$ ↑ Immediate (iii) Increasing Perpetuity ↖ due $= \frac{1}{d^2}$

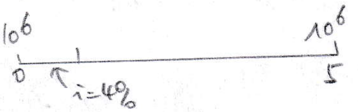
Give: (1) An individual is to receive 10^6 today, 10^6 five years from today

(2) These payments are to be converted to an increasing annual perpetuity, with the first payment X, paid today and each succeding payment 1000 more than the previous payment.

(3) At an annual rate of 4%, the present value of the two payments is equal to the present value of the perpetuity.

Question: What is X?

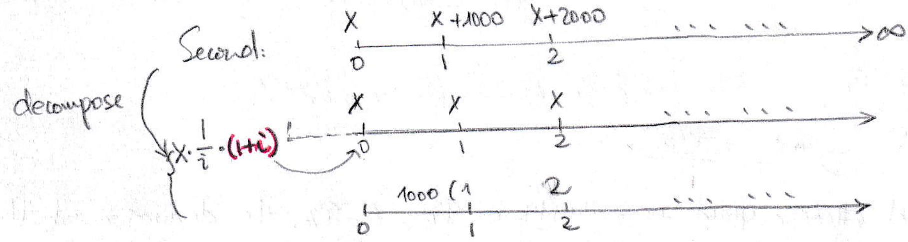
Solve: First: FV



$\Rightarrow FV = 10^6 + 10^6 v^5$ where $i=4\%$ $\Rightarrow FV = 1821927.107$.

Second:

decompose



$$FV = X\cdot\frac{1}{i}\cdot(1+i) + 1000\cdot\left(\frac{1}{i}+\frac{1}{i^2}\right)$$

where $i=4\%$

$$1821927.107 = \underbrace{X\cdot\frac{1}{i}\cdot(1+i)}_{26} + 1000\cdot\left(\underbrace{\frac{1}{i}}_{25}+\underbrace{\frac{1}{i^2}}_{625}\right)$$

$$\Rightarrow 1821927.107 = 26X + 650000$$

$$\Rightarrow X \cong 45074.12 \text{ Ⓐ}$$

Exam FM Question 356. $(a^m)^n = a^{m\times n}$, $\ln a - \ln b = \ln\frac{a}{b}$

Give: (1) At a force of interest $\delta_t = \frac{0.5}{5+0.5t}$ $(0 \le t \le 6)$ an investment of 1000 at time $t=2$ will accumulate to X at time $t=6$,

(2) At an annual nominal rate of discount of 8% convertible quarterly. an investment of Y will accumulate to X at the end of two years.

Question: What is X.

$$AV_1 = 1000e^{\int_2^6 \delta_t\,dt}.$$

$$AV_2 = Y\times\underbrace{\left[\left(1-\frac{8\%}{4}\right)^{-4}\right]}_{1\text{-year}}^{2} = Y\times\left(1-\frac{8\%}{4}\right)^{-8}$$

Solve:

$$AV_1 = AV_2 \Rightarrow \underbrace{1000e^{\int_2^6 \delta_t\,dt}}_{} = Y\times\left(1-\frac{8\%}{4}\right)^{-8}$$

$$1000e^{\ln(5+3)-\ln(5+1)}$$

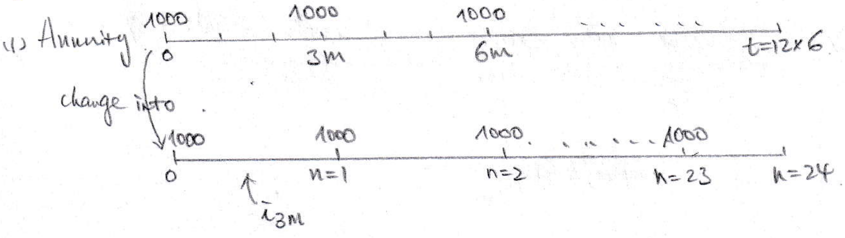
$$1000\underbrace{e^{\ln\frac{8}{6}}}_{\frac{8}{6}}$$

$$\Rightarrow 1000\times\frac{8}{6} = Y(0.98\%)^{-8} \Rightarrow Y \cong 1134.35 \text{ Ⓒ}$$

Exam FM Question 357 $\ddot{a}_{\overline{n}|} = (1+i)\, a_{\overline{n}|}$

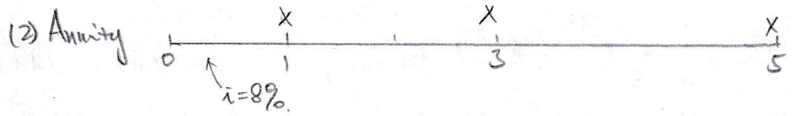
Give:
(1) An annuity has payments of 1000 at the beginning of every 3 months for 6 years.
(2) An other annuity has payments of X at the end of first, 3rd, and 5th years.
(3) Annual rate 8%, the PVs are the same.

Question: What is X?



$PV_1 = \ddot{a}_{\overline{24}|\, i_{3m}} = a_{\overline{24}|\, i_{3m}} \times (1+i_{3m})$.

where $(1+i_{3m})^4 = 1.08$



$PV_2 = X v_{8\%} + X v_{8\%}^2 + X v_{8\%}^5$

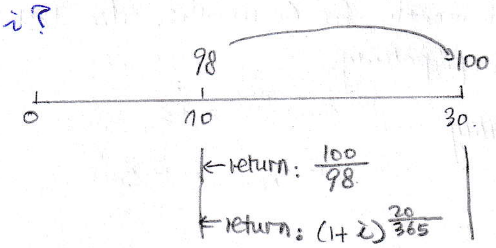
Solve: $PV_1 = PV_2 \Rightarrow \underbrace{1000 \times a_{\overline{24}|\, \overset{1.94265\%}{i_{3m}}} \times (1+i_{3m})}_{19407.21043} = \underbrace{X \times \frac{1}{1.08} + X\frac{1}{1.08^3} + X\frac{1}{1.08^5}}_{2.40034X.}$ where $(1+i_{3m})^4 = 1.08 \Rightarrow i_{3m} = 1.94265\%$

$\Rightarrow X \doteq 8085.1922$ Ⓓ //

Exam FM Question 358

Give:
(1) A retailer offers two payment plans:
(i) A 2% discount if paid within 10 calendar days after purchase
(ii) Pay the full amount on the 30th day after purchase
Assume a 365-day day
(ii) The implied annual yield the buyer is charged for delaying payment day 1 to day 30 is i

Question: What is i?

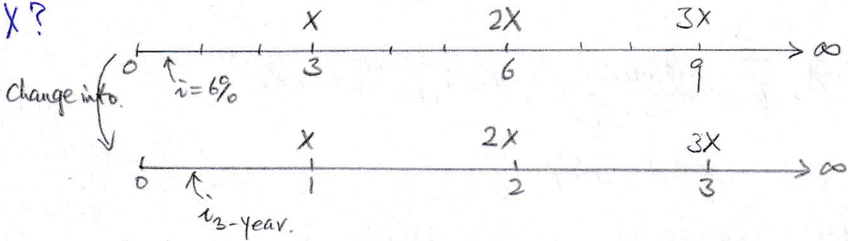


Solve: return $= \boxed{\frac{100}{98} = (1+i)^{\frac{20}{365}}} \Rightarrow i \doteq 44.585\%$ Ⓔ //

Exam FM Question 359 increasing perpetuity immediate $= \frac{1}{i} + \frac{1}{i^2}$

Give: (1) A perpetuity pays X at the end of the third year, 2X at the end of 6th year, 3X at the end of 9th year, etc
(2) At a 6% annual rate, the PV of perpetuity is 655.56

Question: What is X?



$(1+i_{3\text{-year}}) = (1+6\%)^3$.

$\Rightarrow i_{3\text{-year}} = 19.1016\%$

Solve: $X \cdot \left(\frac{1}{i_{3\text{-year}}} + \frac{1}{i^2_{3\text{-year}}}\right) = 655.56$

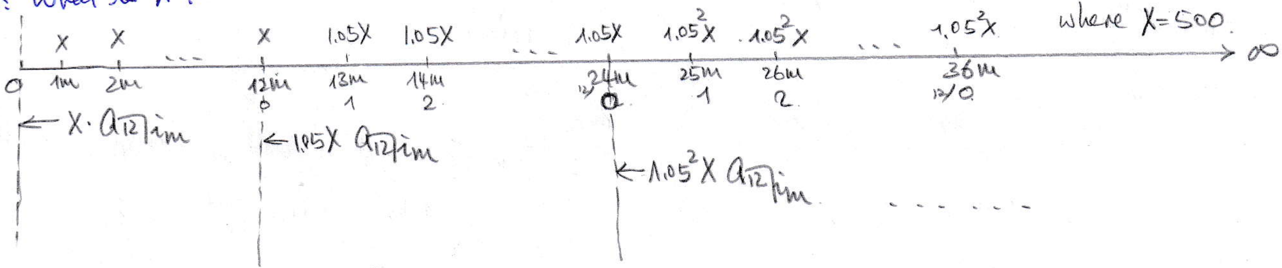
$\Rightarrow X \cdot \left(\underbrace{\frac{1}{19.1016\%}}_{5.23516} + \underbrace{\frac{1}{19.1016\%^2}}_{27.4009}\right) = 655.56 \Rightarrow X \cong 20.08$ Ⓒ //

Exam FM Question 360

Give: (1) A pepetuity-immediate has monthly payments that increase by 5% every 12 payments. The initial 12 payments are 500 each.

(2) The present value of this perpetuity-immediate, using an annual effective rate of 8% is X.

Question: What is X?



Solve: $PV = X \cdot a_{\overline{12}|i_m} + v^{12}\left(1.05X\, a_{\overline{12}|i_m}\right) + v^{24}\left(1.05^2 X\, a_{\overline{12}|i_m}\right) + \cdots$ where $(1+i_m)^{12} = 1.08$

$\Rightarrow i_m = 0.6434\%$

$$= \underset{500}{X}\, a_{\overline{12}|\underset{0.6434\%}{i_m}} \left(\underbrace{1 + v^{12} \times 1.05 + v^{24} \times 1.05^2 + \cdots}_{\frac{1}{1 - 1.05 v^{12}_{1m}}} \right) \quad v_{1m} = \frac{1}{1+0.6434\%}$$

$= 20723|.48$ Ⓓ//

Exam FM Question 361

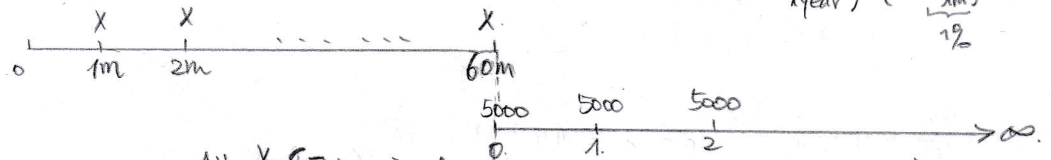
* annual nominal rate is 12% compounded monthly $\overset{\text{tells}}{\Leftrightarrow}$ ① $i_{-1m} = \frac{12\%}{12} = 1\%$ ② i_{-1year}: $(1+\underbrace{i_{1m}}_{1\%})^{12} = 1+i_{1-year}$

* perpetuity due $= \frac{1}{d}$ * $s_{\overline{n}|} = \frac{(1+i)^n - 1}{i}$

Give: ① A level monthly contribution of X is required to purchase an "annual perpetuity" of 5000 that commences five year from today

② The contributions are made at the end of each month for 60 months. The last contribution is made at the same time as the first payment from the perpetuity.

③ The annual nominal rate is 12% compound monthly. → ① $i_{1m} = \frac{12\%}{12} = 1\%$. ② $(1+i_{1year}) = (1+\underbrace{i_{1m}}_{1\%})^2$



$AV = X \cdot s_{\overline{12}|i_{1m}}, \; i_{1m} = 1\% \rightarrow$ $\leftarrow PV = \frac{5000}{d_{1\text{-}year}}$ where $d = \frac{i_{1year}}{1+i_{1year}}$

Solve: $X \cdot \frac{(1+i_{1m})^{\overset{1\%}{60}} - 1}{\underset{1\%}{i_{1m}}} = \frac{5000}{d_{1\text{-}year}}$

$i_{1m} = 1\% \Rightarrow (1+i_{1m})^{12} = (1+i_{1year})$

$\Rightarrow i_{1year} = 12.6825\%$

where $d_{1\text{-}year} = \frac{i_{1year}}{1+i_{1year}}$, $(1+i_{1year}) = (1+1\%)^{12}$

$= 11.25508\%$

$\Rightarrow 81.67X = 44424.39 \Rightarrow X \fallingdotseq 543.95$ Ⓓ//

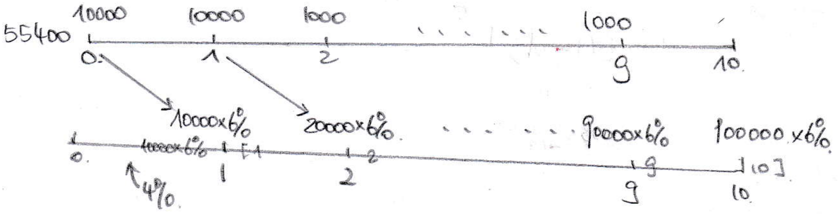
Exam FM Question 362 Reinvest: $I\ddot{s}_{\overline{n}|} = \frac{\ddot{s}-n}{i}$; $\ddot{s}_{\overline{n}|} = \frac{(1+i)^n-1}{d} = (1+i)\,s_{\overline{n}|}$; $d = \frac{i}{1+i}$

Give: (1) A payment of 55.400 made today returns payments of 10,000 at the beginning of each year for 10 years. There payments are deposited into an account that earns interest at an annual effective rate of 6% payable at the end of each year.

(2) The interest is immediately reinvested at an annual effective rate of 4%

(3) The original payment earns an annual effective yield rate of i over the 10-year period.

Question: What is i?



Solve: $AV_1 = 55400(1+i)^{10}$

$AV_2 = 10000 \times 10 + 10000 \times 6\%\ \underbrace{IS_{\overline{10}|4\%}}_{\frac{\ddot{s}_{\overline{n}|4\%}-10}{4\%}}$ where $\underbrace{\ddot{s}_{\overline{10}|4\%}}_{12.486} = \frac{(1+4\%)^{10}-1}{d}$, $d = \frac{4\%}{1+4\%}$

$\underbrace{\frac{\ddot{s}_{\overline{n}|4\%}-10}{4\%}}_{62.158}$

$\cong 100000 + 37295.27$

$\cong 137295.27$

$\because AV_1 = AV_2$

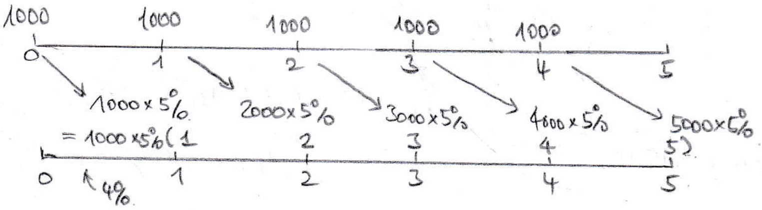
$\therefore \Rightarrow (1+i)^{10} = \frac{137295.27}{55400} = 2.478 \Rightarrow i \cong 9.5\%$ Ⓑ

Exam FM Question 363

Give: (1) An investor deposits 1000 at the beginning of each year for 5 years in a fund earning a 5% annual effective interest rate.

(2) The interest from this fund can be reinvested at a 4% annual rate

Question: What is the total accumulated value at the end of 5 years.



Solve: Total AV = $\underbrace{1000 \times 5}_{\#1} + \underbrace{1000 \times 5\% \times IS_{\overline{5}|4\%}}_{\substack{\#2.\\ \cong 791.50.}}$ where $\overset{15.83.}{IS_{\overline{5}|4\%}} = \frac{\ddot{s}_{\overline{5}|4\%} - 5}{4\%}$, $\ddot{s}_{\overline{5}|4\%} = \frac{(1+4\%)^5-1}{d}$

$\cong 5791.50$ Ⓔ

$d = \frac{4\%}{1+4\%} \cong 0.03846.$

Exam FM Question 364 total payments: n ; total interest paid: $n - a_{\overline{n}|}$; principal repaid total : $a_{\overline{n}|}$;

Give: Consider an amortization schedule for a loan at interest rate i per period, $i>0$, being repaid with payments of 1 at the end of each period for n periods. principal repaid at t: v^{n-t+1}

Question: Determine which of the following statements about this schedule is true.

(A) The total interest paid equals $n - a_{\overline{n}|}$

(B) The total interest paid equals ~~$i a_{\overline{n}|}$~~ $n - a_{\overline{n}|}$

(C) The total principal repaid equals ~~$n - i a_{\overline{n}|}$~~ $a_{\overline{n}|}$

(D) The principal repaid in payment t equals ~~v^{n-t}~~ v^{n-t-1}

(E) The total payment amount equals ~~$a_{\overline{n}|}$~~ n.

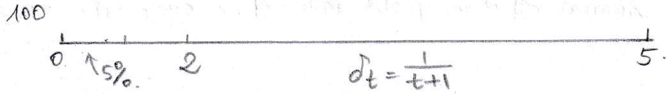
Exam FM Question 365

Give: (1) A loan of 100 is to have all principal and accrued interest paid at the end of five years.

(2) Interest accrues at an annual effective rate of 5% for the first two years.

(3) force of Interest at time t in years (t>2) of $\delta_t = \frac{1}{1+t}$

Question: What is the equivalent annual effective rate of discount for the five year period?



Solve: $100(1+5\%)^2 \times e^{\int_2^5 \frac{1}{1+t} dt} = \underbrace{100\, [\underbrace{(1-d)^{-1}}_{1\text{-year}}]^{5}}_{100(1-d)^{-5}}$ ← 5-years.

$\Rightarrow 100(1+5\%)^2 \times e^{\underbrace{\ln 6 - \ln 3}_{\frac{6}{3}}}$

$\Rightarrow 100(1+5\%)^2 \times \frac{6}{3} = 100(1-d)^{-5} \Rightarrow 1-d \cong 0.8537 \Rightarrow d \cong 0.1463$ Ⓒ//

Exam FM Question 366.

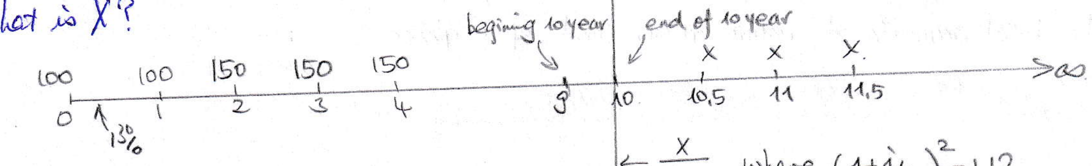
Give: ① The following deposits are made into a fund at the beginning of each year:

Year	Deposit
1	100
2	100
3	150
4	150
5	150

② The fund earns effective rate 13%

③ At the end of the 10th year, the fund is used to purchase a perpetuity-immediate with semi-annual paymets of X. The perpetuity earns an annual effective rate of 12%.

Question: What is X?



$\leftarrow \frac{X}{i_{6m}}$ where $(1+i_{6m})^2 = 1.12$. ②

$\Rightarrow i_{6m} = 5.83\%$

① $100(1+i)^{10} + 100(1+i)^9 + 150(1+i)^8 + 150(1+i)^7 \longrightarrow$

$+150(1+i)^6$ where $i = 13\%$

or: $(100\,\ddot{s}_{\overline{5}|13\%} + 50\,\ddot{s}_{\overline{3}|13\%})(1+13\%)^5$

Solve: $\underbrace{\underbrace{100(1+13\%)^{10}}_{339.456} + \underbrace{100(1+13\%)^9}_{300.404} + \underbrace{150(1+13\%)^8}_{398.766} + \underbrace{150(1+13\%)^7}_{352.891} + \underbrace{150(1+13\%)^6}_{312.293}}_{1703.34}$

$= \frac{X}{i_{6m}}$ ← 5.83%

$\Rightarrow 1703.34 = \frac{X}{5.83\%} \Rightarrow X \cong 99.304$ Ⓐ//

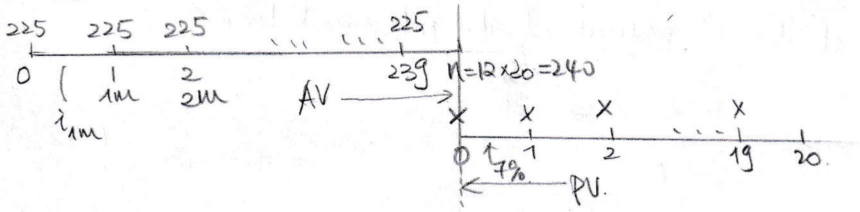
Exam FM Question 367

$\ddot{s}_{\overline{n}|} = \frac{(1+i)^n - 1}{d}$, $\ddot{a}_{\overline{n}|} = \frac{1-v^n}{d}$, $d = \frac{i}{1+i}$

Give:
(1) An investor deposits 225 into a bank at the beginning of each month for 20 years. The bank pays interest at an annual rate 8%.

(2) At the end of 20 years, the investor uses the money in the bank to buy a 30-year annuity-due with annual payments of X. These annual payments are based on an annual effective rate of 7%.

Question: What is X?



0.006434

Solve. $$\begin{cases} AV = 225\,\ddot{s}_{\overline{240}|i_{1m}} \text{ where } \ddot{s}_{\overline{240}|i_{1m}} = \frac{(1+i_{1m})^{240}-1}{d_{1m}}, \ d_{1m} = \frac{i_{1m}}{1+i_{1m}}, \ (1+i_{1m})^{12} = 1.08 \\ PV = X \cdot \ddot{a}_{\overline{20}|7\%} \text{ where } \ddot{a}_{\overline{20}|7\%} = \frac{1-v_{7\%}^n}{d} \text{ where } v_{7\%} = \frac{1}{1+7\%}, \ d = \frac{7\%}{1+7\%} \end{cases}$$

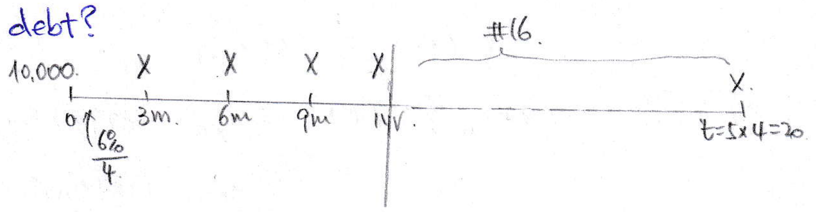
$$\Rightarrow 225\,\ddot{s}_{\overline{240}|0.6434\%} = X \cdot \ddot{a}_{\overline{20}|7\%} \Rightarrow X \cong 9706.99 \text{ Ⓑ}$$

Exam FM Question 368

Give:
(1) Harry borrows 10.000 from Sally and agrees to repay the five-year loan with equal payments at the end of each quarter. Interest on the loan is charged at an annual rate of 6% convertible quarterly.

(2) Immediately after the fourth payment, Harry and Sally get married and Sally forgives the remaing debt

Question: What is the total interest payments the Sally forgoes receiving by having forgiven the remaining debt?



Solve: $$\text{Total Interest Payments} = \underbrace{\text{Future level Payment (Total)}}_{16X} - \text{Outstanding}_{t=1\text{year}}$$

where: $$\begin{cases} \text{Outstanding}_{t=1\text{year}} = X \cdot a_{\overline{16}|\frac{6\%}{4}} \Rightarrow \text{Outstanding}_{t=1\text{year}} = 8230.858 \\ 10000 = X\,a_{\overline{20}|\frac{6\%}{4}} \Rightarrow X \cong 582.45 \end{cases}$$

$$\text{Thus. total Interest Payments} = 16 \cdot \overset{582.45}{X} - 8230.858 \cong 1088.342 \text{ Ⓒ}$$

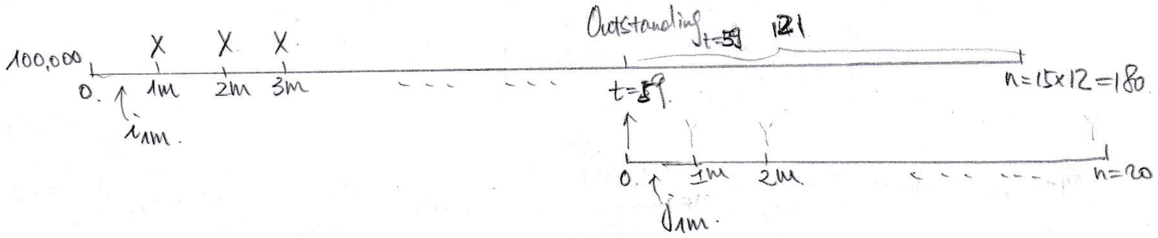
Exam FM Question 369

Give: ① A loan of 100,000 at an annual effective rate 9.12% is repaid with level payments at the end of each month over 15 years.

② Immediately after the 39th payment is made, the outstanding balance is refinanced at an annual effective rate of 5.76%.

③ The term of the refinanced loan is 20 years with level payments at the end of each month.

Question: What is the interest portion of the 1st payment of the refinanced loan?



$(1+i_{1m})^{12}=1.0912$

$(1+j_{1m})^{12}=1.0576$

Solve: $\underbrace{X a_{\overline{121}|}i_{1m}}_{80175.076} = \text{Outstanding}_{t=59}$, where $\underbrace{X a_{\overline{180}|}i_{1m} = 100.000}_{\Rightarrow X \cong 1000,039}$, $(1+\underbrace{i_{1m}}_{0.0073})^{12}=1.0912$, $(1+\underbrace{j_{1m}}_{0.0046777})^{12}=1.0576$.

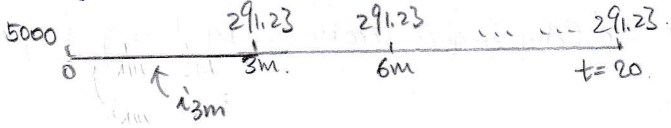
$\Rightarrow$ Interest Portion of the 1st payment $= \underbrace{\text{Outstanding}}_{80175.076} \times \underbrace{j_{1m}}_{0.0046777} \cong 375,035$ Ⓐ

Exam FM Question 370.

Give: ① 5000 is borrowed at an annual nominal rate of interest of j convertible monthly.

② Loan is repaid with payments of 291.23 at the end of each quarter for 5 years.

Question: What is j?



① $i_{1m} = \frac{j}{12}$.

② $(1+i_{1m})^3 = 1+i_{3m}$

Solve: $5000 = 291.23\, a_{\overline{20}|}i_{3m} \Rightarrow i_{3m} \cong 1.5\% \Rightarrow (1+i_{1m})^3 \cong 1.015 \Rightarrow i_{1m} \cong 0.004975$

$\Rightarrow i_{year} = 12 \times i_{1m} \cong 0.0597$ Ⓐ

Exam FM Question 371

The prices for four 1000 face amount zero-coupon bonds are as follows.

Price	Term (in years)
943.40	1
747.26	5
558.39	10
311.80	20

Determine which of the following statements describes the yield curve underlying these prices.

(A) The yield curve is increasing with constant slope.

✓(B) The yield curve is flat.

(C) The yield curve is inverted with constan slope.

(D) The yield curve is concave upward.

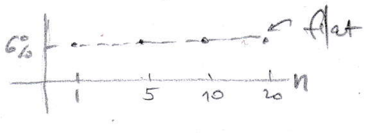
(E) The yield curve is concave downward.

Solve: (1) $943.40 = 1000\frac{1}{(1+i)} \Rightarrow i \cong 6\%$

(2) $747.26 = 1000\frac{1}{(1+j)^5} \Rightarrow j \cong 6\%$

(3) $558.39 = 1000\frac{1}{(1+m)^{10}} \Rightarrow m \cong 6\%$

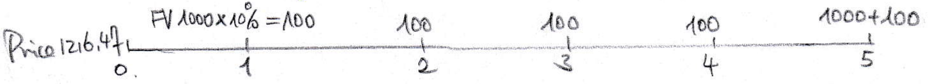
(4) $311.80 = 1000\frac{1}{(1+n)^{20}} \Rightarrow n \cong 6\%$



Exam FM Question 372

Give: A five-year 1000 face amount bond with 10% annual coupons is purchased at time of issue for 1216.47 based on an annual effective interest rate of 5%.

Question: What is the Macaulay duration?



$i = 5\%$.

Solve: $$D^{mac} = \frac{(100v)\times 1 + (100v^2)\times 2 + (100v^3)\times 3 + (100v^4)\times 4 + (1100v^5)\times 5}{1216.47}$$

$$= \frac{95.238 + 181.4059 + 259.15128 + 329.081 + 4309.3939}{1216.47} \cong 4.2535 \text{ Ⓔ}$$

Exam FM Question 373.

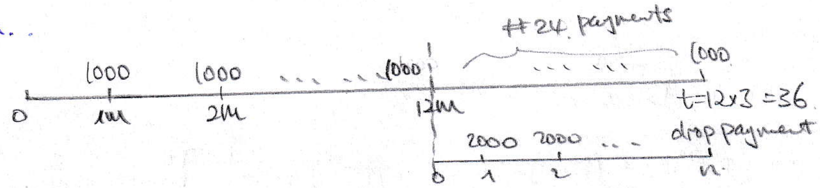
Give:

① A loan is originally scheduled to be paid in installments of 1000 payable at the end of each month for 3 years.

② The amortization is calculated with an annual nominal interest rate of 9%. convertible monthly.

③ After paying one full year of scheduled installment, the borrower increases monthly payments to 2000, resulting in a final drop payment.

Question: What is the number of months after the first year that it takes for the borrower to pay off the loan..



Solve: $$PV_{t=12m} = \boxed{1000\, a_{\overline{24}|i_{1m}}^{\frac{9\%}{12}} = 2000 \cdot a_{\overline{n}|i_{1m}}^{\frac{9\%}{12}}} \Rightarrow n = 11.46$$

∵ final payment is drop payment

∴ N=12 Ⓒ (11 level payments 2000, 12th drop payment P).

Exam FM Question 374.

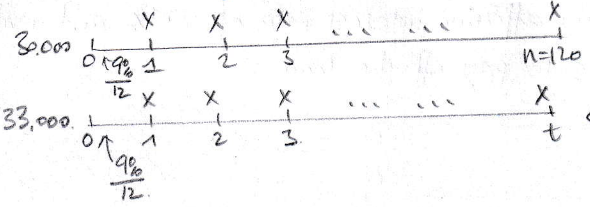
Give:

① A car dealership offers a 120-month loan for a blue car costing 30,000, with an annual nominal interest rate of 9% compounded monthly and level end-of-month payments.

② The dealership also offers a loan for a red car costing 33,000, with the same interest rate and end-of-month payments as for the loan for the blue car.

Question: What is the number of months needed to payoff loan for the red car?

Solve:



$$\Leftrightarrow X \cdot a_{\overline{120}|\frac{9\%}{12}} = 30{,}000 \Rightarrow X \cong 380.0273$$

$$\Leftrightarrow X \cdot a_{\overline{t}|\frac{9\%}{12}} = 33{,}000 \Rightarrow n \cong 140.9867 \cong 141 \text{ Ⓔ}$$

Exam FM Question 375

Give: (1) Two bonds have the same annual effective yield rate, where r>0. The bonds have Macaulay duration of 5 year and 6 year w.r.t r.

(2) One of the bonds has modified duration of 5.76 years while the other bond has modified duration of d years.

Question: what is d?

Solve: give: $D^{mac}_{Bond 1} = 5$, $D^{mac}_{Bond 2} = 6$

∵ $D^{mac} = (1+i)\, D^{MOD}$

∴ $D^{MOD} = 5.76$ must belong to Bond #2.

Thus: $\underbrace{D^{mac}_{Bond 2}}_{6} = (1+i)\underbrace{D^{MOD}}_{5.76} \Rightarrow i \cong 4.1667\%$

So: $D^{mod}_{Bond 1} = \frac{D^{mac}_{Bond 1}}{1+i} = \frac{5}{1.041667} \cong 4.8$ Ⓑ

Exam FM Question 376

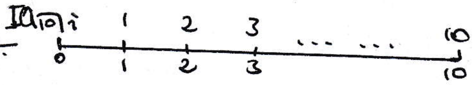
Give: (1) The modified duration of a ten-year zero-coupon bond with an annual effective yield rate of i is 9.26 years.

(2) Based on an annual rate of i, the Macaulay duration of a ten-year annuity-immediate with level annual payments is D

Question: What is D?

Solve: (1) $D^{mac} = \boxed{10 = (1+i)\, D^{mod.}_{\nwarrow 9.26}} \Rightarrow i \cong 8\%$

(2). $D = \frac{(\cancel{D}v)\times 1 + (\cancel{D}v^2)\times 2 + (\cancel{D}v^3)\times 3 + \cdots + (\cancel{D}v^{10})\times 10}{\cancel{D}\cdot a_{\overline{10|}i \nwarrow 8\%}} \leftarrow$



$= \frac{Ia_{\overline{10|}i \swarrow 8\%}}{a_{\overline{10|}i \swarrow 8\%}}$

$\ddot{a}_{\overline{10|}8\%} = (1+8\%)\underbrace{a_{\overline{10|}8\%}}_{6.71}$

$= \frac{\frac{\ddot{a}_{\overline{10|}8\%} - 10v^{10}_{8\%}}{i=8\%}}{\underbrace{a_{\overline{10|}8\%}}_{6.71}} = \frac{\frac{7.2468-4.632}{0.08}}{6.71} \cong 4.87$ Ⓓ

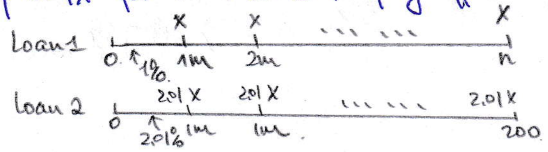
Exam FM Question 377

Give: (1) A bank offers a loan to each of two borrowers with different credit scores. Both loans are for the same amount

(2) The first borrower is charged a monthly effective interest rate 1% and makes level end-of-month payments of X for n months to pay off the loan.

(3) The second borrower is charged a monthly effective interest rate of 2.01% and makes level end-of-month payments of 2.01X for 200 months to pay off the loan.

Question: What is n?



Solve: $\cancel{X} a_{\overline{n|}1\%} = 2.01\cancel{X}\, a_{\overline{200|}2.01\%}$

$\Rightarrow n \cong 400$ Ⓔ

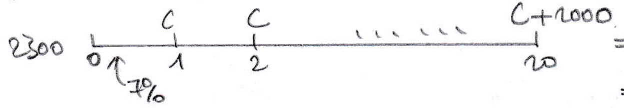
Exam FM Question 378.

Give: (1) A 20-year non-callable bond that pays coupons annually has a face amount of 2000. The bond was bought at a price of 2300 and has an annual effective yield rate of 7%.

(2) A 20-year callable bond with the same annual coupon rate and face amount is callable for 2000 at the end of 18th or 19th year.

Question: What is the maximum price of the callable bond that guarantees an annual effective yield of at least 7%?

Solve: non-callable



$\Rightarrow 2300 = C \cdot a_{\overline{20|}7\%} + 2000\, v_{7\%}^{20}$

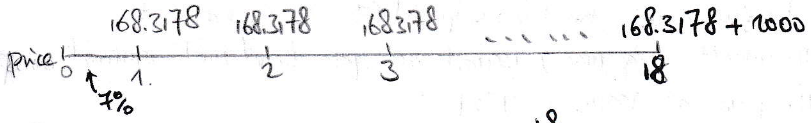
$\Rightarrow C \doteqdot 168.3178$

$\Rightarrow$ Coupon rate $= \frac{C}{2000} \doteqdot 8.416\%$

$\because$ Coupon rate $= 8.416\% > 7\% =$ yield rate

$\therefore \Rightarrow$ Premium case, to get "maximum price / lowest yield", should call at earlist.

$\Rightarrow$ callable:



Thus: $\text{Price} = \underbrace{168.3178\, a_{\overline{18|}7\%}}_{1693.1233} + \underbrace{2000\, v_{7\%}^{18}}_{591.7278}$

$\doteqdot 2284.85$ Ⓒ//

Exam FM Question 379

Give: (1) An investor deposits 50,000 into a new saving account, which earns an annual effective discount rate of 3.2%.

(2) The investor then withdraws an amount X at the end of every two-year period.

(3) The balance at the end of 12 years, just after the withdrawal is 45,000.

Question: Determine which of the following is an equation of value that can be used to solve for X?

(A) $\frac{45{,}000}{(1.032)^{12}} + X \sum_{k=1}^{7} \frac{1}{(1.032)^{2(k-1)}} = 50{,}000$

(B) $\frac{45{,}000}{(1.032)^{12}} + X \sum_{k=1}^{6} \frac{1}{(1.032)^{2k}} = 50{,}000$

✓ (C) $45{,}000(0.968)^{12} + X \sum_{k=1}^{6} (0.968)^{2k} = 50{,}000$

(D) $\frac{45000}{(1.032)^{12}} + X \sum_{k=1}^{6} (0.968)^{2k} = 50{,}000$

(E) $45{,}000(0.968)^{12} + X \sum_{k=1}^{6} \frac{1}{(1.032)^{2k}} = 50{,}000$

Solve. First: 50,000 is PV, so should use "discount factor".

Second: Disount factor $= \underbrace{(1 - \text{Disount rate}_{\uparrow 3.2\%})}_{0.968^{+}}{}^{+i}$

$\Rightarrow$ (C)

Exam FM Question 380.

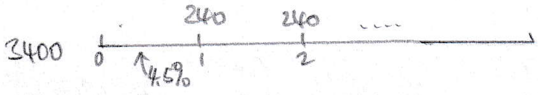
Give: ① Annual payments of 240 are made at the end of each year to repay a loan of 3400.

② The payments are based on an annual effective rate of 4.5%.

③ the loan is settled with a drop payment of X.

Question: What is X?

Solve:



First: $240\,a_{\overline{n}|4.5\%}=3400 \Rightarrow n \cong 23.0532.$

Second: ∵ final payment is a drop payment $\Rightarrow N=24$ {(23 level payments of 240. / 24th drop payment P.

$$\Rightarrow 3400=240\,a_{\overline{23}|4.5\%}+P\,v^{24}_{4.5\%} \Rightarrow P \cong 13.0398 \text{ Ⓔ}//$$

Exam FM Question 381. Premium Case: minimum yield ⇔ call at earlist

Give: You have decided to invest in a newly issued 20-year bond with annual coupons and the following characteristics:

i) The price at issue is 1321

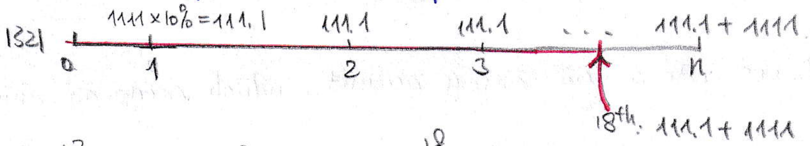
ii) the face amount is 1111 ← no mention, then redemption value = 1111

} ⇒ Premium Case (Price > redemption value)

iii) The annual coupon rate is 10%

iv) The bond is callable for 1111 imediately after the payment of either the 18th or 19th coupon

Question: What is the "minimum possible annual yield rate"?



Solve: $\underbrace{1321}_{-PV}=\underbrace{111.1}_{AMT}\,a_{\underbrace{\overline{18}|}_{N}i}+\underbrace{1111}_{FV}\,v_i^{18}$

$\Rightarrow i=7.9852\%$ Ⓒ//

Liability's CFs

Exam FM Question 382 $\boxed{\text{Fully immunize}}$: (1) PV the same (2) Duration the same (3) Asset's CFs happen before & after ✓

Give: A company would to be protected from large interest rate changes. It has a liability of 5000 due at time 3. and three possible sets of asset cash flows:

I. $A_1=1511.72$, $A_4=3500.00$

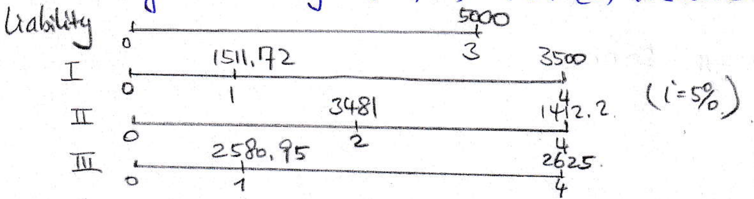
II $A_2=3481.00$, $A_4=1412.20$.

III $A_1=2580.95$. $A_4=2625.00$. where A_t is the asset cash flow at time t.

The effective interest rate is 5% per time period

Determine which set(s) of asset cash flows full immunize the liability.

Question: [✓ (A)] I only (B) II only (C) III only (D) I, II, and III (E) The correct answer is not given by (A), (B), (C), (D)



Solve: I ✓ {Check PV ✓: $1151.72(1+i)^2+3500(1+i)^{-1}=5000$ (i=5%) ✓

Check D^{mac} ✓: $2\times1151.72\times(1+i)-3500(1+i)^{-2}=0$ ✓

Check ✓ Asset's CFs happen before & after Liability's CF (single)

II {Check PV: $3481(1+i)+1412.2(1+i)^{-1}=5000$ ✓

Check D^{mac}: $3481-1412.2(1+i)^{-2}=0$ X

III {Check PV: $2580.95(1+i)^2+2625(1+i)^{-1}=5000$ X

Check D^{mac}: $2\times2580.95(1+i)-2625(1+i)^{-2}=0$ X

Exam FM Question 383.

Note: (1) if only 1 CF at t=5 ⇒ $D^{mac}=5$
(2) 2 CFs, one at t=0, one at t=5 ⇒ $D^{mac}\neq 5$

Give: A company's liabilities are 20,000 today and 100,000 five years from today. The Macaulay duration of the company's liabilities w.r.t. the market annual effective yield rate is 3.7 years. ⇔ $D^{mac}=3.7$

Question: What is the modified duration of the company's liabilities, in years.

Solve: $D^{mac} = \frac{(100{,}000\,v^5)\times 5}{20{,}000+(100{,}000v^5)} = 3.7 \Rightarrow v^5 = 0.56923 \Rightarrow v \cong 0.893424 \Rightarrow 1+i \cong 1.119289.$

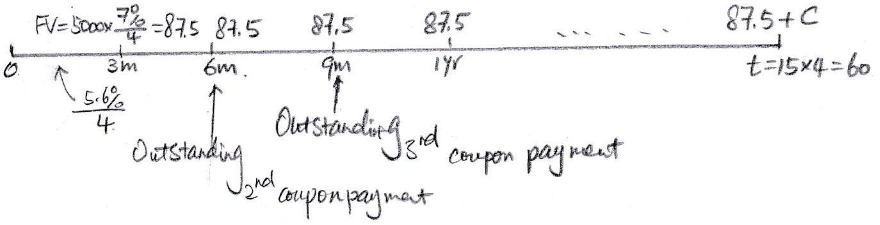
Thus: $\underset{\overset{\uparrow}{3.7}}{D^{mac}} = (1+i)\,D^{mod} \Rightarrow D^{mod} = \frac{3.7}{1+i} = \frac{3.7}{1.119289} \doteqdot 3.3057$ Ⓑ

Exam FM Question 384.

Amortization Amt in 3rd coupon payment = $\text{Outstanding}_{2^{nd}\text{ payment}} - \text{Outstanding}_{3^{rd}\text{ payment}}$

Give: (1) A 15-year bond with a face amount of 5000, a redemption value of C, and an annual coupon rate of 7% paid quarterly is purchased at a price that yields an annual nominal rate of 5.6% convertible quarterly.
(2) the amount for amortization of premium in the third coupon payment is 6.88.

Question: What is C?



Solve: Amortization Amt in the 3rd coupon payment = 6.88

$= \text{Outstanding}_{2^{nd}\text{ coupon payment}} - \text{Outstanding}_{3^{rd}\text{ coupon payment}} = 6.88$

$= \left(\underbrace{87.5\,a_{\overline{58}|\frac{5.6\%}{4}}}_{3459.5218} + \underbrace{C\cdot v^{58}_{\frac{5.6\%}{4}}}_{0.4464765C}\right) - \left(\underbrace{87.5\,a_{\overline{57}|\frac{5.6\%}{4}}}_{3420.4551} + \underbrace{C\cdot v^{57}_{\frac{5.6\%}{4}}}_{0.452727176C}\right) = 6.88$ where $i_{3m}=1.4\%$

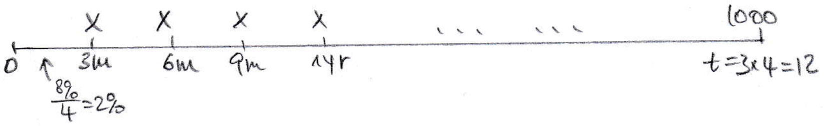
$39.0667 - 0.006251C = 6.88 \Rightarrow C \cong 5149.05$ Ⓐ

Exam FM Question 385

Give: ① A 3-year 1000 face amount bond pays coupons of X quarterly. It is bought at a price to yield an annual nominal rate of 8% convertible quarterly.
② If the amount of each coupon were doubled, the purchase price would have to increase by 500 for the bond to maintain the same yield rate.

Question: What is X?

Solve:



Ⓐ $X\,\underset{N}{a_{\overline{12}|}}\underset{2=I/Y}{2\%} + \underset{FV}{1000}\,v^{12} = \underset{-PV}{P}$

Ⓑ $2X\,a_{\overline{12}|2\%} + 1000\,v^{12} = P+50$

Ⓑ-Ⓐ: $X\,a_{\overline{12}|2\%} = 500 \Rightarrow X \cong 47.28$ Ⓔ