SOA and CAS: Exam FM[1]
Written Solutions: 319-325

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This document only provides written solutions to official example problems 319-325. For official sample questions, check out the official websites of Society of Actuaries and the Casualty Actuarial Society.

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Exam FM Question 319

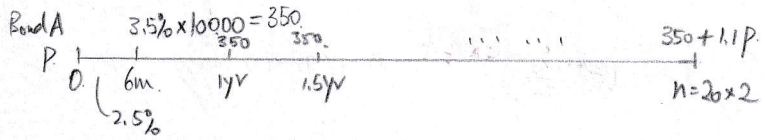
① Bond A and Bond B.

Give: ② Bond A: 20-year bond, 10000 face value, annual coupon rate 7% paid semi-annually, redemption value 10% higher than its price. Bond A is bought to yield an annual nominal interest rate of 5% convertible semi-annually.

③ Bond B: n-year bond with semi-anual coupons and same face value, redemption value, coupon rate. yield rate. The price of Bond B is 158.33 less than the price of Bond A.

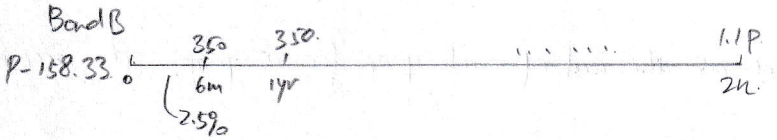
Question: What is n?

Solve:



$$P = 350\,a_{\overline{40}|2.5\%} + 1.1P\,v^{40}_{2.5\%} \Rightarrow P = 14883.24$$

($a_{\overline{40}|2.5\%} = 28.5597$, $v^{40}_{2.5\%} = 0.3724$)



$$14883.24 - 158.33 = 350\,a_{\overline{2n}|2.5\%} + 1.1P\,v^{2n}$$

$$14724.91 = 14000\left(1 - v^{2n}\right) + 16371.56\,v^{2n} \Rightarrow v^{2n} \cong 0.305668$$

$$\Leftrightarrow 2n = \frac{\ln(0.305668)}{\ln\left(\frac{1}{1.025}\right)} \Rightarrow 2n \cong 48 \Rightarrow n \cong 24$$

Exam FM Question 320

① An investor purchases two bonds having the same positive annual effective rate.

Give: ② One "modified duration" is a, Another "modified duration" is b. $(0<a<b)$

③ One of those bonds has a "$D^{mac} = d$" with $a<d<b$.

Question: Determine which of the following is the "D^{mac} for another bond".

(A) bd/a (B) ad/b (C) ab/d (D) $b+d-a$ (E) $a+d-b$.

Solve: Assume: Bond A: $d \leftarrow D_A^{mac} = (1+i)\,D_A^{mod.} \leftarrow a$ eq(A)

Bond B: $D_B^{mac} = (1+i)\,D_B^{mod.} \leftarrow b$ eq(B)

$\because D^{mac} = d$, where $d<b$. (if $D_B^{mac} = d$, then d must $> b$, since $d = (1+i) \times b$)

$\therefore D^{mac} = D_A^{mac} = d$

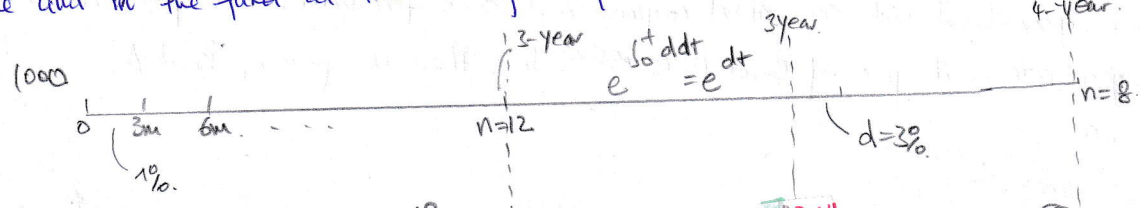
Thus: $\frac{eq(A)}{eq(B)} = \frac{d}{D_B^{mac}} = \frac{(1+i)a}{(1+i)b} \Rightarrow D_B^{mac} = \frac{db}{a} \Rightarrow (A)$ //

Exam FM Question 321

John deposits 1000 into a fund. The fund earns:

Give: (i) an annual nominal rate of interest of 4% convertible quarterly for the first 3-year.
(ii) a constant annual force of interest of 5% for the next 3-year.
(iii) an annual nominal discount rate of 6% convertible semi-annually thereafter.

Question: the amt in the fund at the end of 10-year



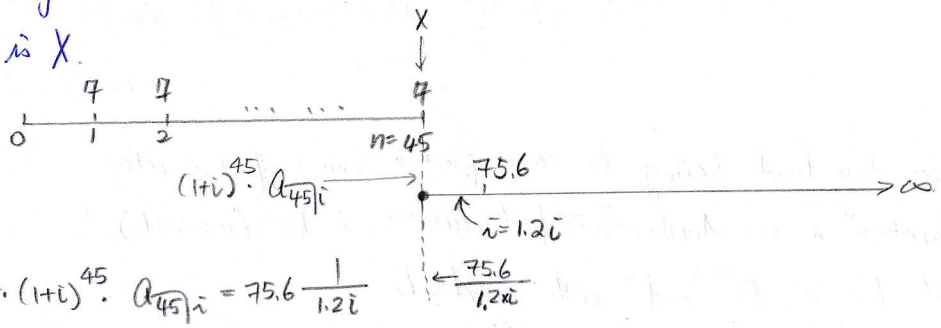
Solve: $AV_{t=10} = 1000(1+1\%)^{12} \times (\delta \times 3) \times \left(1 - \frac{6\%}{2}\right)^{2\times 4} \cong 1670.42$ (C)

Exam FM Question 322

Give: ① A payment of 7 is deposited into an account at the end of each year for 45-year. The account earns an annual rate i

② Immediately after the 45th deposit of 7, the account has accumulated to X, which is used to purchase a perpetuity-immediate. At an rate $1.2i$, the perpetuity will make annual payments of 75.6

Question: What is X.



Solve: $7 \cdot (1+i)^{45} \cdot a_{\overline{45}|i} = 75.6 \frac{1}{1.2i}$

$\Leftrightarrow 7 \cdot (1+i)^{45} \cdot \frac{1-v^{45}}{i} = 75.6 \cdot \frac{1}{1.2i} \Leftrightarrow 7(1+i)^{45} - (1+i)^{45} \cdot \frac{7}{(1+i)^{45}} = \frac{75.6}{1.2} \Leftrightarrow 7(1+i)^{45} = \frac{75.6}{1.2} + 7$

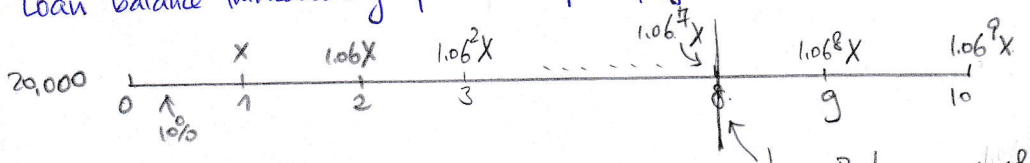
$\Rightarrow (1+i)^{45} = 10 \Rightarrow i = 5.25\%$

Thus: $AV = X = \frac{75.6}{1.2 \times 5.25\%} = 1200$ (C)

Exam FM Question 323

Give: ① A loan of 20,000 is to be paid at the end of year for 10-year.
② The first payment is X. Each subsequent payment is 6% greater than the preceding payment.
③ Loan payments are based on an annual effective rate of 10%.

Question: Loan balance immediately after the eight payment?



Solve First: get X: $20{,}000 = Xv + 1.06Xv^2 + 1.06^2Xv^3 + \dots + 1.06^9Xv^{10} = Xv \cdot \frac{1-(1.06v)^{10}}{1-1.06v} \Rightarrow X \cong 2584.14$

Then: Loan Balance $= (1.06^8 X) \cdot v + (1.06^9 X) \cdot v^2$ where $i = 10\%$.

$\cong 4352.42$ (C)

Exam FM Question 324

Give: force of interest: $\delta_t = \frac{1}{t+8}$

Question: What is the "annual effective rate" in "year 5"?

Solve: i in "year five" $\Leftrightarrow$ $(1+i) = e^{\int_4^5 \delta_t \, dt} = e^{\int_4^5 d\ln(8+t)} = e^{\ln(8+5) - \ln(8+4)} = e^{\ln\frac{13}{12}}$

$= \frac{13}{12}$

$\Rightarrow i = 8.3\%$ (D) //

Exam FM Question 325

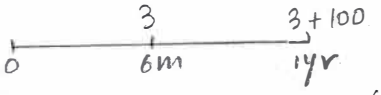
Give: ① An investor buys a one-year bond, Face value 100, annual coupon rate 6% paid semi-annually.

② Bond: purchased at a discount of 1.5 $\Leftrightarrow$ Price = Face Value − 1.5 = 98.5

③ Yield: j convertible semi-annually $\Leftrightarrow$ $j = 2 \times i_{6m}$.

Question: What is j?

Solve:



$\Leftrightarrow 98.5 = 3a_{\overline{2}|i_{6m}} + \frac{100}{100\,(FV)} v^2_{6m} \Rightarrow i_{6m} = 3.79\%$

Calculator: -98.5 (PV), 3 (PMT), n, 100 (FV)

Thus: $j = 2 \times i_{6m} \cong 7.586\%$ (D) //