SOA and CAS: Exam FM[1]
Written Solutions: 311-317

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This document only provides written solutions to official example problems 311-317. For official sample questions, check out the official websites of Society of Actuaries and the Casualty Actuarial Society.

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Exam FM Question 111

Give:

① A credit card company charges an annual rate 16.8%.

② Credit card balance was X at the beginning of month 1. Starting with month 1, the borrower purchased satellite internet service, resulting in 79.99 charge on the credit card at the middle of each month

③ borrower paid 250 at the end of each month, Immediate after payment in month 15, balance was 3000.

Question: Which of the following equation can be used to solve for X.

(A) $X + \sum_{n=1}^{15} \frac{79.99}{(1.014)^{n-0.5}} = \frac{3000}{(1.014)^{15}} + \sum_{n=1}^{15} \frac{250}{(1.014)^{n}}$

(B) $X + \sum_{n=1}^{16} \frac{79.99}{(1.168)^{\frac{n+0.5}{12}}} = \frac{3000}{(1.168)^{5/4}} + \sum_{n=1}^{16} \frac{250}{(1.168)^{\frac{n}{12}}}$

(C) $X + \sum_{n=1}^{16} \frac{79.99}{(1.168)^{\frac{n-0.5}{2}}} = \frac{3000}{(1.168)^{4/3}} + \sum_{n=1}^{16} \frac{250}{(1.168)^{n/12}}$

(D) $X + \sum_{n=1}^{15} \frac{79.99}{(1.168)^{\frac{n+0.5}{12}}} = \frac{3000}{(1.168)^{5/4}} + \sum_{n=1}^{15} \frac{250}{(1.168)^{n/12}}$

(E) $X + \sum_{n=1}^{15} \frac{79.99}{(1.168)^{\frac{n-0.5}{2}}} = \frac{3000}{(1.168)^{5/4}} + \sum_{n=1}^{15} \frac{250}{(1.168)^{n/12}}$

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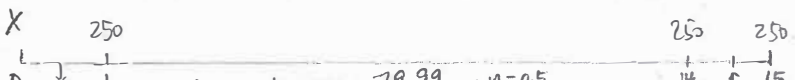


① 79.99 ← the first should be $\frac{79.99}{(1.014)^{0.5/12}}$ $n-0.5$ and last $\frac{79.99}{(1.014)^{14.5/12}}$ $n=14.5\ (n-0.5)$ ⇒ A or E ; ② last 250. : $\frac{250}{(1.168)^{15/12}}$ $n=15$. ⇒ (E)

Exam FM Question 312

Give: ① Deposits of 100, made today and one year from today.
② Force of interest at time t: $\delta_t = 0.03 + 0.005t \quad (0 \le t \le 3)$

Question: What is the account balance two years from today?

Solve: $AV = 100 \cdot e^{\int_0^2 (0.03+0.005t)dt} + 100e^{\int_1^2 (0.03+0.005t)\,dt}$

$= 100 \cdot e^{(0.03t + 0.005 \cdot \frac{t^2}{2})\Big|_0^2} + 100e^{(0.03t + 0.005\frac{t^2}{2})\Big|_1^2}$

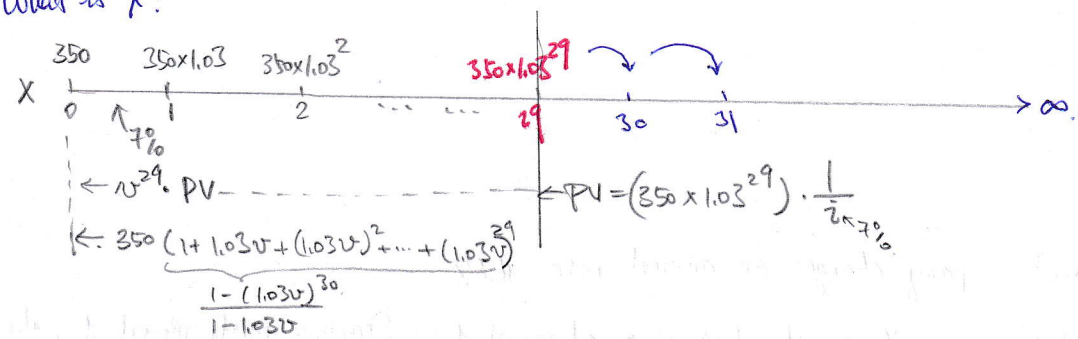
$e^{(0.03\times 2 + 0.005 \times \frac{4}{2}) - (0)}$ $\qquad$ $e^{(0.03^{\times 2} + 0.005 \times \frac{4}{2}) - (0.03\times 1 + 0.005 \times \frac{1}{2})}$

$\cong 211.072$ (A)

Exam FM Question 313

Give: ① Perpetuity-due price at X, annual rate 7%
② First payment 350, each payment 3% larger than the previous payment
③ Starting with the 31st payment, each payment is equal to the 30th payment

Question: What is X?



Solve: $X = 350 \times \frac{1-(1.03v)^{30}}{1-1.03v} + v^{29}\left(350 \times 1.03^{29} \times \frac{1}{i}\right)$ where $i = \frac{1}{7\%}$, $v = \frac{1}{1+i}$

$= 6376.79875 + 1656.22741 \cong 8033.02616$ (E)

Exam FM Question 314

Give: ① A student takes out a loan for 30,000, i = 9% convertible semi-annually.
② Student pays off loan in 5-year, monthly payment beginning one month from today.
③ First payment is 500, each subsequent is X more than the previous one.

Question: Which of the following can be used to solve for X?

(A) $30{,}000 = \sum_{n=0}^{60} \frac{500+nX}{(1.015)^{n/2}}$

(B) $30{,}000 = \sum_{n=0}^{60} \frac{500+nX}{(1.045)^{n/6}}$

(C) $30{,}000 = \sum_{n=1}^{60} \frac{500+nX}{(1.045)^{n/6}}$

(D) $30{,}000 = \sum_{n=1}^{60} \frac{500+(n-1)X}{(1.015)^{n/2}}$

(E) $30{,}000 = \sum_{n=1}^{60} \frac{500+(n-1)X}{(1.045)^{n/6}}$

Solve:
① i_{1m}: $(1+i_{1m})^6 = \frac{9\%}{2}$: first 500 should be: $\frac{500}{1.045^{1/6}}$ with $n=1$
$\Rightarrow$ C or E.

② X starts at $t=2m$, thus, can only has $(n-1)X$. $\Rightarrow$ (E)

Exam FM Question 315

Give: ① Each bonds A, B, C all sells for 10,000, has the same annual effective yield rate, same term n.

② The par values and coupon rate are shown below

	Bond A	Bond B	Bond C
Par Value	20,000	10,835.58	X
Annual Coupon rate	0%	4%	3%

③ Each bond redeemed at par. coupon paid annually

Question: What is X?

Solve: Bond A: 10,000 at 0, 20,000 at n

$10{,}000 = 20{,}000 \cdot v^n \Rightarrow v^n = \frac{1}{2}$

Bond B: 10,000 at 0; $10835.58 \times 4\% = 433.4232$ at 1, 2, ..., n; $433.4232 + 10835.58$ at n

$10000 = 433.4232\, a_{\overline{n}|i} + 10835.58\, v^n \Rightarrow a_{\overline{n}|} \cong 10.5721$

Bond C: 10,000; $X \cdot 3\%$ at 1, 2, ...; $X \cdot 3\% + X$ at n

$10{,}000 = (X \cdot 3\%)\, a_{\overline{n}|i} + X \cdot v^n = X \times \left(3\% \times a_{\overline{n}|} + v^n\right)$

($a_{\overline{n}|} \approx 10.5721$, $v^n = \frac{1}{2}$)

$\Rightarrow X \cong 12237.51$ (A)

Exam FM Question 316

Give: ① Perpetuity-immediate with annual payments consists of 10 level payments of K. followed by a series of increasing payments.

② Beginning with the 11th payment. each payment is 200 larger than the preceding payment.

③ Annual rate 5.2%. PV of the perpetuity is 50,000.

Question: What is K?

50,000 at 0; K at 1, 2, ..., 10; K+200 at 11; K+200×2 at 12; K+200×3 at 13; ... ∞

5.2%

K at 1, 2, 3, ... ∞ : $K \cdot \frac{1}{i} = a_{\overline{\infty}|}$

+ 200 at 1, 2, 3, ... ∞ : $200 \cdot \left(\frac{1}{i} + \frac{1}{i^2}\right) = I a_{\overline{\infty}|}$

$PV = K \cdot \frac{1}{i} + 200\left(\frac{1}{i} + \frac{1}{i^2}\right)$

Solve: $50{,}000 = K \cdot \underbrace{a_{\overline{10}|5.2\%}}_{7.64728} + \underbrace{v^{10}}_{0.60234}\left(K \cdot \frac{1}{i} + 200\left(\frac{1}{i} + \frac{1}{i^2}\right)\right)$ where $i = 5.2\%$.

$\Rightarrow K \cong 162.85$ (C)

Exam FM Question 317.

Given: ① Bank issues two 30-year bonds: A and B, each with annual coupons.
② Yield = 7%. Face Amount = 1000. Total Price of two bonds = 3000.
③ Bond's B annual coupon rate = Bond's A annual coupon rate + 0.5%.

Question: What is annual coupon rate of Bond A.

Solve: Bond A: Price assumed to be "X"

Bond B: Price assumed to be "3000 − X"

$i = 7\%$.

(A) $(1000 \times r)\, a_{\overline{30}|7\%} + 1000 v^{30} = X$

(B) $[1000 \times (r + 0.5\%)]\, a_{\overline{30}|7\%} + 1000 v^{30} = 3000 - X$

$\Rightarrow$ (A) + (B) $1000(2r + 0.5\%) \cdot a_{\overline{30}|7\%} + 2000 \underbrace{v^{30}}_{(\frac{1}{1.07})^{30}} = 3000$ where $i = 7\%$

$\Rightarrow (2r + 0.5\%)\, a_{\overline{30}|7\%} = 2.737266 \Rightarrow 2r + 0.5\% = 0.220586 \Rightarrow r = 10.7793\%$. (E)