SOA and CAS: Exam FM[1]
Written Solutions: 261-309

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January 14, 2024

This document only provides written solutions to official example problems 261-309. For official sample questions, check out the official websites of Society of Actuaries and the Casualty Actuarial Society.

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[2] Email: liyifinhub@outlook.com The written solutions were drafted when I was preparing for the exam. Please email me if you find any errors. My personal website: `https://yilifinhub.com/`

Exam FM Question 261

Give

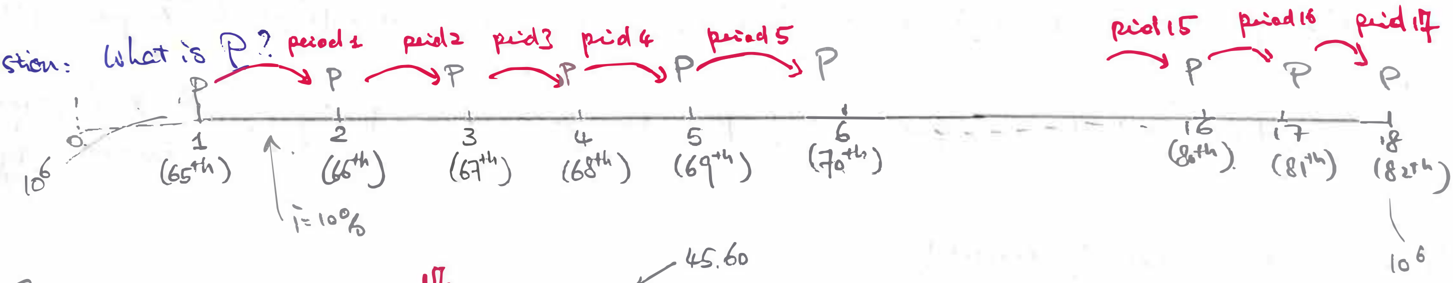
① 65th birthday, has a fund of 10^6

② withdraw P at 65th birthday & each birthday after

③ 82th birthday, fund value $=10^6$

④ $i=10\%$

Question: What is P?



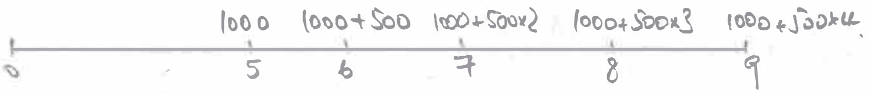
Solve:

$$\underbrace{10^6 \times (1+10\%)^{17}}_{\text{AV of Fund at 82th}} = \underbrace{P \cdot \overset{45.60}{a_{\overline{18}|10\%}}}_{\text{AV of P}} + \underbrace{10^6}_{\text{fund value at 82th birthday}}$$

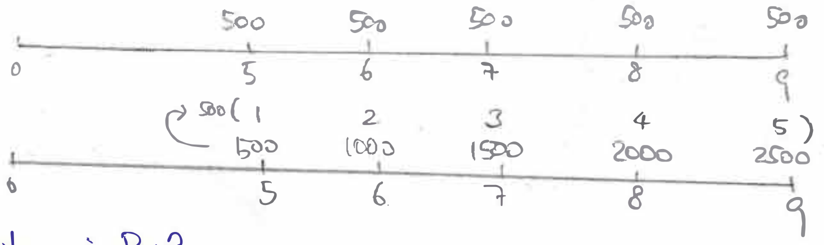
$\Rightarrow P \cong 88913.82$ Ⓑ//

Exam FM Question 262

Give



From Answer, we know to decompose into



$\leftarrow (500\ddot{a}_{\overline{5}|i}) \cdot v^5$

$\leftarrow (500\, I\ddot{a}_{\overline{5}|i}) \cdot v^5$

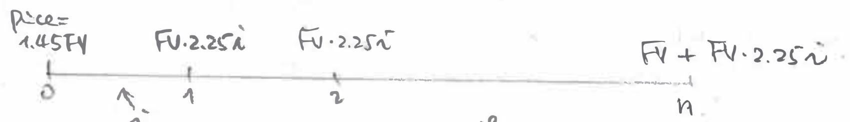
Question: What is PV?

$\Rightarrow$ Thus. $PV = (500\ddot{a}_{\overline{5}|i}) \times v^5 + (500\,(I\ddot{a}_{\overline{5}|i})) \times v^5$ Ⓓ//

Exam FM Question 263

Give ① 18-year bond

② n-year bond



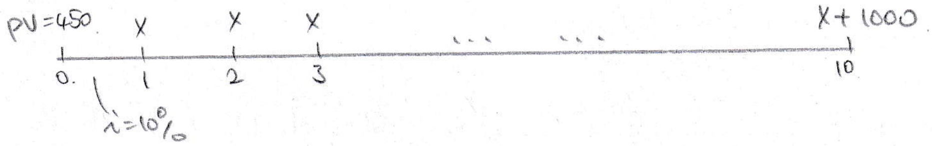
Question: What is n?

Solve. we knew: $1.61FV = (2.25 \times i \times FV) \cdot \overset{\frac{1-v^{18}}{i}}{a_{\overline{18}|i}} + (FV) \cdot v^{18} \Rightarrow 1.61 = 2.25(1-v^{18}) + v^{18} \Rightarrow v \cong 0.9635$

Then, we get: $1.45FV = (2.25 \times i \times FV) \cdot a_{\overline{n}|i} + FV \cdot v^n \Rightarrow v^n = 0.64 \Rightarrow n = \frac{\ln(0.64)}{\ln(v)} = 11.997$ Ⓑ//

Exam FM Question 264

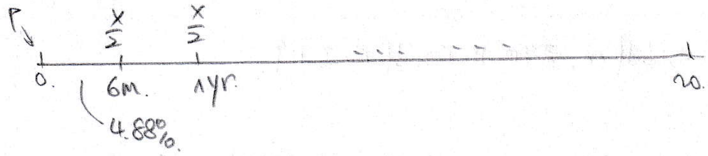
Give: ① 10-year, paying annual coupons of X, FV 1000, price 450, $i=10\%$



② 10-year, annual rate 10%, semi-coupon $\frac{X}{2}$, price "P"

(a) $(1+i_{6m})^2 = 1.1 \Rightarrow i_{6m} = 4.88\%$

(b)



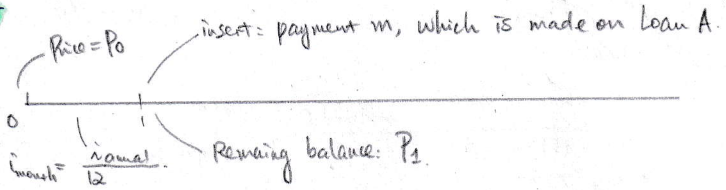
Question: What is P?

Solve: ① $450 = X \cdot a_{\overline{10}|10\%} + 1000 \cdot v^{10} \Rightarrow X \cong 10.49$

② $P = \underbrace{\left(\frac{X}{2}\right)}_{5.245} \cdot a_{\overline{20}|4.88\%} + 1000 \cdot v^{20} \Rightarrow P = 451.64$ Ⓒ//

Exam FM Question 265

Give: Loan A:



① $\text{Interest earn}_{\underset{1\text{month}}{\underline{(\text{during } t=0,\, t=1)}}} = \underbrace{AV_{\text{at } t=1}}_{P_1+m} - \underbrace{PV_{\text{at } t=0}}_{P_0} = (P_1+m) - P_0$

② $\text{Interest Rate}_{\underline{1\text{month}}} = \frac{\text{Interest earn}}{P_0} = \frac{(P_1+m)-P_0}{P_0}$

$\Rightarrow \text{Interest Rate}_{\underline{1\text{year}}} = 12 \times i_{1\text{month}} = 12 \cdot \frac{(P_1+m)-P_0}{P_0}$

Loan B: ① has the same "annual rate" as Loan A $\Leftrightarrow \text{Interest Rate}_{1\text{year}} = 12 \cdot \frac{(P_1+m)-P_0}{P_0}$

② covertible daily

Question: What is the j_{monthly} of "Loan B"

Solve: Loan B: Ⓐ $i_{\text{daily}} = \frac{i_{1\text{year}}}{365} = \frac{12}{365} \cdot \frac{(P_1+m)-P_0}{P_0}$

no monthly convertible

Ⓑ $(1+i_{\text{daily}})^{365} = 1 + i_{\text{year}} = (1+j_{\text{month}})^{12} \Rightarrow j_{\text{month}} = \left(1 + \frac{12}{365} \cdot \frac{(P_1+m)-P_0}{P_0}\right)^{\frac{12}{365}} - 1$ Ⓔ//

no mention how many days in a month

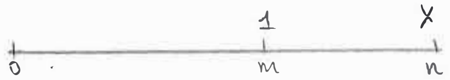
Exam FM Question 266

Give ① $P(0,t)$: current price of a zero-coupon bond, pay 1 at time t

② X: value at time n, of an investment of 1 made at m, where $m<n$

Determine X.

Solve:



they should have the same $PV_{t=0}$

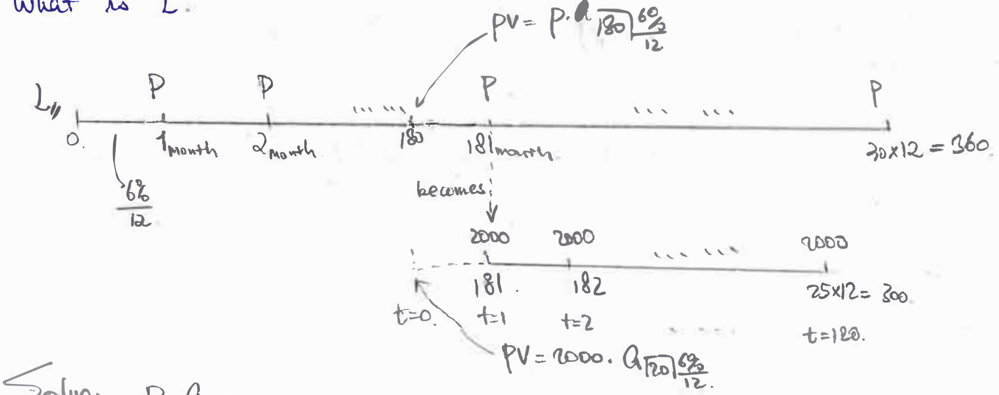
By ① definition: $1 \cdot P(0,m) = X \cdot P(0,n) \Rightarrow X = \dfrac{P(0,m)}{P(0,n)}$ Ⓓ//

Exam FM Question 267.

Give: ① Loan: Level payment (assume P), end of each month, 30-year, i annual 6%, convert monthly
L

② starting at 181st payment, monthly payment becomes 2000, loan can be repaid 5 years earlier.

Question: what is L.



Solve: $P \cdot a_{\overline{180}|\frac{6\%}{12}} = 2000 \cdot a_{\overline{120}|\frac{6\%}{12}} \Rightarrow P \approx 1520.18$

Then: $L = P \cdot a_{\overline{360}|\frac{6\%}{12}} \approx 253553.61$ Ⓐ//

Exam FM Question 268.

Give: (1) $PV_{asset} = PV_{liability}$

(2) $D^{mac}_{asset} = D^{mac}_{liability}$

(3) $C_{Asset} > C_{liability}$

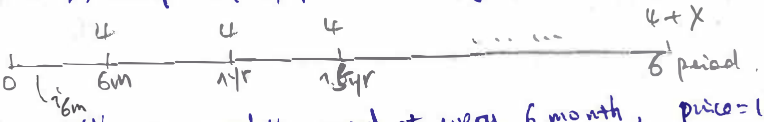
Question: which term precisely describes the company's position.

✓(E) Redington immunized

Exam FM Question 269

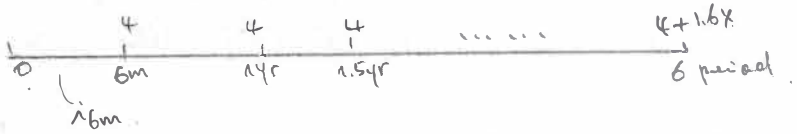
Give: ① 3-year, face amount X, coupon of 4, paid at every onths, price = 90.17

① price= 90.17



② 3-year, face value 1.6X, coupon of 4, paid at every 6 month, price = 132.47

② price=132.47



③ Same yield rate

Question: what is i_{year}, convert semi-annully

Solve: ①: $4a_{\overline{6}|i_{6m}} + X \cdot v_{6m}^{6} = 90.17$

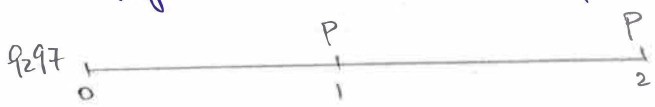
②: $4a_{\overline{6}|i_{6m}} + 1.6X \cdot v_{6m}^{6} = 132.47$ $\Big\} \Rightarrow$ ② − ① $\Rightarrow (X \cdot v_{6m}^{6}) = 70.5$ plug into ①

Then we have: $4a_{\overline{6}|i_{6m}} + 70.5 = 90.17 \Rightarrow \underline{a_{\overline{6}|i_{6m}} \doteq 4.92} \Rightarrow i_{6m} = 5.98\%$ $\Big\} \Rightarrow i_{year} \doteq 11.96\%$ Ⓔ//

$\because \frac{i_{year}}{2} = i_{6m}$

Exam FM Question 270.

Give: ① 2-year bond: level payment (assume: P) at the end of each year, price = 9297



② immunize $\Big\{$ 1-year zero-coupon bond

2-year zero coupon bond.

③ $i = 5\%$

Question: what is the \$ amount invest in 2-year zero-coupon bond today?

Solve: Assume: \$X invest in 1-year zero-coupon bond

\$Y invest in 2-year zero-coupon bond

immunize: $\left\{ \begin{array}{l} X + Y = 9297 \quad (PV) \\ \dfrac{\overbrace{(Pv)}^{4761.9} \times 1 + \overbrace{(Pv^2)}^{4535.15} \times 2}{9297} = X \times 1 + Y \times 2 \quad (D^{mac}) \end{array} \right.$

$X = 9297 - Y$

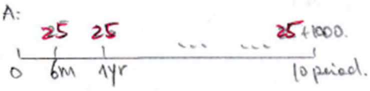
$\Big\} \Rightarrow Y = 4535.2$ Ⓑ//

where: $P \cdot a_{\overline{2}|5\%} = 9297 \Rightarrow P \doteq 5000.$

Exam FM Question 271

Give: Bond A: 3-year, FV=1000, annual coupon rate 5% semi-annually, price = P, $i_{year} = 4\%$

A:



$\Leftrightarrow$ $(1+i_{6m})^2 = 1.04 \Rightarrow i_{6m} = 1.98\%$ } $\Rightarrow P = 1046.76$

$P = 25 \cdot a_{\overline{10}|1.98\%} + 1000 v^{10}$ }

25 $\boxed{PMT}$ 10 $\boxed{N}$ 1.98 $\boxed{I/Y}$ 1000 $\boxed{FV}$ $\Rightarrow \boxed{PV} = -1046.76.$ { FV: positive; PV: negative

Bond B: 3-year, FV=1000, annual coupon 3%, price = P−100, i^{B}_{annual}

Question: What is i^{B}_{annual}

Solve: B: $P-100 = 30 \times a_{\overline{5}|i} + 1000 v^5 \Rightarrow i = 4.203\%.$

30 $\boxed{PMT}$ 5 $\boxed{N}$ 1000 $\boxed{FV}$ −946.76 $\boxed{PV}$ $\Rightarrow i = 4.203$

→ PV: negative
→ FV: positive
→ i: contains %.

Exam FM Question 272

(注意是 "Ballon Payment", 不是 "drop payment")

Give: ① Loan: 100,000, $i_{annual} = 8\%$ convert quarterly

② Quarterly Payment, first 5-year each period will be 2500, afterthat, 5000 except final "Ballon payment" (which is less than 10,000)

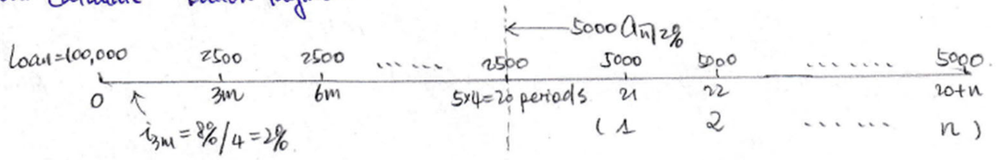
Question: Calculate "ballon Payment".

Exam FM Question 272

Give: ① Loan: 100,000, $i_{annual} = 8\%$ convert quarterly

② Quarterly Payment, first 5-year each period will be 2500, after-that, 5000 exept final "Ballon payment" (which is less than 10,000)

Question: Calculate "ballon Payment".



Solve. $100{,}000 = \underbrace{2500 \cdot a_{\overline{20}|2\%}}_{40878.58} + v^{20}_{3m} \cdot (5000 \cdot a_{\overline{n}|2\%}) \Rightarrow a_{\overline{n}|2\%} = 17.563 \Rightarrow n = 21.85$ where $i_{3m} = 2\%$

∵ Ballon Payment

∴ n=21: first n=20 payments: 5000, last n=21th payment > 5000

Thus: re-write: $100{,}000 = 2500 \cdot a_{\overline{20}|2\%} + v^{20}_{3m} \cdot (5000 \cdot a_{\overline{20}|2\%}) + v^{21}_{3m} \cdot P$ where $i_{3m} = 2\%$

$\Rightarrow P \cong 9231.64$ Ⓔ//

Exam FM Question 273

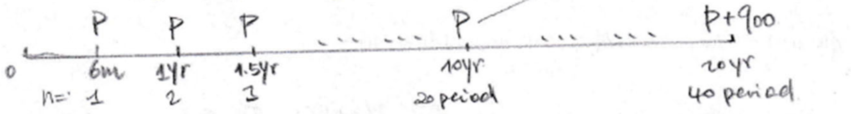
Give: ① investor purchases: 20-year, 1000 FV, semi-annual coupon, price = 900, redemption = 900
yield 10% convertible semi-annually

② After 10-year, investor sells the bond, annual nominal rate 8% convertible semi-annually
10-year, 1000 FV. redemption value 1100, semi-coupon.

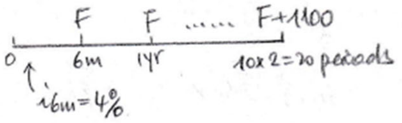
Question: semi-annual coupon payment of the new bond.

$PV = 45 a_{\overline{20}|4\%} + \underline{\underline{900 v_{6m}^{20}}}$

Solve:



$PV = F \cdot a_{\overline{20}|4\%} + 1100 v_{4\%}^{20}$



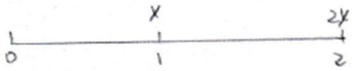
① $P a_{\overline{40}|5\%} + 900 v_{5\%}^{40} = 900 \xrightarrow[PM=-900]{FV=900} P = 45$

② $\underbrace{45 a_{\overline{20}|4\%} + 900 v_{6m\,(4\%)}^{20}}_{1022.31} = F \cdot a_{\overline{20}|4\%} + \underbrace{1100}_{FV} \cdot v_{6m\,(4\%)}^{20} \Rightarrow F = 38.28$ Ⓑ//

$(PV = -1022.31)$

Exam FM Question 274

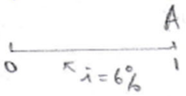
Give: ① Liability:



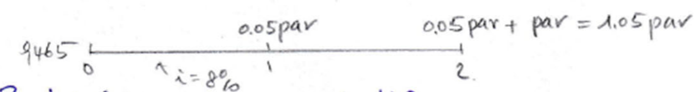
⇔ liablity due at the end of 2nd year is twice the 1st year

② Use: 2 bonds to match liabilities.

Bond 1: 1-year zero-coupon bond, par, $i_{effective} = 6\%$



Bond 2: 2-year, coupon rate 5%, par, $i_{effectiv} = 8\%$, price = 9465



Question: Price of Bond 1 ($ invest in one-year bond)?

Solve: $9465 = (0.05 \cdot par) v + (1.05 par) \cdot v^2$ where $v = \frac{1}{1.08}$

$\Rightarrow par = 10{,}000$

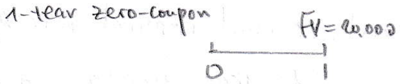
Comparing to t=2's liability: $2X = 1.05 \overset{10,000}{par} \Rightarrow X \doteq 5250$

Then. for t=1, $\overset{5250}{X} = A + 0.05 \overset{10,000}{par}$ (2-year bond) $\Rightarrow A \doteq 4750. \Rightarrow Price_{Bond 1} = \frac{A}{1.06} = 4481$ ©//

Exam FM Question 275

Give: ① Liability: single, due in 3-year

② Fully immunizes:



Question: Amount of liability

Solve. Fully immunize:

① $P_{Liability} = P_{Asset} \Leftrightarrow \boxed{liability = 20.000(1+i)^2 + 50.000(1+i)^{-1}}$ ← plug in.

② $P'_{liability} = P'_{Asset} \Leftrightarrow \boxed{\underbrace{liability'}_{0} = \underbrace{20,000(1+i)^{2'}}_{40.000(1+i)} + \underbrace{50000(1+i)^{-1'}}_{-50,000(1+i)^{-2}}} \Rightarrow (1+i)^{-3} = \underbrace{\frac{4}{5}}_{0.8} \Rightarrow \boxed{(1+i) = \underbrace{0.8^{\frac{1}{-3}}}_{1.07721}}$

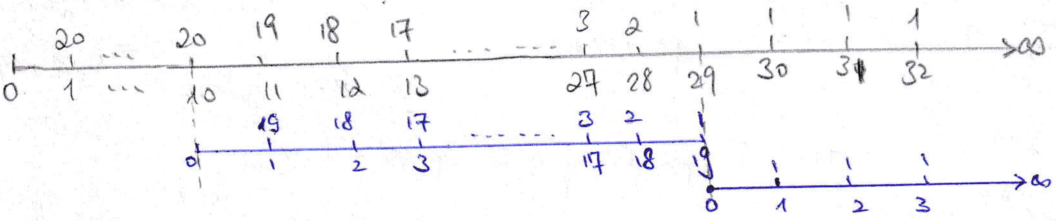
Then: ① becomes: $liability = 20.000 \times 1.07721^2 + 50.000 \times 1.07721^{-1} \Rightarrow liability \simeq 69623.83$ Ⓒ //

Exam FM Question 276

$Da_{\overline{n}|i} = \frac{n - a_{\overline{n}|}}{i}$

Give:
① Perpetuity - immediate, $i_{annual} = 6\%$, $PV = X$
② Pays 20 for 10-year, decreases 1 per year for 19-year, pays 1 thereafter

Question. X ?



Solve: $PV = X = 20 \cdot a_{\overline{10}|} 6\% + v^{10} \cdot (Da_{\overline{19}|} 6\%) + v^{29} (1 \cdot (\frac{1}{i}))$ where $i = 6\%$

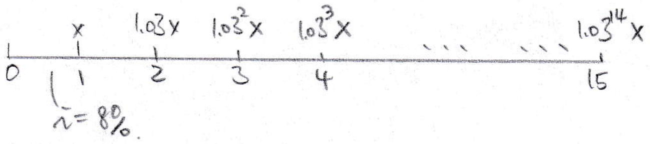
$= 147.20 + v^{10} \cdot \frac{\overset{19}{n} - a_{\overline{\overset{19}{n}}|}6\%}{i} + v^{29} \cdot \frac{1}{6\%} = 147.20 + 72.98 + 3.07 = 223.25$ Ⓓ

Exam FM Question 277

Give:
① Actuary's child: needs 2000, 15-year from now
② Deposit into a fund, for next 15-year, first deposit X, next deposit 3% larger than previous
③ $i = 8\%$

Question: What is X?

Solve:



$AV = \boxed{2000 = (1+8\%)^{15} \cdot PV}$ where $PV = Xv + 1.03Xv^2 + 1.03^2Xv^3 + \cdots + 1.03^{14}Xv^{15}$ where $i = 8\%$

$= Xv\,(1 + 1.03v + (1.03v)^2 + \cdots + (1.03v)^{14})$

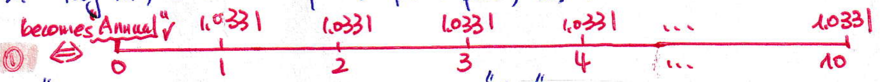
$= Xv \underbrace{\frac{1 - (1.03v)^{14} \cdot (1.03v)}{1 - 1.03v}}_{\frac{0.509}{0.046}} \nwarrow \frac{a_1 - a_n q}{1 - q} = 10.25X.$

Thus: $2000 = (1+8\%)^{15} \cdot (10.25X) \Rightarrow X \cong 61.52$ Ⓒ

Exam FM Question 278

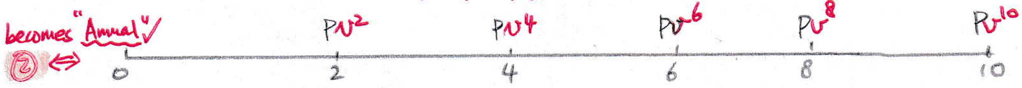
Give: Two annuities ① Annuity X: "Pay $\frac{1}{m}$, m times a year, for 10 year" & "AV at time 1 is 1.0331"

$PV = 1.0331\, a_{\overline{10}|} i_{annual}$



② Annuity Y: "Pay P, at the end of $\underset{\Leftrightarrow v^2,\ v^4,\ v^6,\ v^8,\ v^{10}}{\underline{2, 4, 6, 8, 10}}$" & "PV of Y = PV of X"

$PV = \underbrace{Pv^2_{annual} + Pv^4_{annual} + Pv^6_{annual} + Pv^8_{annual} + Pv^{10}_{annual}}_{等比數列}$



③ $s_{\overline{2}|} = 2.075$

Question: What is P?

Solve: $\boxed{1.0331\, \underbrace{a_{\overline{10}|i}}_{\frac{1 - v^{10}}{i}} = P\,\underbrace{(v^2 + v^4 + \cdots + v^{10})}_{\frac{v^2 - v^{12}}{1 - v^2}} \nwarrow \frac{a_1 - a_n q}{1 - q}}$ $\xrightarrow[③.1]{\text{change into }(1+i)} 1.0331 \frac{\cancel{(1+i)^{10} - 1}}{i \cdot (1+i)^{10}} = P \frac{\cancel{(1+i)^{10} - 1}}{(1+i)^{12} - (1+i)^{10}} \overset{③.2}{\Rightarrow} 1.0331 = P \cdot \underset{\uparrow \frac{1}{s_{\overline{2}|}}}{\boxed{\frac{i}{(1+i)^2 - 1}}} \Rightarrow P = 1.0331 \cdot \overset{2.075}{\overset{\downarrow}{s_{\overline{2}|}}} = 2.14$ Ⓔ

Exam FM Question 279.

Give: ① debt. 60 peiod, monthly payment, $i_{annual} = 12\%$

② Principal repay: in the 3^{rd} period = 900

Question: Principal repay in the 33^{rd} peiod?

Solve: Recall Table below:

t	Amt	Principal repay	Interest repay
1	1	v^n	$1-v^n$
2	1	v^{n-1}	$1-v^{n-1}$
3	$P \leftrightarrow 1$	$P\cdot v^{n-2}$	$1-v^{n-2}$
⋮	⋮	⋮	⋮
33	$P \leftrightarrow 1$	$P\cdot v^{n-32}$	$1-v^{n-32}$

eq(1) $\boxed{900 = P\cdot v_{1m}^{n-2}}$ where: $n=60$; $(1+i_m)^{12} = 1.12 \Rightarrow i_{1m} \cong 0.0095$

$\Rightarrow P \cong 1556.466$ $\qquad \Rightarrow v_{1m} \cong 0.9906$

Thus: Pricipal Repay in 33^{rd} peiod $= P\cdot \underset{0.9906}{v}{}_{1m}^{\overset{60}{n}-32}$

$= 1556.466 \times (0.9906^{28})$

$\cong 1194.79$ Ⓓ

Exam FM Question 280.

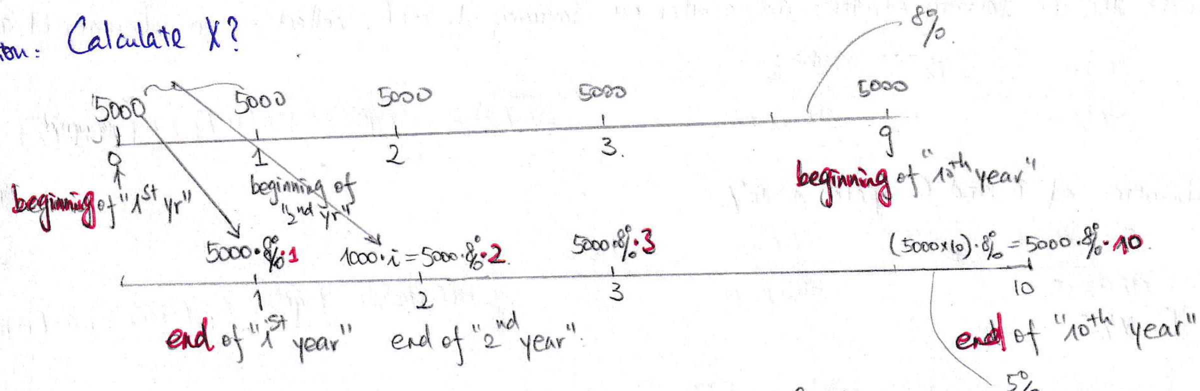
$IS_{\overline{10}|} = \frac{\ddot{s}-n}{i}$ or $(1+i)^{10} \cdot \frac{\ddot{a}_{\overline{10}|} - 10v^{10}}{i}$ where $\ddot{a}_{\overline{10}|} = (1+i)\cdot a_{\overline{10}|}$

Give: ① Beginning of year, receive 5000, which earns interest at 8%.

② End of year, " the interest of this Amt" is reinvest at 5%

③ AV of this fund at the end of 10-year is X

Question: Calculate X?



Solve: $X = \underbrace{5000 \times 10}_{50000} + 5000 \cdot 8\% \cdot IS_{\overline{10}|5\%}$

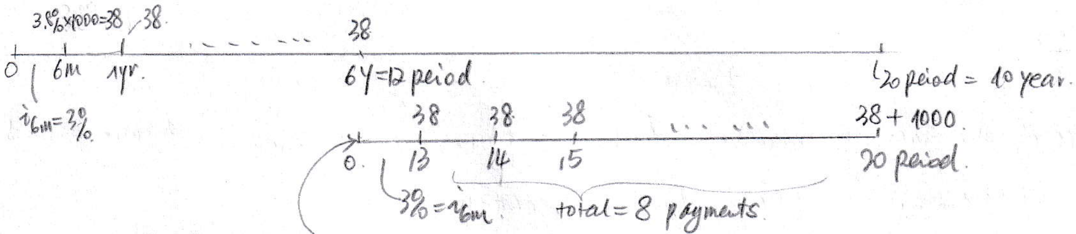
$$\frac{\ddot{s}_{\overline{10}|}-10}{i} \text{ or } \underbrace{(1+5\%)^{10} \cdot \frac{\overset{\frac{8.11-6.14}{5\%}}{\ddot{a}_{\overline{10}|} - 10 \cdot v^{10}_{5\%}}}{i \leftarrow 5\%}}_{64.18} \text{ where } \ddot{a}_{\overline{10}|5\%} = (1+5\%)\cdot a_{\overline{10}|5\%}$$

$= 50000 + 5000 \cdot 8\% \cdot 64.18 \cong 75671.38$ Ⓒ//

Exam FM Question 281.

Give: ① Bond: coupon semi-annual; r coupon rate 7.6% convertible semi-annually; 10-year; 1000 par value

Question: What is "Book Value" at the end of year 6 after the coupon payment?



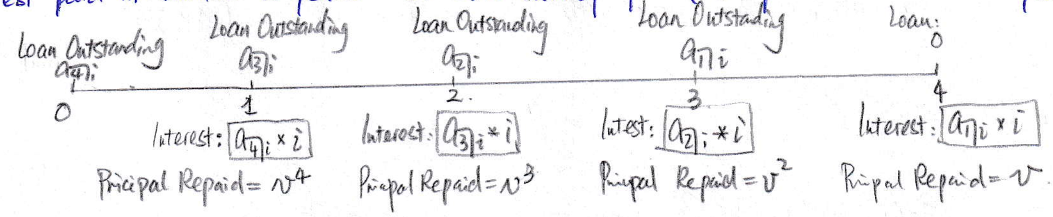
Solve: $PV = 38\, a_{\overline{8}|3\%} + 1000\, v^{8}_{6m}$ $\xrightarrow{\text{computer: FV=1000, I/Y=3, PMT=38, N=8}}$ $PV = 1056.16$ Ⓑ//

Exam FM Question 282.

Give: ① Loan: $a_{\overline{4}|i}$, repaid with payment 1, for 4-year.

② The sum of interest paid in the last 2 years = $(1+i)^2 \times$ sum of principal repaid in the 1st 2 years.

Question: What is i?



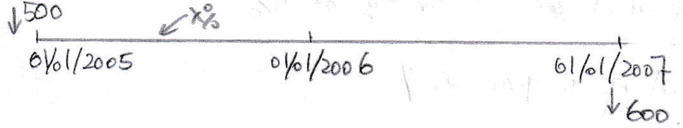
Solve: $(\underset{\uparrow \frac{1-v^2}{i}}{a_{\overline{2}|i}} \times i) + (\underset{\uparrow \frac{1-v}{i}}{a_{\overline{1}|i}} \times i) = (1+i)^2 \times (v^4 + v^3).$

$$\Rightarrow 2 - \frac{1}{(1+i)^2} - \frac{1}{(1+i)} = (1+i)^2 \times \left(\frac{1}{(1+i)^4} + \frac{1}{(1+i)^3}\right) \Rightarrow \underbrace{\frac{1}{(1+i)^2}}_{x^2} + \underbrace{\frac{1}{(1+i)}}_{x} - 1 = 0 \Rightarrow x \cong \overset{0.618}{\boxed{\frac{\sqrt{5}-1}{2}}} \Rightarrow \frac{1}{1+i} \cong 0.618 \Rightarrow i \cong 0.618$$

Ⓓ//

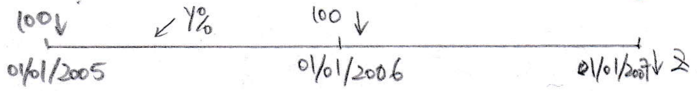
Exam FM Question 283

Give: ① Paul makes one investment of 500 on January 1, 2005, and collects 600 on January 1, 2007, $i = X\%$



(eq①) $500(1+X\%)^2 = 600 \Rightarrow X\% = 0.0954$

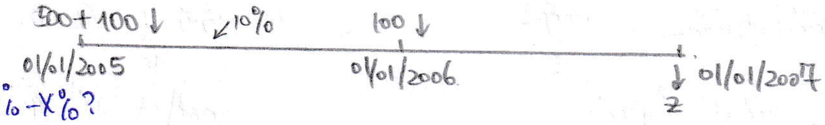
② Toby invests 100 on January 01, 2005 and another on January 01, 2006, collect Z on January 01, 2007. $i = Y\%$



(eq②) $100(1+Y\%)^2 + 100(1+Y\%) = Z$

plug in.

③ The combination of ① and ② yield $i = 10\%$



(eq③) $600(1+10\%)^2 + 100(1+10\%) = 600 + Z$

$\Rightarrow Z = 236$

Question: What is $Y\% - X\%$?

Solve: First. from eq(1), we know: $X\% = 0.0954$

Second: from eq(3), we have $Z = 236 \xrightarrow{\text{plug in}}$ eq(2), we have: $100(1+Y\%)^2 + 100(1+Y\%) = 236$.

$\Rightarrow (1+Y\%) = \frac{-1+\sqrt{1+4\times 2.36}}{2} \Rightarrow Y\% = 0.1155$

Thus: $Y\% - X\% = 0.1155 - 0.0954 = 0.0201 = 2.01\%$ Ⓒ

Exam FM Question 284

Give: ① ten-year certificate of deposite, annual effective rate 8%, if balance is withdrawn before end of 10-year, the purchaser can face two penalties:

i) a loss of the last 9 month's interest

ii) a reduction in the annual effective rate of interest to j

② Now. the purchaser withdraws the fund after 3 years. then, the two paneltias are equivalent

Question: What is j?

Solve: i) Loss of the last 9 $\overset{\text{Interest}}{\text{month}} = AV_{t=3} - AV_{t=2\text{year}\&3\text{month}}$; Assume PV=1

$\because (1+i_{\text{month}})^{12} = 1.08 \Rightarrow i_{\text{month}} \hat{=} 0.006434$

$\therefore AV_{2\text{year}\&3\text{month}} = 1 \cdot \underbrace{1.08^2}_{1.1604} \cdot \underbrace{(1+0.006434)^3}_{1.019426}$

Thus: "loss of 9 month Interest" $= (1+0.08)^3 - 1 \times 1.08^2 \times (1+0.006434)^3 \hat{=} 0.070652$.

ii). Reduction in the rate $(8\% \to j) = (1+0.08)^3 - (1+j)^3$

Since i) = ii)

Thus: $0.070652 = (1+0.08)^3 - (1+j)^3 \Rightarrow j \hat{=} 0.05941 9$ Ⓑ

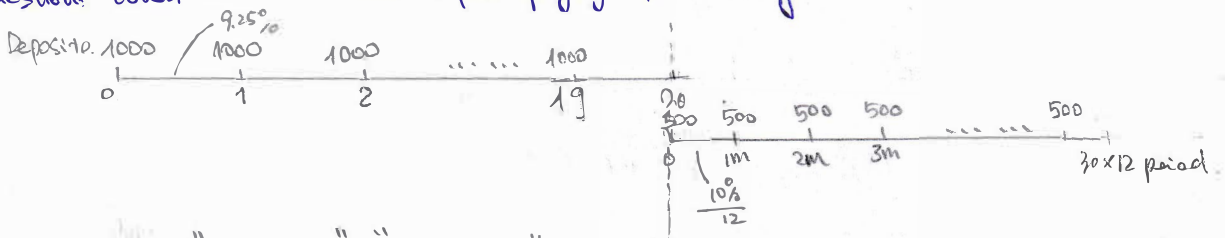
Exam FM Question 285

$\ddot{s}_{\overline{n}|} = \frac{(1+i)^n - 1}{d}$, $\ddot{a}_{\overline{n}|} = \frac{1-v^n}{d}$, $d = \frac{i}{1+i} = iv$

Give: ① An investor deposits 1000 at the beginning of each year for 20 years. The fund earns interest at an annual rate of 9.25%

② At the end of 20 years, investor wishes to use the fund to purchase a 30-year annuity-due with monthly payments of 500, based on annual rate of 10% convertible monthly

Question: What is the balance after paying for annuity?



Solve: "AV of 1000" − "FV of 500"

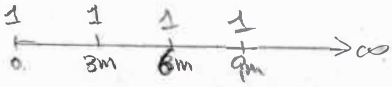
$$= 1000 \cdot \underbrace{\ddot{s}_{\overline{20}|9.25\%}}_{\underbrace{\frac{(1+9.25\%)^{20}-1}{\frac{9.25\%}{1+9.25\%}}}_{57485.26249}} - \underbrace{500 \cdot \underbrace{\ddot{a}_{\overline{360}|\frac{10\%}{12}}}_{\underbrace{\frac{1-v_{1m}^{360}}{d_{1m}}}_{114.90}}}_{57450.20507}$$

where: $v_{1m} = \underbrace{\frac{1}{1+\frac{10\%}{12}}}_{0.9917}$, $d_{1m} = \underbrace{\frac{\frac{10\%}{12}}{1+\frac{10\%}{12}}}_{0.00826}$

$= 35,057$ Ⓒ//

Exam FM Question 286

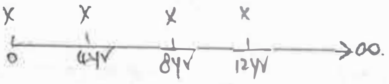
Give: ① Perpetuity pays 1 at the beginning of each 3-month period. PV = 300.



$300 = \frac{1}{d_{3m}} \Rightarrow \underbrace{d_{3m}}_{\frac{i_{3m}}{1+i_{3m}}} = 0.003333 \Rightarrow i_{3m} = 0.00334448$ } $i_{1\text{-year}} = 0.013445$

$(1+i_{3m})^4 = 1 + i_{1\text{-year}}$

② Perpetuity pays X at the beginning of each 4-year, PV = 300



$(1+i_{1\text{-year}})^4 = 1 + i_{4\text{-year}} \Rightarrow i_{4\text{-year}} \simeq 0.054875$

Then $d_{4\text{-year}} \simeq 0.052$

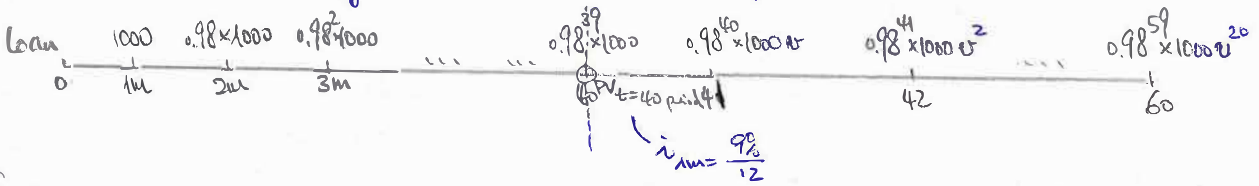
Thus: $\frac{X}{0.052} = 300 \Rightarrow X \simeq 15.606$ Ⓑ//

Exam FM Question 287

Give: ① A loan, annual $i = 9\%$ convertible monthly, repaid with 60 payments.

② First payment = 1000, each succeeding payment is 2% less.

Question: What is "outstanding balance after the 40th payment"?



Solve: $PV_{t=40\text{ period}} = (0.98^{40} \times 1000) v_{1m} + (0.98^{41} \times 1000) v_{1m}^2 + \cdots + (0.98^{59} \times 1000) v_{1m}^{20}$ where $v_{1m} = \frac{1}{1+i_{1m}}$, $i_{1m} = \frac{9\%}{12}$

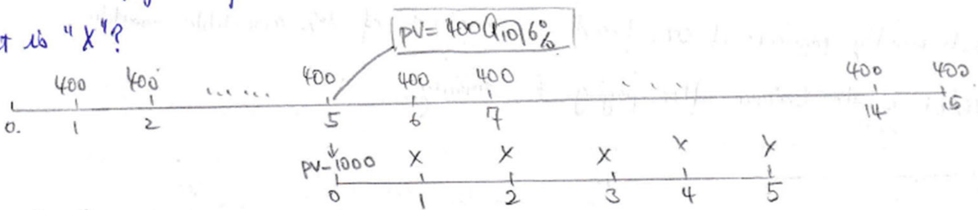
$= (0.98^{40} \times 1000 \times v_{1m}) \frac{1-(0.98v)^{19}(0.98v)}{1-0.98v} \overset{\nwarrow}{} \frac{a_1 - a_{n}q}{1-q} \simeq 6889.1428$ Ⓐ//

Exam FM Question 288

Give: ① 15-year loan, annual rate 6%, payment 400 at the end of each year.

② At the end of 5th payment, borrowers pays extra 1000, then repays the balance over 5 years with revised annual payment of X

Question: What is "X"?



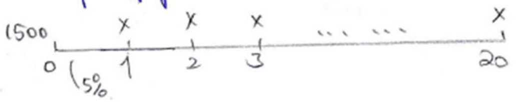
Solve: $\underbrace{400 a_{\overline{10}|6\%}}_{2944.03} - 1000 = X \cdot a_{\overline{5}|6\%} \Rightarrow X \cong 461.505$ Ⓒ

Exam FM Question 289

Give: ① loan of 1500, repaid with payment at the end of year, for 20 years.

② Two payment opinion, under both, sum of payments are the same.

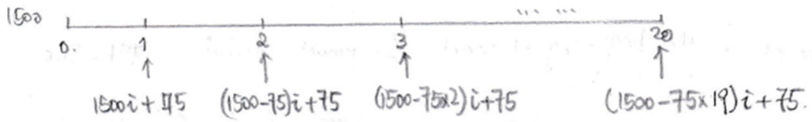
i) equal payment at $i_{annual} = 5\%$.



$X \cdot a_{\overline{20}|5\%} = 1500 \Rightarrow X \cong 120.36$

$\Rightarrow$ Sum of payments $= 20X \cong 2407.2776$

ii) nonlevel payment of interest on the unpaid balance at $i_{annual} = i$, plus 75 of principal



$$\text{Sum of payment} = 1500i \times 20 + 75 \times 20 + (-75i - 75\times2i - 75\times3i - \cdots - 75\times19i)$$

$$= 30000i + 1500 - 75i\underbrace{(1+2+3+\cdots+19)}_{\frac{19(1+19)}{2} \nwarrow \frac{n(a_1+a_n)}{2}}$$

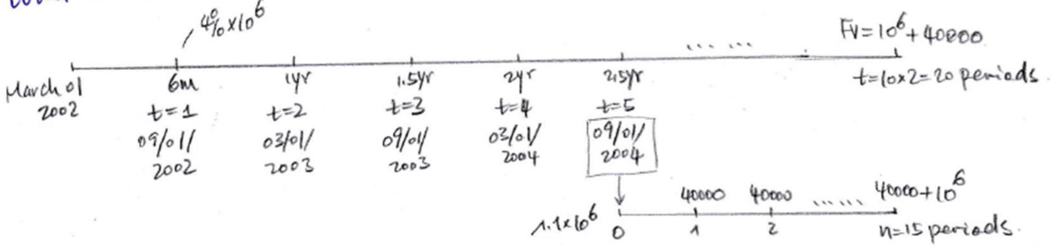
Thus: $30000i + 1500 - 75i \cdot \frac{19(1+19)}{2} = 2407.2776 \Rightarrow 15750i = 907.2776 \Rightarrow i \cong 5.7605\%$ Ⓒ

Exam FM Question 290

Give: ① 10-year, 10^6 Face Value, Bond issued on March 01, 2002, coupon rate $r_{annual} = 8\%$ semi-annually,

② On September 2, 2004, paid 1.1×10^6 for this bond to yield annual rate of i convertible semi-annually

Question: What is i?



Solve: $1.1 \times 10^6 = 40000\, a_{\overline{15|}}\, i_{6m} + 10^6 \cdot v_{6m}^{15} \Rightarrow i_{6m} \cong 3.153\% \Rightarrow i_{1\text{-year}} = 2 \cdot i_{6m} \cong 6.306\%$ ©

Exam FM Question 291.

Give. An investor deposits 150 today, receives 100 in 1-year, 80 in 2-year.

Question: Calculate the $i^{(4)}$: annual rate convertible quarterly that investor earns

Solve: First. get i_{annual} : $150 = 100v + 80v^2 \Rightarrow 80v^2 + 100v - 150 = 0$

$$v = \frac{-100 \pm \sqrt{100^2 + 4 \times 80 \times 150}}{2 \times 80} \Rightarrow v \cong 0.88$$

$$\Rightarrow 1 + i_{annual} \cong 1.1361$$

Second. get $i^{(4)}$: $\boxed{\underbrace{(1 + i_{annual})}_{1.1361} = \left(1 + \frac{i^{(4)}}{4}\right)^4} \Rightarrow i^{(4)} \cong 0.1296 \cong 13.0\%$ Ⓑ//

Exam FM Question 292.

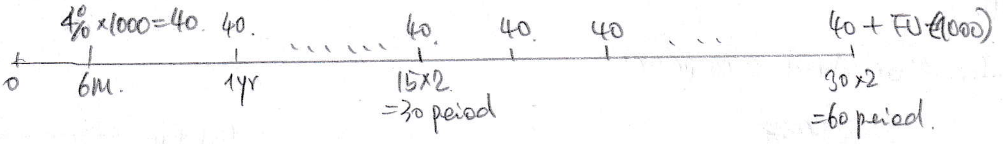
Give: ① Investor purchases 30-year bond, face value 1000, annual coupon rate 8% paid semi-annually.

② Callable: at face value 1000, occuring at the end of 15th year.

③ Bond: bought at a premium based on an annual effective yield rate of 7%.

Question: What is the "purchase price" of this bond?

Solve:



Call at earlist

∵ Bond: boughat a premium ⇔ "Bought price > Face value" or "Coupon rate > Yield rate".
8% 7%

∵ Call at earlist, that is $n = 30$ period

Thus: $\text{Price} = 40\, a_{\overline{30}|\, i_{6M}} + 1000\, v_{6m}^{30}$ where: $(1 + i_{6m})^2 = 1 + 7\% \Rightarrow i_{6m} = 3.44\%$

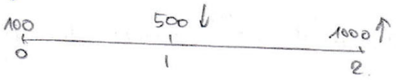
Using Calculator, we know: Price ≅ 1103.77 Ⓑ//

Exam FM Question 293.

$D^{mac} = (1+i)\cdot D^{mod}$; $D^{mod} = -\dfrac{\frac{\partial P(i)}{\partial i}}{P(i)}$

Give ① the present value of CFs at effective rate i, is given by.

$$P(i) = 100 + 500(1+i)^{-3} - 1000(1+i)^{-4}$$



Question: which gives the "modified duration"?

Solve: First, "Macaulay Duration" $D^{mac} = \dfrac{[500(1+i)^{-3}]\times 3 + [-1000(1+i)^{-4}]\times 4}{P(i)}$ ← this is Ⓑ.

Second, $D^{mac} = (1+i)D^{mod} \Rightarrow \boxed{D^{mod}} = v\cdot D^{mac} = \dfrac{[500(1+i)^{-3}]\times 3v + [-1000(1+i)^{-4}]\times 4v}{P(i)}$ ← $\boxed{Ⓐ}_{/\!/}$

Alternative, recall: $D^{mod} = -\dfrac{\frac{\partial P(i)}{\partial i}}{P(i)} = -\dfrac{0 + -3[500(1+i)^{-4}] + 4[1000(1+i)^{-5}]}{P(i)}$ Ⓐ// as well.

Exam FM Question 294

Terms	Annual Yield Rate
1	3.5%
2	4.0%
3	4.0%
4	5.0%

Give: The current yield rates for zero-coupon bonds are the following.

Question: What is the $\underbrace{\text{"implied 2-year spot" at the end of year two}}_{f[2,4]}$?

Solve: $\underbrace{(1+r_2)^2}_{4\%} \times (1+f[2,4])^2 = \underbrace{(1+r_4)^4}_{5\%} \Rightarrow f[2,4] \cong 0.06$ Ⓔ//

Exam FM Question 295

Give: the following information about 3 companies

	Asset			Liability		
Company	PV	Modified Duration	Convexity	PV	Modified Duration	Convexity
U	200,000	10.2	150	200,000	9.3	148
✓ V	300,000	8.6	85	300,000	8.6	80
W	400,000	15.8	300.	400,000.	15.8	305

Determining which of the following statements are true?

(I) Company U's position is immunized against small i changes.

(II) " V's " " " " "

(III) " W's " " " " "

⇒ Ⓑ. II only.

Solve: (I) is false, since "Duration NOT Match", Attention: here is "Modified", but

$D^{mac} = D^{mod}\cdot(1+i)$, thus, same as "$D^{mac}$ should be the same"

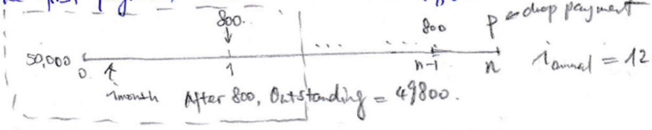
Ⓑ// ⇐ (II) is OK., (III) C^{Asset} should > $C^{liability}$, thus (III) is false

Exam FM Question 296

Give. ① on the first month, Chuck took out a business loan for 50,000, payment at the end of month, annual nominal rat compounded monthly. Each month 800, except for a final drop payment

② Immediately after the first payment, Balance owned was 49,800

Question: # of payment n?



$i_{annual} = 12 \cdot i_{month}$

Solve: $50{,}000 = 800v_{1m} + 49800v_{1m} \Rightarrow i_{1m} = 0.012$

Thus: $\underbrace{50{,}000}_{PV=-50{,}000} = 800\,a_{\overline{n}|i_{m}=1.2\%} \Rightarrow \underset{\text{drop payment}}{n \cong 116.216} \Big\} \Rightarrow \boxed{n_{total} = 117}$ Ⓑ// with 116 paymants of 800, & 1 final drop Amt.

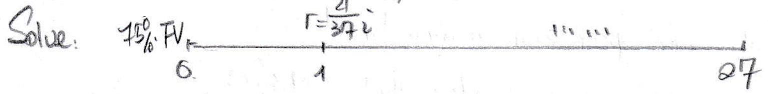
Exam FM Question 297.

Give: ① 27-year bond, price = 75% Face Value, Face Value (F), annual coupon with coupon rate = $\frac{21}{37}$ annual yield rate

② n-year, zero-coupon bond, same price, face value, yield rate. (same: FV, price, $i \Leftrightarrow v$)

Question: What is n? coupon = $\frac{21}{37} \cdot i \cdot Fv$

Solve:



$$75\% FV = \left[\left(\frac{21}{37} \cdot i\right) \cdot FV\right] \cdot a_{\overline{27|}i} + FV \cdot v^{27}$$

$$\Rightarrow 75\% FV = \left(\frac{21}{37} \cdot \cancel{i} \cdot FV\right) \cdot \frac{1 - v^{27}}{\cancel{i}} + FV \cdot v^{27}$$

$$\Rightarrow \quad v = 0.96854$$

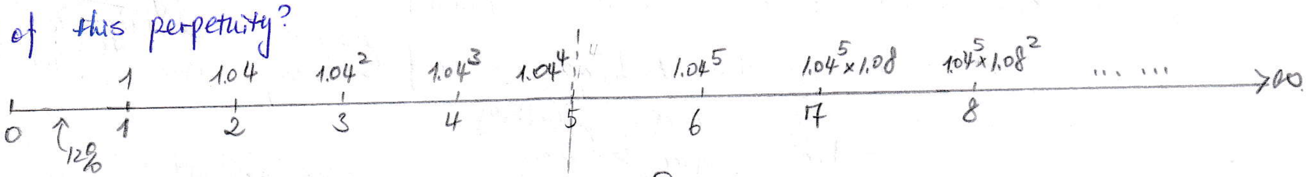
Thus: For the zero-coupon bond: $\cancel{FV} \cdot \underset{\underset{0.96854}{\uparrow}}{v}^{n} = 0.75 \cdot \cancel{FV} \Rightarrow n = \frac{\ln(0.75)}{\ln(0.96854)} \Rightarrow n \cong 8.99976$ ⓒ //

Exam FM Question 298

Give: ① An annual perpetuity pays 1 one year from now. Payments will then increase by 4% per year for 5 years and 8% per year thereafter ; $i_{annual} = 12\%$

Question: PV of this perpetuity?

Method 1



$$PV^{①}_{t=0} = v + 1.04v^2 + 1.04^2 v^3 + \cdots + 1.04^4 v^5$$
$$= v\left(1 + 1.04v + (1.04v)^2 + \cdots + (1.04v)^4\right)$$
$$= v \, \frac{1 - (1.04v)^5}{1 - 1.04v}$$
$$= \frac{1 - (1.04v)^5}{1.12 - 1.04}$$
$$PV^{①}_{t=1} = \frac{1 - \left(\frac{1.04}{1.12}\right)^5}{0.12 - 0.14} = 3.87$$

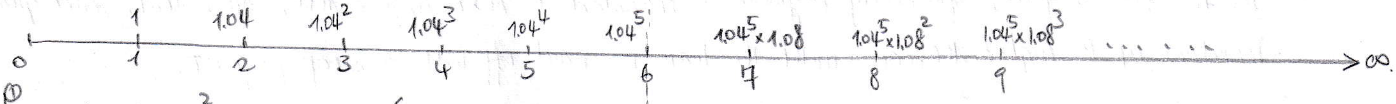
$$\xleftarrow{\quad} PV^{②}_{t=5} = 1.04^5 v + 1.04^5 \times 1.08 v^2 + 1.04^5 \times 1.08^2 v^2 + \cdots$$
$$= 1.04^5 \underset{\underset{\frac{1}{v}}{\uparrow}}{v} \, \underbrace{\left(1 + 1.08v + (1.08v)^2 + \cdots\right)}_{\frac{1}{1 - 1.08v}}$$
$$= 1.04^5 \cdot \underbrace{\frac{1}{\frac{1}{v}(1 - 1.08v)}}_{\frac{1}{\underset{1.12}{\boxed{\frac{1}{v}}} - 1.08}}$$
$$= 1.04^5 \cdot \frac{1}{1.12 - 1.08}$$

$$PV^{②}_{t=0} = v^5 \cdot \left(1.04^5 \cdot \frac{1}{1.12 - 1.08}\right)$$

$$\boxed{PV^{②}_{t=0} = \left(\frac{1.04}{1.12}\right)^5 \cdot \frac{1}{0.12 - 0.08}} = 17.26$$

Thus: $PV^{①}_{t=0} + PV^{②}_{t=0} \cong 3.87 + 17.26 = 21.13$ Ⓐ

Method 2



$$PV^{①}_{t=0} = v + 1.04v^2 + \cdots + 1.04^5 v^6$$
$$= v\,\underbrace{\left(1 + 1.04v + \cdots + (1.04v)^5\right)}_{\frac{1 - (1.04v)^6}{1 - 1.04v}} \cong 4.48$$

$$\xleftarrow{\quad} PV^{②}_{t=6} = 1.04^5 \times 1.08 v + 1.04^5 \times 1.08^2 v^2 + 1.04^5 \times 1.08^3 v^3 + \cdots$$
$$= 1.04^5 \times 1.08 v \, \underbrace{\left(1 + 1.08v + 1.08^2 v^2 + \cdots\right)}_{\frac{1}{1 - 1.08v}} \cong 32.85$$

$$PV^{②}_{t=0} = v^6 \cdot 32.85 \cong 16.64$$

Thus: $PV^{①}_{t=0} + PV^{②}_{t=0} \cong 4.48 + 16.64 \cong 21.1269$ Ⓐ //

Exam FM Question 299

$$C^{mac} = \frac{\Sigma t^2 \cdot pv}{PV\ of\ Portfolio}$$

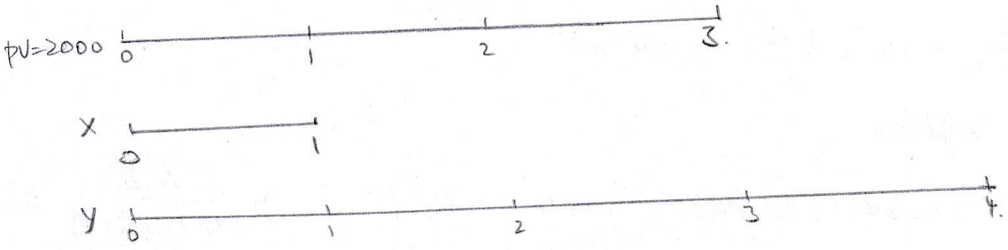
Give: ① insurance company has a liability of 2662, which is due at the end of 3 year. The PV is 2000.

② There are 2 investments avaliable: $\left\{ \begin{array}{l} \text{one-year zero-coupon bond} \\ \text{four-year zero-coupon bond.} \end{array} \right.$

③ The investment plan satisfied "Redington Immunization".

Question: Calculate the Amt the company invests in the one-year zero-coupon bond.

(A) 400 (B) 667 ✓ (C) 858 (D) 1000 (E) No investment plan that satisfies



Solve: (1) PV Matches: $X + Y = 2000$

(2) D^{mac} Matches: $3 = \underbrace{\frac{X}{2000}}_{weight1} \times 1 + \underbrace{\frac{Y}{2000}}_{weight2} \times 4$ $\Bigg\} \Rightarrow \left\{ \begin{array}{l} X \doteq 666.67 \\ Y \doteq 1333.33. \end{array} \right.$

(3) $C^{Asset}_{mod} > C^{Liability}_{mod}$: Recall $C_{mod} = \frac{C_{mac} + D_{mac}}{(1+i)^2}$, we already know D_{mac} are equal.

Thus: As long as we proof $\boxed{C^{Asset}_{mac} \geq C^{Liability}_{mac}}$, we have $C^{Asset}_{mod} > C^{Liability}_{mod}$

→ Proof: $C^{Asset}_{mac} > C^{Liability}_{mac}$

$$\to C^{Asset}_{mac} = + \frac{\frac{\partial^2}{\partial \delta^2} P(i)}{P(i)} = \frac{\Sigma t^2 (v^2 A_t)^{\to PV}}{P(i)}$$

Thus: $C^{Asset}_{mac} = \frac{1^2 \times 666.67}{2000.} + \frac{4^2 \times 1333.33}{2000} = 11.$

$C^{Liability}_{mac} = \frac{3^2 \times 1000}{2000} = 9$

$\Rightarrow C^{Asset}_{mac} > C^{Liability}_{mac}$

$\Leftrightarrow C^{Asset}_{mod} > C^{Liability}_{mod}$

$\Leftrightarrow$ holds.

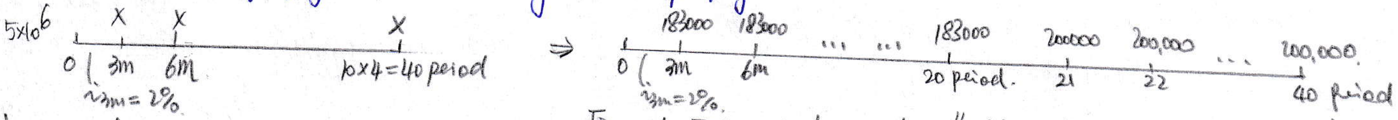
$\Rightarrow$ (B). X=667. & "Redington Immunization" holds

Exam FM Question 300.

Give: ① loan of 5×10^6, repaid by installments of X at the end of each quarter, 10-year period, annual rate 8% compounded quarterly

① For the first 5-year, quarterly payment is rounded to next higher 1000, After that, each quarterly X rounded up to higher 100,000, until the loan is paid off with a drop payment.

Question: the total # of payments, including the drop payment.



Solve: $5\times10^6 = X \cdot a_{\overline{40}|2\%} \Rightarrow X = 182778.739$

→ First: 4x5=20 period: round to. "183000"

→ Second: 21-40 period: round to "200,000"

Thus: $5\times10^6 = \underbrace{183000\, a_{\overline{20}|2\%}}_{2992312.302.} + (200{,}000\, a_{\overline{n}|2\%}) \cdot v^{20}_{3m.} \Rightarrow \underset{\text{drop payment}}{n \cong 17.89} \Big\} \Rightarrow n = 18$ payment $\Leftrightarrow$

Ⓒ//

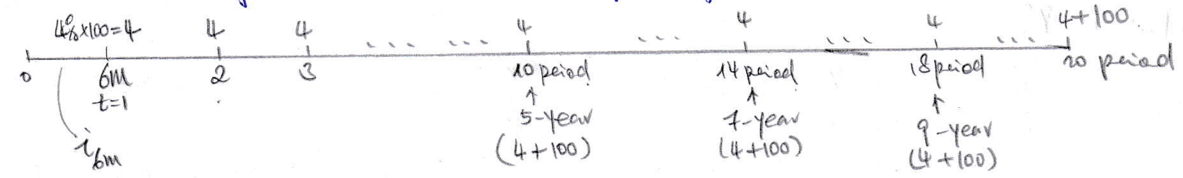
- Total 20+18=38 payment
- 37 regular payment
- Final drop payment

Exam FM Question 301

Give:
① 10-year, face value = 100, annual coupon rate 8% semi-annually.
② Call: bond is callable at the end of the 5th, 7th, 9th years at par immediately after the coupon paymet

Question " Maximum Price" of the Bond that ensurs "yield of 6%".

Solve:



∵ Coupon rate 8% > Yield rate 6% ⇔ Premium Case

∴ Call at earlist: 5th year (n=10)

(call at 5-year) $\text{Price} = \underset{(FV=100,\ i=2.956\%,\ PMT=4)}{\underline{4\cdot a_{\overline{10}|}\overset{2.956\%}{i_{6m}} + 100\,v_{6m}^{20}}}$ where $(1+i_{6m})^2 = 1.06 \Rightarrow i_{6m} = 2.956\%$

$= 108.92556$

(If not call) $\text{Price} = 4\,a_{\overline{20}|2.956\%} + 100\,v_{6m}^{20} = 115.9546$

⇒ "Maximum Price" = "Lowest PV/price" = Should Call at 5-year

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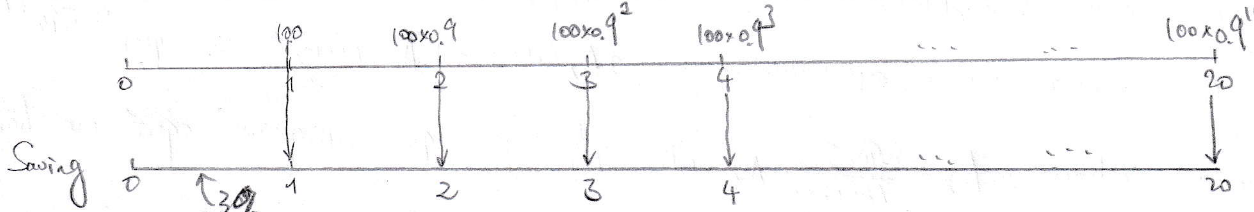
Exam FM Question 302.

Give:
① Susan Receives annual payment. 20-year, annuity-immediate. The payment in year 1 is 100, 90% in the each succeeding year.
② Upon recaipt of each payment, Susan invests the payment in a savings account, interest 3%.

Question: The balance in saving account immediatly after Susan invests the last annuity payment.

Solve: ① tells, once Susan receive the annual payment, she immediate puts this money into saving account. No time gap.



$$\text{Balance} = FV = \Big(\underbrace{100v + 100\times0.9v^2 + 100\times0.9^2v^3 + \cdots + 100\times0.9^{19}v^{20}}_{100v(\underbrace{1+0.9v+0.9v^2+\cdots+0.9^{19}v^{19}}_{\frac{1-(0.9v)^{20}}{1-0.9v}\ \wedge\ \frac{a_1-a_nq}{1-q}})}\Big) \times (1+3\%)^{20}$$

where $i=3\%$. $v=\frac{1}{1.03}$.

$= 717.4666 \times (1+3\%)^{20} \cong 1295.8244$ Ⓓ//

Exam FM Question 303

Give: ① Two deposits are made into a fund: 300 at time 0, X at time 4.

② force of interest $\delta_t = \frac{t}{50}$ $(t \geq 0)$, "the amount of interest earned" from 0 to 10 is 4X.

Question: What is X?

Solve:

4X = "Interest amount of 300" (from t=0→10) + "Interest amount of X" (from t=4→10).

$$= \underbrace{\left(300 \cdot e^{\int_0^{10} \frac{t}{50} dt} - 300\right)}_{FV_{10} - PV_0} + \underbrace{\left(X \cdot e^{\int_4^{10} \frac{t}{50} dt} - X\right)}_{FV_{10} - PV_6} = 1.3164X$$

$= 815.4845 - 300$

$$\Rightarrow 4X - 1.3164X = 815.4845 - 300 \Rightarrow X \cong 192.0869$$

Exam FM Question 304

Give: ① Full immunization ⇔ ① "at reference point", Values are equal; ② take derivative "over i",

② Liability: 750,000, at t=4

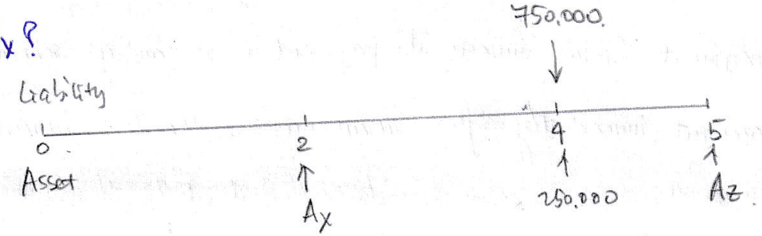
③ Asset: #1: cash flow of A_X, at t=2

#2: cash flow of 250,000, at t=4

#3: cash flow of A_Z, at t=5.

④ $v = 0.95$

Question: what is A_X?



Solve: ① at t=4: $A_X(1+i)^2 + 250{,}000 + A_Z \cdot (1+i)^{-1} = 750{,}000$ ← eq(1). ∵ $v = 0.95$ ∴ $\frac{1}{0.95} = (1+i)$

② derivative over i eq(1) becomes: $2A_X(1+i) - 1 A_Z \cdot \underbrace{(1+i)^{-2}}_{v^2} = 0$ ← eq(2)

from (2) we know: $A_X = \frac{0.9025}{(1+i) \times 2} \cdot A_Z$ where $\frac{1}{1+i} = v = 0.95$. plug into eq(1). we have:

$$0.4287 A_Z \times \underbrace{(1+i)^2}_{(\frac{1}{0.95})^2} + 0.95 A_Z = 500{,}000 \Rightarrow A_Z \cong 350873.78$$

with $A_X = \frac{0.9025}{(1+i) \times 2} A_Z$ where $\frac{1}{1+i} = 0.95$ $\Bigg\} \Rightarrow A_X \cong 150415.20$

Ⓐ

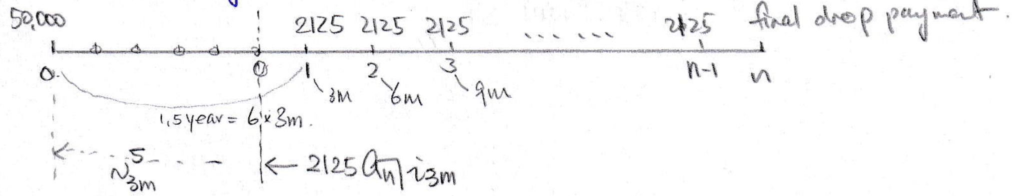
Exam FM Question 305

Give: ① Loan: 50,000, $i_{annual} = 6\%$.

② Repaid { quarterly: 2125 & one final drop payment

First payment is 1.5 year from now.

Solve:



$$\underbrace{50{,}000}_{} = v^5_{3m} \times \left(2125 \cdot a_{\overline{n}|i_{3m}}\right) \text{ where } (1+i_{3m})^4 = 1.06 \Rightarrow i_{3m} \cong 1.46738\%$$

(Calculator:) PV = −50,000

$\Rightarrow n \cong 31.865 \Rightarrow$ { 31: 2125 payment ← Ⓓ

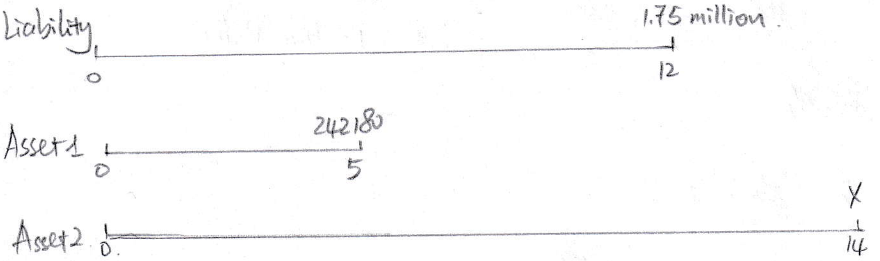
32#: last one is a drop payment

Exam FM. Question 306

Give. ① Fully Immunized.
② Liability: 1.75 million, t=12 years from now
③ Asset { zero-coupon bond: 242,180 maturing in 5-year
zero-coupon bond: X maturing in 14-year
④ $i=7\%$

Question: PV(Asset) − PV(liability) if $i \to 4\%$.

Solve:



At t=12, $242180 \times (1+7\%)^7 + X \cdot \left(\frac{1}{1+7\%}\right)^2 = 1.75 \times 10^6 \Rightarrow X \cong 1558336.948$

When $i \to 4\%$. PV(Asset) − PV(liability)

$= 242180 \cdot v^5 + X \cdot v^{14} - (1.75 \times 10^6) v^{12}$ where $v = \frac{1}{1+4\%}$

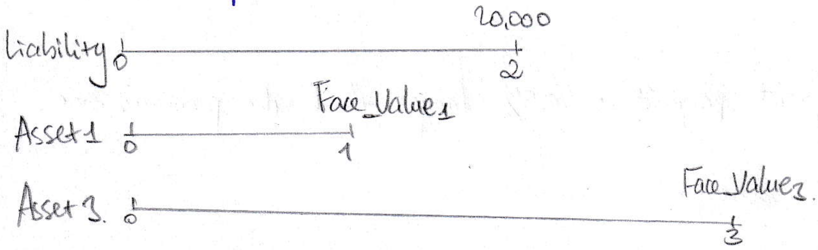
$= 5910.2278$ Ⓐ//

Exam FM. Question 307

Give. ① Fully immunized.
② Liability: 20,000, t=2-year from now.
③ Asset: { zero-coupon bond: maturing in 1-year
zero-coupon bond: maturing in 3-year.
④ $i=5.5\%$.

Question: Face Value of 1-year zero-coupon bond.

Solve:



eq(1) $\text{Face\_Value}_1 \times (1+i) + \text{Face\_Value}_3 \times (1+i)^{-1} = 20{,}000$ where $i = 5.5\%$

eq(2). derivative over i: $\text{Face\_Value}_1 + (-1) \cdot \text{Face\_Value}_3 \times (1+i)^{-2} = 0$.

eq(2) ⇒ $\text{Face\_Value}_3 = \text{Face\_Value}_1 \times (1+i)^2$. plug into eq(1) ⇒

$\text{Face\_Value}_1 \times (1+i) \times 2 = 20{,}000$

⇒ $\text{Face\_Value}_1 \cong 9478.673$ Ⓐ//

Exam FM Question 308

Give:

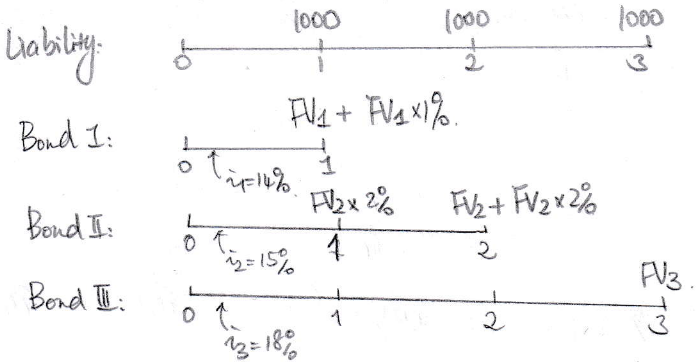
① Liability: 3 liabilities of 1000, due at periods 1, 2, 3.

② Assets:

3 Assets to match liabilities
- Bond I: due at period 1, coupon rate 1%, $i_1 = 14\%$
- Bond II: due at period 2, coupon rate 2%, $i_2 = 15\%$
- Bond III: due at period 3, zero-coupon bond, $i_3 = 18\%$

Question: "Total Purchase Price" of the 3 bonds?

Solve:



Let FV: Face Value.

at t=3: $FV_3 = 1000 \Rightarrow$ Price of Bond III $= \frac{1000}{1.18^3} \cong 608.63087$

at t=2: $1.02 FV_2 = 1000 \Rightarrow FV_2 \cong 980.392$

at t=1: $1.01 FV_1 + 0.02 FV_2 = 1000 \Rightarrow FV_1 = 970.6853$

$\Rightarrow$ Bond Price II $= \frac{FV_2 \times 2\%}{1.15} + \frac{1.02 FV_2}{1.15^2} \cong 17.0503 + 756.1435$

Bond Price I $= \frac{1.01 FV_1}{1.14} \cong 859.9931$

$$\Rightarrow \text{Total Price} = \underbrace{608.63087}_{\text{Bond III}} + \underbrace{17.0503 + 756.1435}_{\text{Bond II}} + \underbrace{859.9931}_{\text{Bond I}} \cong 2241.82$$

Exam FM Question 309

Give:

① Annuity-immediate: annual payments of 10, 20-year

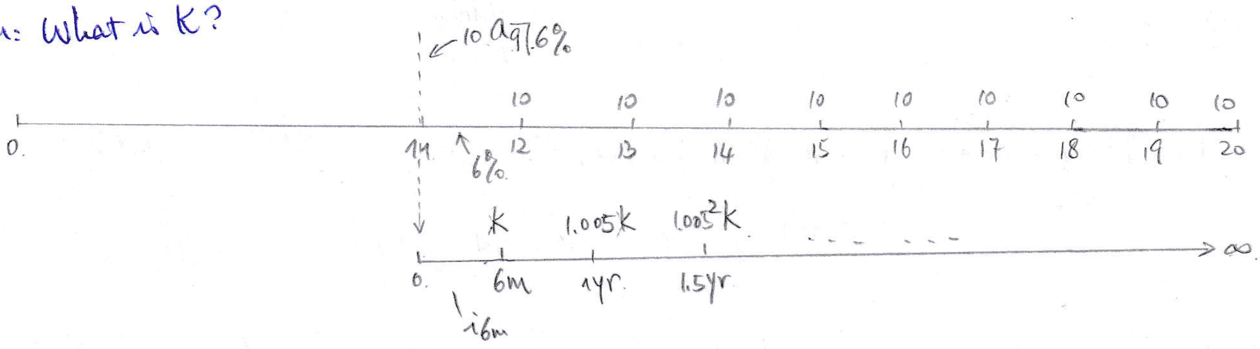
② Immediately following the 11th payment, annuity is exchanged for a perpetuity-immediate with semi-annual payments.

③ $i = 6\%$

④ The 1st payment is K, subsequent payment is 0.5% larger than the previous one.

Question: What is K?

Solve:



$$PV = 10 a_{\overline{9}|6\%} = \underbrace{K v_{6m} + 1.005 K v_{6m}^2 + 1.005^2 K v_{6m}^3 + \cdots}_{K v_{6m} \left(\underbrace{1 + 1.005 v_{6m} + (1.005 v_{6m})^2 + \cdots}_{\frac{1}{1 - 1.005 v_{6m}}} \right)}$$

where $(1 + i_{6m})^2 = 1.06$

$\Rightarrow 40.71162 K = 68.016923 \Rightarrow K \cong 1.67$ Ⓑ