SOA and CAS: Exam FM[1]
Written Solutions: 171-260

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January 14, 2024

This document only provides written solutions to official example problems 171-260. For official sample questions, check out the official websites of Society of Actuaries and the Casualty Actuarial Society.

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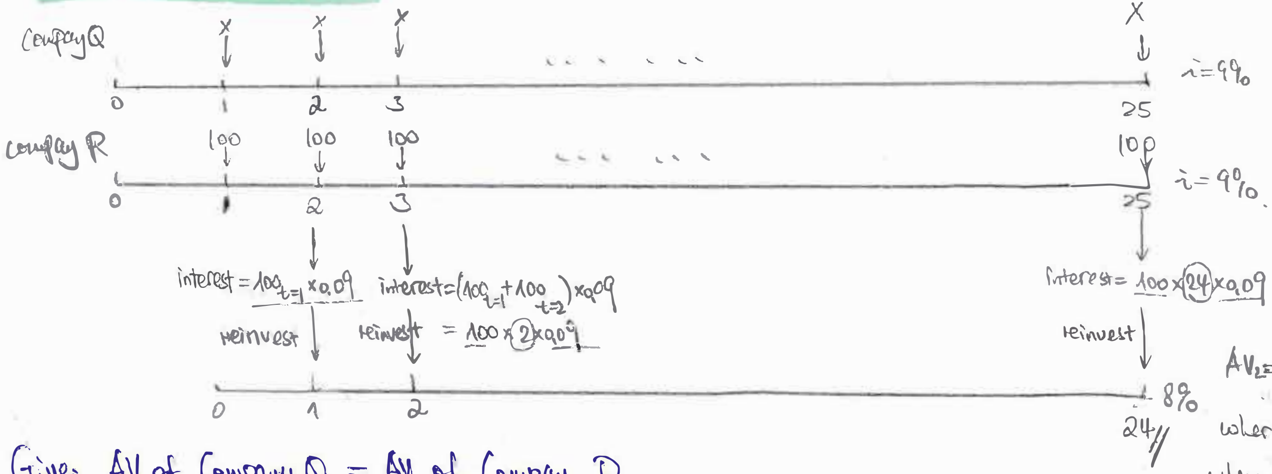
[2] Email: liyifinhub@outlook.com The written solutions were drafted when I was preparing for the exam. Please email me if you find any errors. My personal website: `https://yilifinhub.com/`

Exam FM Question 171 (Similar to Q148)



$\boxed{AV_1 = 100 \times 25}$

$AV_2 = 100 \times 0.09 \times IS_{\overline{n=24}|8\%}$

where $IS = (1+i)^{24} \cdot Ia_{\overline{24}|8\%}$

where $Ia = \frac{\ddot{a} - nv^n}{i}$

where $\ddot{a}_{\overline{n}|} = (1+i)a_{\overline{n}|}$

Given: AV of Company Q = AV of Company P

Question: X

Solve: ① AV of Company R = $AV_1 + AV_2$

$$= (100 \times 25) + (100 \times 0.09) \times \left[(1+i)^{24} \cdot \underbrace{\underbrace{\frac{(1+i)\overset{10.5287}{a_{\overline{n}|}} - \overset{3.78478}{nv^n}}{i}}_{94.8277}}_{601.31958} \right]$$ where $i = 8\%$, $n = 24$

$$= 2500 + 5411.8763 = 7911.8762$$

② AV of Company Q = $X \cdot s_{\overline{25}|9\%} = X \cdot \frac{(1+i)^{25} - 1}{i} = 84.7X$

∵ AV equals each other

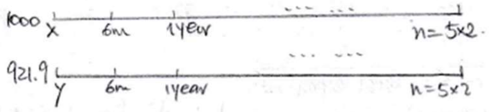
∴ $84.7X = 7911.8762 \Rightarrow X = 93.4095$ Ⓓ

Exam FM Question 172:

Fund X: a deposit of 1000 at time 0. Accumulated at a rate of k compounded semi-annually. Fund Y: a deposit of 921.90 at time 0. Accumulated at a rate of discount k, compounded semi-annually. At the end of 5 year, the AV of Fund X and Fund Y are both equal to P

Question: what is P?

Exam FM Question 172



Solve: $1000\left[\underbrace{\left(1+\frac{k}{2}\right)^2}_{\text{1-year}}\right]^5 = 921.9\left[\underbrace{\left(1-\frac{k}{2}\right)^{-2}}_{\text{1-year}}\right]^5$

$\Rightarrow \left(1+\frac{k}{2}\right)\left(1-\frac{k}{2}\right)=0.99190$

$\Rightarrow 1+\cancel{\frac{k}{2}}-\cancel{\frac{k}{2}}-\frac{k^2}{4}=0.99190$

$\Rightarrow k \cong 0.18$

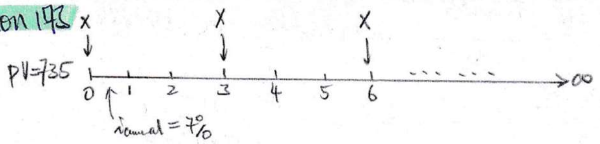
Then: $AV=P=1000\left[\left(1+\frac{k}{2}\right)^2\right]^5 \cong 2367.22$ Ⓔ

Exam FM Question 173:

The PV of a perpetuity-due with payments of X at the beginning of each 3-year period with an annual rate of 7% is 735

Question: what is X?

Exam FM Question 173



Solve: First: $(1+\underbrace{i_{annual}}_{7\%})^3 = 1+i_{3\text{-year}} \Rightarrow i_{3\text{-year}} = 22.5043\% \Rightarrow d_{3\text{-year}} = \frac{i_{3year}}{1+i_{3year}} = \frac{22.5043\%}{1.225043} = 0.1837$

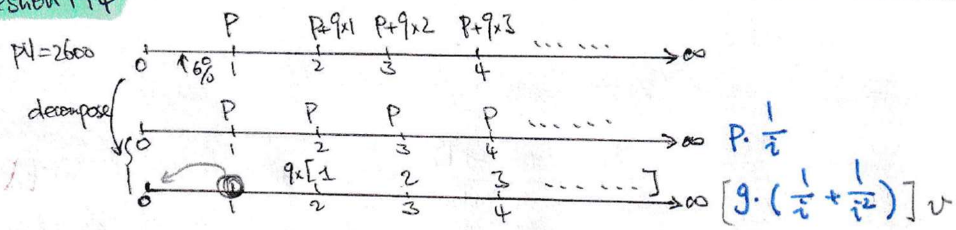
Second: Perpetuity-due $= \frac{X}{d_{3\text{-year}}} = \frac{X}{0.1837} = 735 \Rightarrow X \cong 135.021$ Ⓐ

Exam FM Question 174:

The PV of a perpetuity-immediate with a first payment of P and increase of 9 at an annual rate of 6% is 2600

Question: what is P?

Exam FM Question 174



Solve: $2600 = PV = \left(P\cdot\underset{6\%}{\frac{1}{i}}\right)+v\left(9\cdot\left(\underset{6\%}{\frac{1}{i}}+\underset{6\%}{\frac{1}{i^2}}\right)\right)$ where $i=6\%$

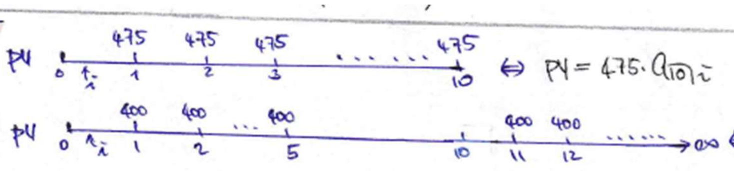
$\Rightarrow 2600 = \frac{P}{0.06}+2500 \Rightarrow P=6$ Ⓑ

Exam FM Question 175:

Two annuities-immediate have the same PV at an annual rate I: (i) A ten-year annuity with annual payments of 475; (ii) A perpetuity with annual payments of 400 in years 1-5, zero in years 6-10, and 400 in years 11 and beyond

Question: what is i?

Exam FM Question 175



$\Leftrightarrow PV = 475 \cdot a_{\overline{10}|i}$

$\Leftrightarrow PV = 400 a_{\overline{5}|i} + v^{10}\left(\frac{400}{i}\right)$

Solve: $PV = 475 \cdot a_{\overline{10}|i} = 400 a_{\overline{5}|i} + v^{10}\left(\frac{400}{i}\right)$

$\Leftrightarrow 475 \cdot \frac{1-v^{10}}{\cancel{i}} = 400 \cdot \frac{1-v^5}{\cancel{i}} + v^{10} \cdot \frac{400}{\cancel{i}}$ Let $X = v^5$

$\Leftrightarrow 875X^2 - 400X - 75 = 0$

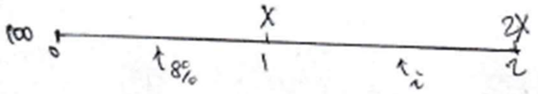
$\Leftrightarrow X = \frac{400 \pm \sqrt{400^2 + 875 \times 75 \times 4}}{2 \times 875} = 0.6 \Rightarrow v^5 = 0.6 \Rightarrow i \cong 10.7566\%$ Ⓑ//

Exam FM Question 176:

Two -year loan of 100 repaid with a payment of X at the end of first year and 2X at the end of the second year. The annual rate charged is 8% in the first year and i in second year. The annual rate for the lender is 10%

Question: what is i?

Exam FM Question 176



Solve: First. $100 = \frac{X}{1+10\%} + \frac{2X}{(1+10\%)^2} \Rightarrow X \cong 39.032258$

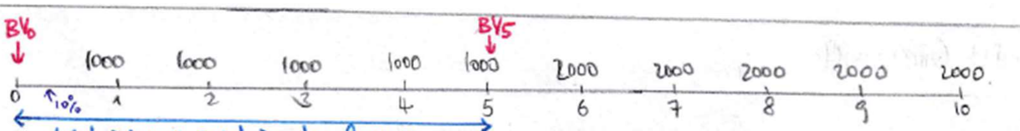
Second. $100 = \frac{X}{1+8\%} + \frac{2X}{(1+8\%)(1+i)} \Rightarrow i \cong 13.1899\%$ Ⓔ//

Exam FM Question 177:

10-year loan repaid with payments at the end of each year. Each of the first 5 payments is 1000 and each of the next 5 payment is 2000. Interest on the loan is charged at an annual rate of 10%

Question: what is the total interest paid in the first 5 payments?

Exam FM Question 177



total interest paid in the first 5 payments

= total level payment (5x1000) − Principal Repaid in the first 5 payments

(BV₀ − BV₅)

Total interest paid in the first 5 payments = total level payment − Pricipal Repaid in the first 5 payments

= 5 x 1000 − (BV₀ − BV₅)

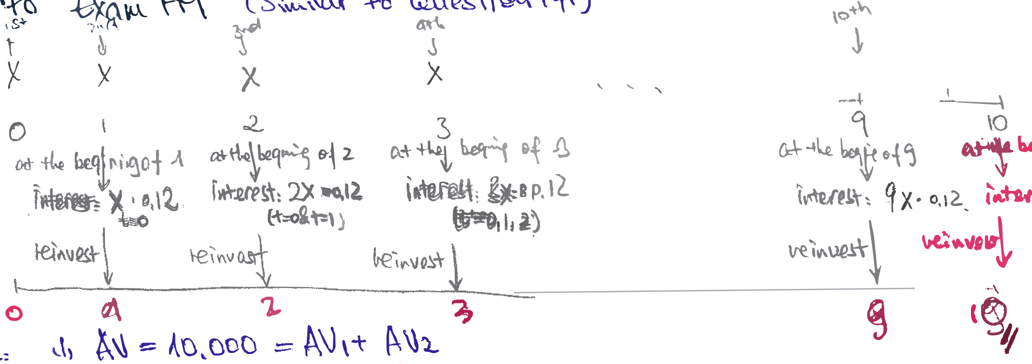
where $BV_0 = 1000 a_{\overline{5}|10\%} + v^5\left(2000 a_{\overline{5}|10\%}\right) = 3790.7867 + v^5 \times 7581.5735 = 8498.347$

$BV_5 = 2000 a_{\overline{5}|10\%} = 7581.5735$

$\Rightarrow BV_0 - BV_5 = 916.7735$

⇒ Total interest paid in the first 5 payments = 5x1000 − 916.7735 = 4083.2265 Ⓓ//

Question 178 Exam FM (Similar to Question 171)



$i = 12\%$ $AV_1 = 10X$

(no interest, since it's all been reinvested)

$i = 8\%$.

$AV_2 = 0.12X \cdot s_{\overline{10|}8\%}$

Give: (1) $AV = 10{,}000 = AV_1 + AV_2$

at the end of 10

Question: X

Solve: $AV = 10000 = AV_1 + AV_2$

where: $AV_1 = 10X$

$AV_2 = 0.12X \cdot s_{\overline{10|}8\%}$

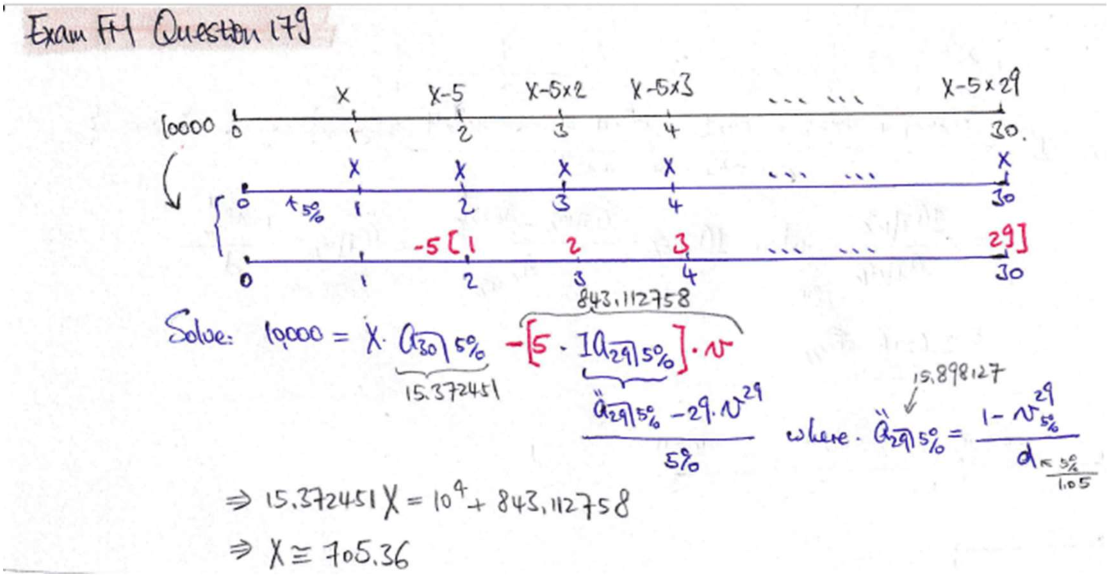
$= 8.4683X$

$\Rightarrow AV_1 + AV_2 = 18.4683X = 10{,}000 \Rightarrow 541.47$

Exam FM Question 179:

An investment of 10,000 is associated with a series of 30 annual payments. The first payment is X, made one year after the investment. Each payment decreases by 5 from the previous one. An annual rate of 5%

Question: what is X?



Exam FM Question 179

Solve: $10000 = X \cdot \underbrace{a_{\overline{30}|5\%}}_{15.372451} - \overbrace{\left[5 \cdot \underbrace{Ia_{\overline{29}|5\%}}_{\frac{\ddot{a}_{\overline{29}|5\%} - 29 \cdot v^{29}}{5\%}}\right] \cdot v}^{843.112758}$

where. $\overset{15.898127}{\ddot{a}_{\overline{29}|5\%}} = \frac{1 - v_{5\%}^{29}}{d_{=\frac{5\%}{1.05}}}$

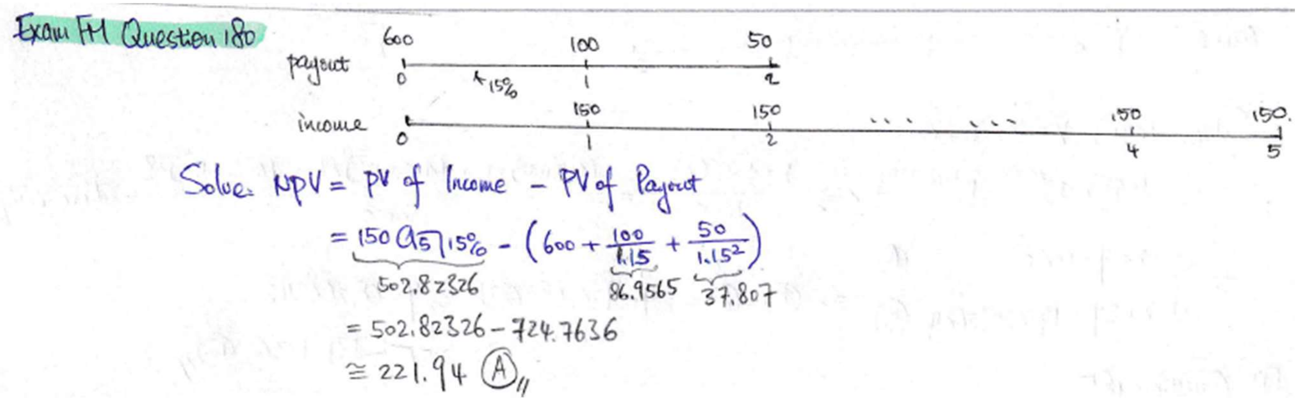
$\Rightarrow 15.372451 X = 10^4 + 843.112758$

$\Rightarrow X \cong 705.36$

Exam FM Question 180:

A project requires an initial investment of 600, additional investments of 100 and 50 at the end of years of one and two. The revenue from this project is 150 per year for 5 years, beginning 1 year from the initial investment. An annual rate of 15%.

Question: what is the net PV of this project?



Exam FM Question 180

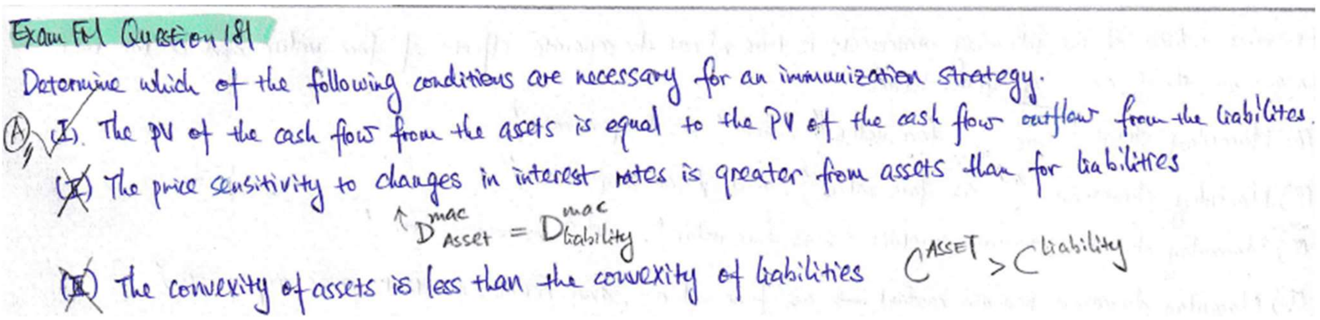
Solve. $NPV = \text{PV of Income} - \text{PV of Payout}$

$= \underbrace{150\, a_{\overline{5}|15\%}}_{502.82326} - \left(600 + \underbrace{\frac{100}{1.15}}_{86.9565} + \underbrace{\frac{50}{1.15^2}}_{37.807}\right)$

$= 502.82326 - 724.7636$

$\cong 221.94$ Ⓐ//

Exam FM Question 181:



Exam FM Question 181

Determine which of the following conditions are necessary for an immunization strategy.

Ⓐ// ✓ ~~(I)~~. The PV of the cash flow from the assets is equal to the PV of the cash flow outflow from the liabilites.

~~(II)~~ The price sensitivity to changes in interest rates is greater from assets than for liabilities

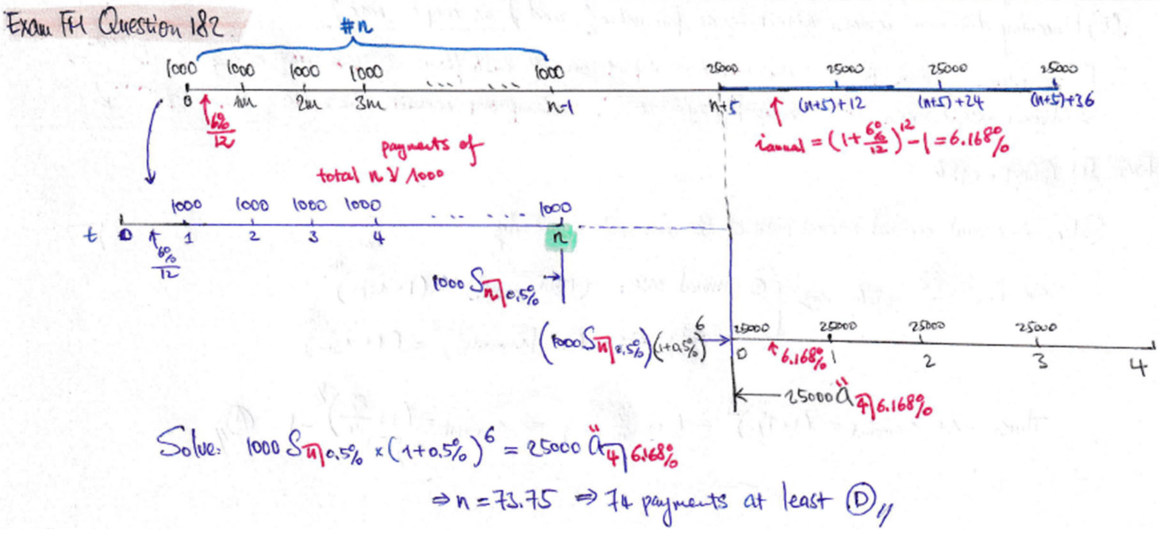
$\uparrow D^{mac}_{Asset} = D^{mac}_{liability}$

~~(III)~~ The convexity of assets is less than the convexity of liabilities $C^{ASSET} > C^{liability}$

Exam FM Question 182:

4 annual tuition payment of 25,000 paid at future time. The payments will be funded by investing 1000 at the beginning of each month. Last deposit will be made 6 months before the first tuition payment. An annual rate of 6% convertible monthly.

Question: what is the minimal number of monthly deposits required to fund the total tuition?



Exam FM Question 182

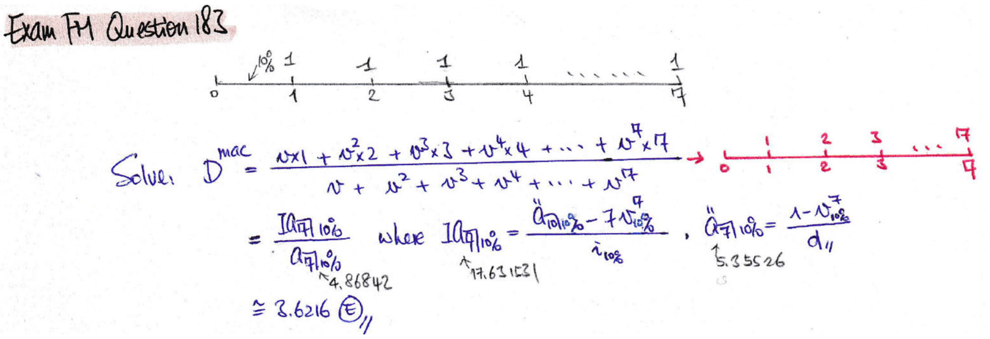
Solve: $1000 s_{\overline{n}|0.5\%} \times (1+0.5\%)^6 = 25000\, \ddot{a}_{\overline{4}|6.168\%}$

$\Rightarrow n = 73.75 \Rightarrow$ 74 payments at least Ⓓ//

Exam FM Question 183:

7-year loan repaid with level payments at end of each year. An annual rate 10%

Question: what is the Macaulay duration of the loan payments?



Exam FM Question 183

Solve: $D^{mac} = \frac{v \times 1 + v^2 \times 2 + v^3 \times 3 + v^4 \times 4 + \cdots + v^7 \times 7}{v + v^2 + v^3 + v^4 + \cdots + v^7}$

$= \frac{Ia_{\overline{7}|10\%}}{a_{\overline{7}|10\%}}$ where $Ia_{\overline{7}|10\%} = \frac{\ddot{a}_{\overline{10}|10\%} - 7v^7_{10\%}}{i_{10\%}}$, $\ddot{a}_{\overline{7}|10\%} = \frac{1 - v^7_{10\%}}{d_{//}}$

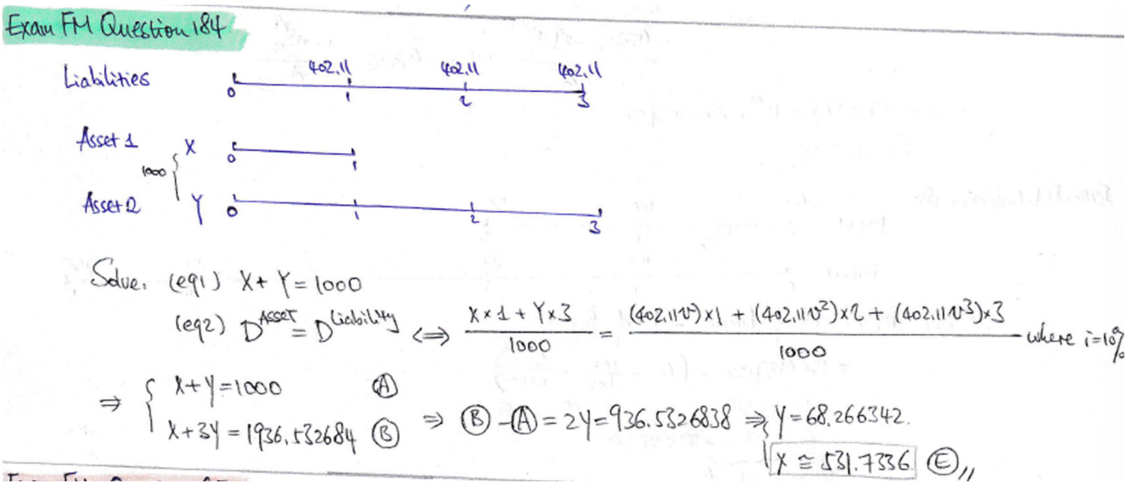
4.86842 17.631531 5.35526

$\cong 3.6216$ Ⓔ//

Exam FM Question 184:

Liabilities of 402.11 due at the end of each of next 3 years. Company matches the duration of its liabilities by investing a total of 1000 in 1-year and 3-year zero-coupon bonds. An annual yield of both bond is 10%

Question: what is the amount the company will invest in 1-year bonds?



Exam FM Question 184

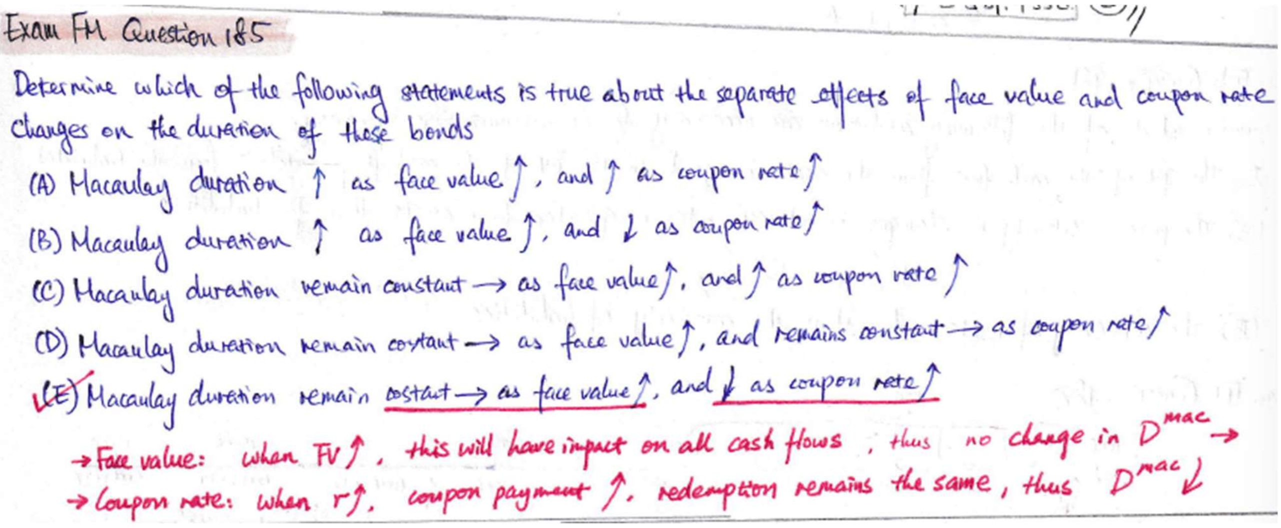
Solve: (eq1) $X + Y = 1000$

(eq2) $D^{Asset} = D^{Liability} \Leftrightarrow \frac{X \times 1 + Y \times 3}{1000} = \frac{(402.11v) \times 1 + (402.11v^2) \times 2 + (402.11v^3) \times 3}{1000}$ where $i = 10\%$

$\Rightarrow \begin{cases} X + Y = 1000 & Ⓐ \\ X + 3Y = 1936.532684 & Ⓑ \end{cases} \Rightarrow Ⓑ - Ⓐ = 2Y = 936.5326838 \Rightarrow Y = 68.266342.$

$\boxed{X \cong 531.7336}$ Ⓔ//

Exam FM Question 185:



Exam FM Question 185

Determine which of the following statements is true about the separate effects of face value and coupon rate changes on the duration of these bonds

(A) Macaulay duration ↑ as face value ↑, and ↑ as coupon rate ↑

(B) Macaulay duration ↑ as face value ↑, and ↓ as coupon rate ↑

(C) Macaulay duration remain constant → as face value ↑, and ↑ as coupon rate ↑

(D) Macaulay duration remain costant → as face value ↑, and remains constant → as coupon rate ↑

✓(E) Macaulay duration remain costant → as face value ↑, and ↓ as coupon rate ↑

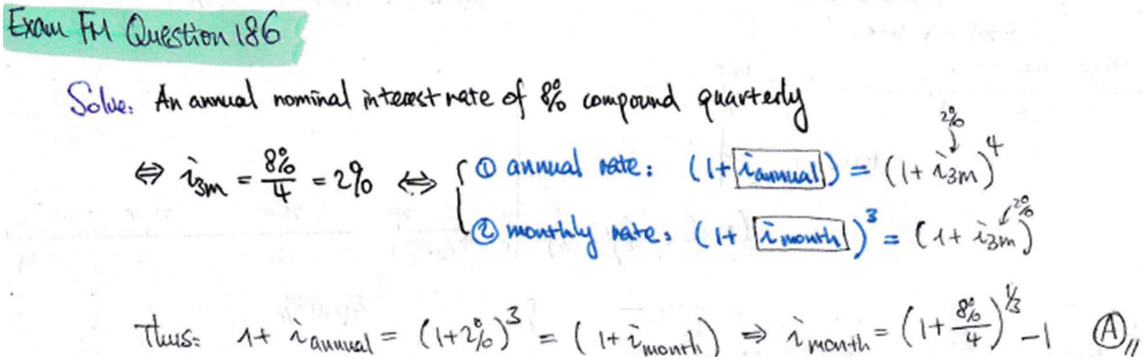
→ Face value: when FV ↑, this will have impact on all cash flows, thus no change in D^{mac} →

→ Coupon rate: when r ↑, coupon payment ↑, redemption remains the same, thus D^{mac} ↓

Exam FM Question 186:

A corporate makes a payment at the end of each month into an account that offers an annual rate of 8% compounded quarterly.

Question: What is the equivalent rate per payment period?



Exam FM Question 186

Solve: An annual nominal interest rate of 8% compound quarterly

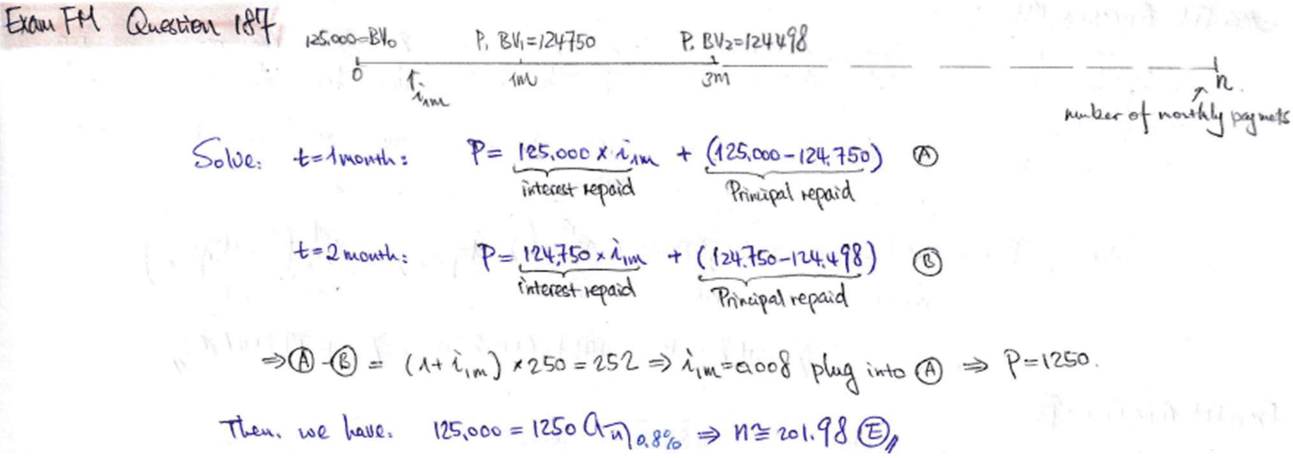
$$\Leftrightarrow i_{3m} = \frac{8\%}{4} = 2\% \Leftrightarrow \begin{cases} \text{① annual rate: } (1+\boxed{i_{annual}}) = (1+\overset{2\%}{i_{3m}})^4 \\ \text{② monthly rate: } (1+\boxed{i_{month}})^3 = (1+\overset{2\%}{i_{3m}}) \end{cases}$$

$$\text{Thus: } 1+i_{annual} = (1+2\%)^3 = (1+i_{month}) \Rightarrow i_{month} = \left(1+\frac{8\%}{4}\right)^{1/3} - 1$$ Ⓐ//

Exam FM Question 187:

125,000 has level payments at the end of each month, an annual rate compounded monthly. The balances owed immediately after the 1st and 2nd payments were 124,750 and 124,498

Question: What is the number of payments needed to pay off the mortgage?



Exam FM Question 187

Solve: t=1 month: $P = \underbrace{125{,}000 \times i_{1m}}_{\text{interest repaid}} + \underbrace{(125{,}000 - 124{,}750)}_{\text{Principal repaid}}$ Ⓐ

t=2 month: $P = \underbrace{124{,}750 \times i_{1m}}_{\text{interest repaid}} + \underbrace{(124{,}750 - 124{,}498)}_{\text{Principal repaid}}$ Ⓑ

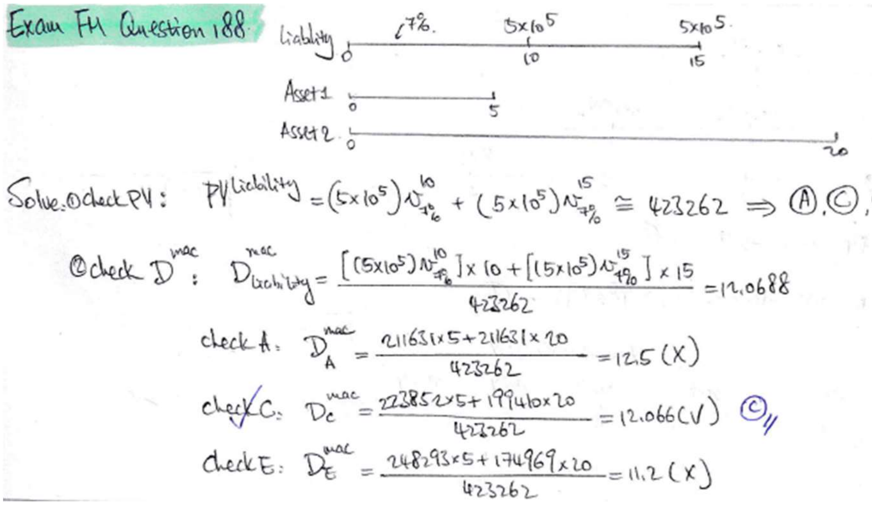
$\Rightarrow$ Ⓐ - Ⓑ $= (1+i_{1m}) \times 250 = 252 \Rightarrow i_{1m} = 0.008$ plug into Ⓐ $\Rightarrow P = 1250.$

Then, we have. $125{,}000 = 1250\, a_{\overline{n}|0.8\%} \Rightarrow n \cong 201.98$ Ⓔ//

Exam FM Question 188:

Need to pay 500,000 10-year from now and 500,000 15-year from now. The company needs to create an investment portfolio using 5-year and 20-year zero-coupon bonds, force of interest 7%, the PV and Macaulay duration of its assets match the liabilities

Question: the amount invested today in each bond?



Exam FM Question 188

Liability: 0, 7%, 5×10^5 at 10, 5×10^5 at 15

Asset 1: 0 to 5

Asset 2: 0 to 20

Solve: ① check PV: $PV^{\text{Liability}} = (5\times10^5)v_{7\%}^{10} + (5\times10^5)v_{7\%}^{15} \cong 423262 \Rightarrow$ Ⓐ, Ⓒ, Ⓔ

② check D^{mac}: $D^{mac}_{\text{Liability}} = \dfrac{[(5\times10^5)v_{7\%}^{10}]\times10 + [(5\times10^5)v_{7\%}^{15}]\times15}{423262} = 12.0688$

check A: $D_A^{mac} = \dfrac{211631\times5 + 211631\times20}{423262} = 12.5\ (X)$

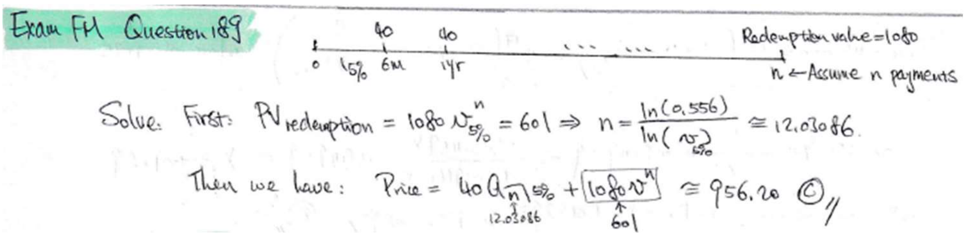
check C: $D_C^{mac} = \dfrac{223852\times5 + 199410\times20}{423262} = 12.066\ (V)$ Ⓒ//

check E: $D_E^{mac} = \dfrac{248293\times5 + 174969\times20}{423262} = 11.2\ (X)$

Exam FM Question 189:

Bond with face value of 1000 and a redemption value of 1080 with annual coupon rate of 8% payable semi-annually. An annual yield rate 10% convertible semi-annually. At this yield rate, PV of redemption is 601

Question: What is the purchase piece of bond?



Exam FM Question 189

40 at 6M, 40 at 1yr, … …, Redemption value = 1080 at n ← Assume n payments; 0, 5%

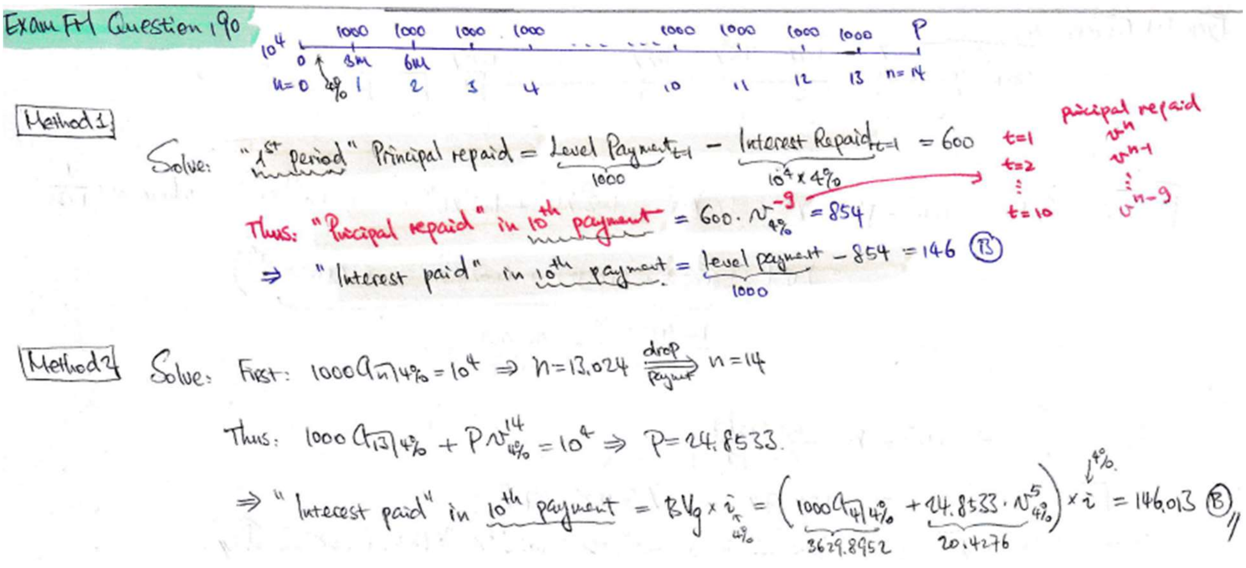
Solve: First: $PV_{\text{redemption}} = 1080\,v_{5\%}^{n} = 601 \Rightarrow n = \dfrac{\ln(0.556)}{\ln(v_{5\%})} \cong 12.03086$

Then we have: $\text{Price} = 40\,a_{\underset{12.03086}{\overline{n}|}5\%} + \boxed{\underset{601}{1080\,v^{n}}} \cong 956.20$ Ⓒ//

Exam FM Question 190:

A loan of 10,000 repaid by payments of 1,000 at the end of each quarter, plus a drop payment. The annual rate of interest is 16% convertible quarter

Question: What is the amount of interest in the 10th payment



Exam FM Question 190

10^4 at 0; 1000 at 3M, 1000 at 6M, 1000, 1000, … …, 1000, 1000, 1000, 1000, P

n = 0, 4%, 1, 2, 3, 4, 10, 11, 12, 13, n = 14

Method 1

Solve: "1st period" Principal repaid $= \underbrace{\text{Level Payment}_{t=1}}_{1000} - \underbrace{\text{Interest Repaid}_{t=1}}_{10^4\times4\%} = 600$

principal repaid: t=1 v^{n}; t=2 v^{n-1}; ⋮; t=10 v^{n-9}

Thus: "Principal repaid" in 10th payment $= 600\cdot v_{4\%}^{-9} = 854$

$\Rightarrow$ "Interest paid" in 10th payment $= \underbrace{\text{level payment}}_{1000} - 854 = 146$ Ⓑ

Method 2

Solve: First: $1000\,a_{\overline{n}|4\%} = 10^4 \Rightarrow n = 13.024 \xrightarrow[\text{payment}]{\text{drop}} n = 14$

Thus: $1000\,a_{\overline{13}|4\%} + P\,v_{4\%}^{14} = 10^4 \Rightarrow P = 24.8533$

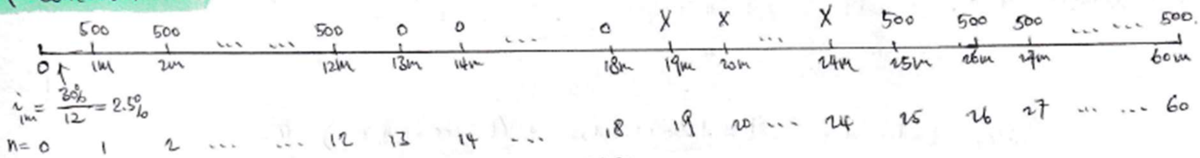
$\Rightarrow$ "Interest paid" in 10th payment $= BV_9 \times \underset{4\%}{i} = \Big(\underbrace{1000\,a_{\overline{4}|4\%}}_{3629.8952} + \underbrace{24.8533\cdot v_{4\%}^{5}}_{20.4276}\Big)\times \overset{4\%}{i} = 146.013$ Ⓑ//

Exam FM Question 191:

5-year loan has an annual rate of 30% convertible monthly. The loan is repaid with level monthly payments of 500 beginning 1 month after the date of loan. The borrower miss the 13th through 18th payments, but increases the next 6 payments to X so that the final 36 payments of 500 will repay the loan

Question: What is X?

Exam FM Question 191



$i_{1m} = \frac{30\%}{12} = 2.5\%$

Solve

$$\text{Price} = \overbrace{\underbrace{500\,a_{\overline{60}|2.5\%}}_{15454.328}}^{\text{original}} = \overbrace{\underbrace{500\,a_{\overline{12}|2.5\%}}_{5128.8823} + \underbrace{v^{18}_{2.5\%}}_{0.64166}\left(X\cdot \underbrace{a_{\overline{6}|2.5\%}}_{5.5081}\right) + \underbrace{v^{24}_{2.5\%}}_{0.552875}\left(\underbrace{500\,a_{\overline{36}|2.5\%}}_{11778.12553}\right)}^{\text{now}}$$

$$\Rightarrow 3.5316X = 10325.4457 - 6511.8311 \Rightarrow X \cong 1079.8546 \text{ Ⓒ} //$$

Exam FM Question 192:

3 liabilities of 1000 due at the times 1, 2, and 3 in years. There are 3 bond available to match these liabilities:

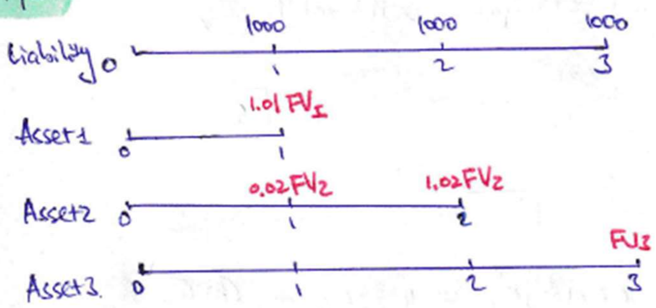
Bond I: bond due at the end of period 1 with a coupon rate of 1% per year, an annual yield rate of 14%

Bond II: bond due at the end of period 2 with a coupon rate of 2% per year, an annual yield rate of 15%

Bond III: zero-coupon bond due at time 3, an annual yield rate of 18%

Question: what is the face value of each bond that needed to exactly match the liabilities

Exam FM Question 192.

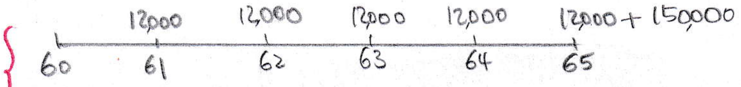


Solve: time 3: $FV_3 = 1000 \Rightarrow$ Ⓐ or Ⓔ

time 2: $1.02FV_2 = 1000 \Rightarrow FV_2 \cong 980.392 \Rightarrow$ Ⓐ //

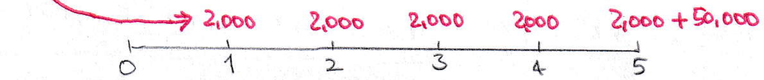
Exam FM Question 193

Give: ① retirment package, 60 yr old retiree, pay 12,000 per year, at the end of each year, till 65yr old, plus a lump sum of 150,000 at age 65



② portfolio: bond 1: 5-year bond, face amount of 100,000, coupon rate 10%

bond 2: 5-year bond.



⇒ coupon = 2,000, Par value = 50,000 } ⇒ $\boxed{\text{coupon rate} = \frac{2{,}000}{50{,}000} = 4\%}$

Question: which one is correct? (since: coupon rate = 4%, can only choose A or B)

(A) Bond with price 42,015, $\boxed{\text{coupon rate } 4\%}$, i is 8%

(B) Bond with Price 50,000, $\boxed{\text{coupon rate } 4\%}$, i is 8%

Solve: Check Bond Price $= 2000\, a_{\overline{5}|8\%} + 50{,}000\, v^{8}$ where $i = 8\%$

$= 42{,}014.58$ Ⓐ //

Exam FM Question 194

Give: ① Par value (F) = 2000

② Redemption Value (C) = 2250

③ 30-year Bond

④ Coupon rate (r) = 2 × Yield rate (i); Coupon paid annually

⑤ Purchase price is 3609.29

⑥ $D_{mac} = 14.41$

$\boxed{r = 2 \cdot i}$

Solve: $3609.29 = 2000 \times \boxed{\overset{r = 2i}{r\%}}\ \boxed{\overset{\frac{1-v^{30}}{i}}{a_{\overline{20}|i}}} + 2250 \cdot v^{30}$

$= 2000 \times (2i) \times \frac{1-v^{30}}{i} + 2250 \cdot v^{30} \Rightarrow i \cong 0.05125$

Question: Modified Duration

$D_{mac} = (1+i) \cdot D_{mod} \Rightarrow D_{mod} = \frac{1}{1+\underset{0.05125}{i}} \overset{14.41}{D_{mac}} \cong 13.707$ Ⓒ //

(1) give: coupon rate & yield i relationship, use $a_{\overline{n}|} = \frac{1-v^{n}}{i} \Rightarrow v^{n} \Rightarrow i$

(2) $D_{mac} = (1+i) \cdot D_{mod}$

Exam FM Question 195

Give: ① Stock: paying annual dividends, starting 5-year from now

② Annual effective rate: 10%, Modified Duration = D

Question: D.

Solve: $D_{mac} = (1+i)\cdot D_{mod} \Rightarrow \boxed{D_{mod} = \frac{1}{1+i}\cdot D_{mac}}$ where $i = 10\%$

$$D_{mac} = \frac{(1\cdot v^5)\times 5 + (1\cdot v^6)\times 6 + (1\times v^7)\times 7 + \cdots}{(1\cdot v^5) + (1\cdot v^6) + (1\times v^5) + \cdots}$$

$$= \frac{-[\underset{0.90909}{\uparrow v} + \underbrace{v^2\times 2}_{1.65289} + \underbrace{v^3\times 3}_{2.25394} + \underbrace{v^4\times 4}_{2.73205}] + [\underbrace{v + v^2\times 2 + v^3\times 3 + v^4\times 4 + \underline{v^5\times 5 + v^6\times 6 + v^7\times 7 + \cdots}}_{I\ddot{a}_{\overline{\infty}|} = \frac{1}{i} + \frac{1}{i^2}}] \quad = \frac{1}{0.1} + \frac{1}{0.1^2} = 10 + 100 = 110.}{v^5 [\underbrace{1 + v + v^2 + \cdots}_{\frac{1}{1-v}}] \quad \leftarrow 0.62092 \times 11}$$

where $i = 10\%$, $v = \frac{1}{1+10\%}$

$\cong 15$

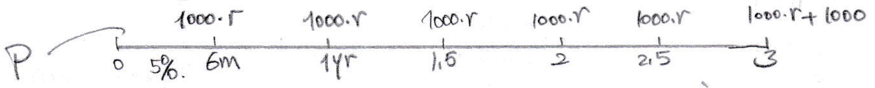
$\Rightarrow D_{mod} = \frac{1}{1+10\%}\times 15 \cong 13.63$ Ⓐ //

Exam FM Question 196

Give: ① 3-year bond; Face value of \$1000; coupon semi-annually; bond is redeemable at face value. It is bought at a price to produce $i_{annual} = 10\%$ convertible semi-annually

② If the term is doubled, yield remains the same, the price will decrease by 49

Question: coupon payment



↑ state: "convertible semi-annually", thus, $i_{6m} = \frac{i}{2}$

Solve: ①: $P = 1000\cdot r\%\cdot a_{\overline{6}|5\%} + 1000\cdot v^6$ where $i = 5\%$, $v = \frac{1}{1+i} = \frac{1}{1.05}$

②: $P - 49 = 1000\cdot r\%\cdot a_{\overline{12}|5\%} + 1000\cdot v^{12}$ where $i = 5\%$.

$\Rightarrow$ ① − ②: $49 = 1000\cdot r\%\,(\underbrace{a_{\overline{6}|5\%}}_{5.0757} - \underbrace{a_{\overline{12}|5\%}}_{8.86325}) + 1000\,(\underbrace{v^6}_{0.746215} - \underbrace{v^{12}}_{0.556837})$ $\Rightarrow r \cong 3.7063\%$.

Thus: Coupon $= 1000\cdot r\% \cong 37.063$ Ⓐ //

Exam FM Question 197

Give: 3 different assets' cash flows

t (in years)	0	1	2	3
X	102,400	-192,000	0	100,000
Y	158,400	-342,000	100,000	100,000
Z	-89,600	288,000	100,000	-300,000

Determine: which set is "Redington Immunized" for $i_{annual} = 25\%$

Solve: "3 Conditions" of "Redingto Immunized": using "NPV version":

Condition 1: $NPV = 0$

Condition 2: $\frac{\partial NPV}{\partial i} = 0$

Condition 3: $\frac{\partial NPV}{\partial i^2} > 0$

Ⓐ// (okay) **Check X**: Condition 1: $NPV = 102{,}400 - \underbrace{192{,}000v}_{153600} + \underbrace{100{,}000v^3}_{51200}$ where $v = \frac{1}{1+25\%}$ $\Rightarrow NPV = 0$ (check "✓")

Condition 2. ∵ $\boxed{\frac{\partial v^n}{\partial i}} = \left[\left(\frac{1}{1+i}\right)^n\right]' = [(1+i)^{-n}]' = [-n(1+i)^{-n-1}] = \boxed{-n \cdot v^{n+1}} \leftarrow \frac{\partial NPV}{\partial i}$

$$\therefore \boxed{NPV'} = \frac{dNPV}{di} = \frac{d(102{,}400 - 192{,}000v + 100{,}000v^3)}{di} = \underbrace{0 - 192000 \cdot \underbrace{(v^1)'}_{-1\cdot v^2}}_{122{,}880} + \underbrace{100{,}000 \underbrace{(v^3)'}_{-3\cdot v^{3+1}}}_{-122{,}800} \overset{v=\frac{1}{1+25\%}}{=} \boxed{0} \text{ (check "✓")}$$

Condition 3. ∵ $\boxed{\frac{\partial(-n \cdot v^{n+1})'}{\partial i}} = [-n(1+i)^{-(n+1)}]' = [+n(n+1) \cdot (1+i)^{-(n+2)}]$

$= \boxed{[+n(n+1) \cdot v^{(n+2)}]} \leftarrow \frac{\partial^2 NPV}{\partial i^2}$

$$\therefore \boxed{NPV''} = \frac{d^2NPV}{di^2} = (102{,}400 - 192{,}000v + 100{,}000v^3)'' = \underbrace{-192{,}000 \times [1 \times 2 \times v^3]}_{-196608} + \underbrace{100{,}000 \times [3 \times 4 \times v^5]}_{393216} \quad v = \frac{1}{1+25\%}$$

$= 196608 \boxed{> 0}$ (check "✓")

Check Y: condition 1 (✓); condition 2 (X); condition 3 (X)

Check Z: condition 1 (x); condition 2 (X); condition 3 (x)

Exam FM Question 198

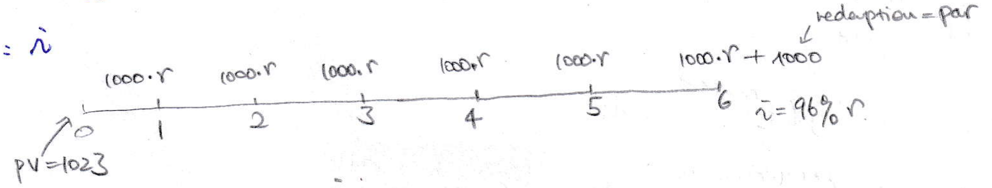
Give: ① 10-year bond, par value 1000, coupon paid annually with coupon rate r ; Price 1023

② Callable: at the end of 6th, 7th, 8th, or 9th year

③ worst case senario for bond investor ⟺ call at earlist 6th, since $\overset{1023}{P} > \overset{par=1000}{C}$, buy at premium

④ $i = 96\% \cdot r$

Question: i



Solve: $1023 = (1000 \cdot r) \overset{\frac{i}{0.96}}{} \boxed{a_{\overline{6}|0.96r}} + 1000 \cdot v^6$ where $i = 0.96r$

$= 1000 \cdot \frac{i}{0.96} \cdot \boxed{\frac{1-v^6}{i}} + 1000 \cdot v^6$

$\Leftrightarrow 1023 = 1041.6667(1 - v^6) + 1000 \cdot v^6$

$\Leftrightarrow 41.6667 v^6 = 1041.6667 - 1023 = 18.6667$

$\Leftrightarrow v = 0.448^{\frac{1}{6}}$

$\Leftrightarrow \overset{\frac{1}{1+i}}{v} \cong 0.87474$

$\Rightarrow i \cong 14.32\%$ Ⓔ//

① $a_{\overline{n}|i} = \frac{1-v^n}{i}$

② Premium: worst cast, call at earlist.

Exam FM Question 199

Question: which of the following is false, regarding to "immunization":

(A) Yield curve is flat (correct)

(B) Parallel shifts in the yield (correct)

(C) Designed only for small changes in the interest rate (correct)

(D) "Modified duration" of "asset" = "Modified duration" of "liability" (correct)

$$\left.\begin{aligned} D_{mac}^{Asset} &= D_{mac}^{liability} \\ D_{mac} &= (1+i)\,D_{mod} \end{aligned}\right\} \Rightarrow D_{mod}^{Asset} = D_{mod}^{liability}$$

(E) "Convexity of Asset" $\overset{}{\underset{>}{\neq}}$ "Convexity of liability" Ⓔ//

Exam FM Question 200.

Give: ① f(x): continuous rate: $100e^{0.5t}$ from t=1 to t=3

② force of interest: $\delta = 8\%$

Question: AV at t=4

∵ "AV", "NOT PV"

Solve: $$AV = \boxed{\int_1^3 \underbrace{f(t)}_{100e^{0,5t}} \cdot e^{+\int_t^4 8\%\,dt}\,dt}$$

$$= \int_1^3 (100e^{0.5t})(e^{0,08(4-t)})\,dt.$$

$$= 100\int_1^3 e^{0,5t+0,32-0,08t}\,dt.$$ $$\boxed{e^a \cdot e^b = e^{a+b}}$$

$$= 100e^{0.32}\int_1^3 e^{0,42t}\,dt$$

$$= 100e^{0.32}\int_1^3 \frac{1}{0.42}\,d\,e^{0.42t}$$

$$= \frac{100e^{0.32}}{0.42}\underbrace{(e^{0.42\times3} - e^{0.42\times1})}_{2.003460}$$ $$\boxed{e^{0.42t}\Big|_1^3 = e^{0.42\times3} - e^{0.42\times1}}$$

≅656.90. Ⓔ//

Exam FM Question 201:

The following information about a perpetuity with annual payments:

(i) First 15 payments are each 2500, first payment made 3 years from now

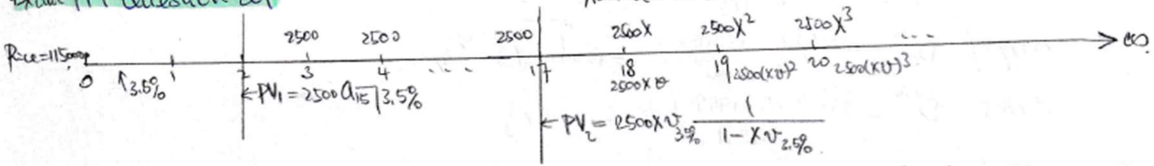
(ii) Beginning with the 16th payment, each payment is k% larger than the previous one

(iii) Using an annual rate of 3.5%, the PV of the perpetuity is 115,000

Question: What is k?

Exam FM Question 201

Assumed $(1+k\%)=X$



$$\text{Solve: Price} = 115.000 = \underbrace{v^2}_{0.93351}\Big(\underbrace{2500a_{\overline{15}|3.5\%}}_{28793.52724}\Big) + \underbrace{v^{17}}_{0.5172}\Big(2500Xv_{2.5\%}\frac{1}{1-Xv_{3.5\%}}\Big) \quad \text{where } v=\frac{1}{1.035}\approx 0.96618$$

$$\Rightarrow 2500Xv\frac{1}{1-Xv} = 158149.59 \Rightarrow \frac{2415.459X}{1-0.96618X} = 158149.59 \Rightarrow X \cong 1.018893.$$

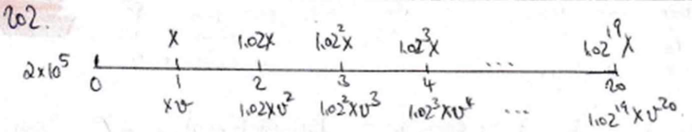
Recall: we assume $1+k\% = X = 1.018893 \Rightarrow k \cong 1.8893\%$ Ⓔ,,

Exam FM Question 202:

An annuity with 20 annual payments, first payment beginning 1 year from today and each subsequent payment 2% greater than the previous one. An annual rate of 3%, the PV of the annuity is 200,000

Question: What is the amount of the final payment from this annuity?

Exam FM Question 202.



$$\text{Solve: } 2\times10^5 = PV = Xv + 1.02Xv^2 + 1.02^2Xv^3 + 1.02^3Xv^4 + \dots + 1.02^{19}Xv^{20} \quad \text{where } v=\frac{1}{1.03}$$

$$= Xv\Big(\underbrace{1+1.02v+(1.02v)^2+(1.02v)^3+\dots+(1.02v)^{19}}_{\frac{1-(1.02v)^{20}}{1-1.02v}\nwarrow\frac{a_1-a_nq}{1-q}}\Big)$$

$$\Rightarrow 2\times10^5 = Xv\cdot\frac{0.177267}{0.0097087}$$

$$= 17.72677X \Rightarrow X \cong 11282.3718$$

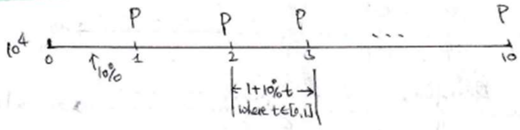
Notice: Question is about "final payment" $\Rightarrow 1.02^{19}X \cong 16436.2853$ Ⓑ,,

Exam FM Question 203: **(204. DELETED)**

A loan of 10,000 repaid by level annual payments at the end of each year for 10 years. The annual rate is 10% for the loan. This is also the rate that bank used to compute the annual payment and the balance on the loan at the end of each year. However, the rate used for balances during each year is 10%

Question: What is the balance of the loan halfway through the 4th year?

Exam FM Question 203



Solve. First, we get "Level payment" P: $10^4 = P\,a_{\overline{10}|10\%} \Rightarrow P \cong 1627.454$.

Second: Balance of the loan halfway through the 4th year $= BV_3 \times (1 + 10\% \times \overset{t}{\boxed{\tfrac{1}{2}}})$

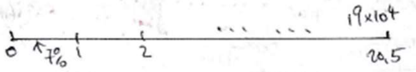
where $BV_3 = \underset{1627.454}{P} \cdot a_{\overline{7}|10\%} = 7923.12743$

Thus: Question $= 7923.12743 \times (1 + 10\% \times \frac{1}{2}) \cong 8319.28$ Ⓔ//

Exam FM Question 205:

Need to pay 190,000 in 20.5 years. An investment portfolio using 3 bonds with annual coupons. Redington immunized, an annual rate of 7%. D_mac of Bond A, B, C is 10, 15, and 30 years. Investment amount in Bond B is twice the amount in Bond B. Question: What is the amount the company invests in Bond A?

Exam FM Question 205



$D_A^{mac} = 10$, $D_B^{mac} = 15$, $D_C^{mac} = 30$

PV Assume: X, PV, $2PV$

Solve: (eq1) $20.5 = \dfrac{X \times 10 + PV \times 15 + (2PV) \times 30}{X + PV + 2PV}$ plug in $\Longrightarrow PV = 11075.49$.

(eq2) $X + PV + 2PV = \dfrac{19\times10^4}{1.07^{20.5}} = 47466.3851 \Rightarrow X = 47466.3851 - 3P$

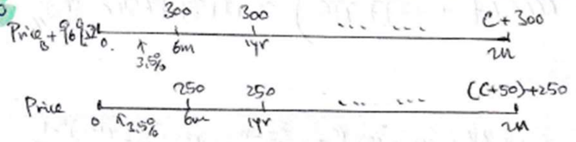
Once we get: $PV = 11075.49$, plug into (eq1) $\Rightarrow X \cong 14239.915$ Ⓔ//

Exam FM Question 206:

Bond X and Y are n-year bond, FV of 10,000, semi-annually coupons, an annual rate of 7% convertible semi-annually. Bond X has 6% annual coupon rate, redemption value c. Bond Y has 5% annual coupon rate, redemption value c+50. Price of Bond X exceeds the price of Bond Y by 969.52

Question: What is n?

Exam FM Question 206



Solve: (eq1) $Price_B + 969.52 = 300\,a_{\overline{2n}|3.5\%} + C \cdot v_{3.5\%}^{2n}$ Ⓐ

(eq2) $Price_B = 250\,a_{\overline{2n}|2.5\%} + (C+50)\,v_{3.5\%}^{2n}$ Ⓑ

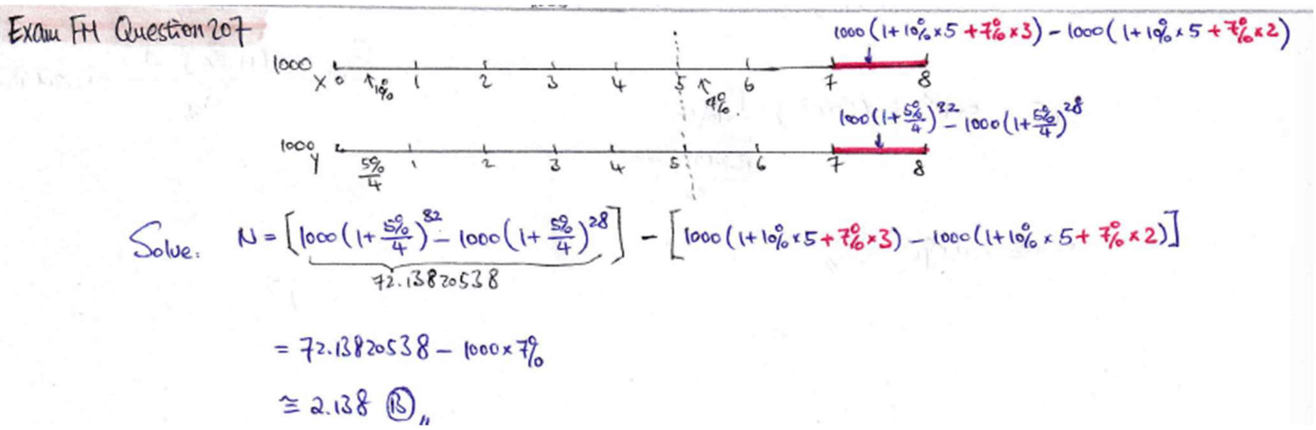
Ⓐ−Ⓑ: $50\,\underbrace{a_{\overline{2n}|3.5\%}}_{\frac{1-v_{3.5\%}^{2n}}{i \leftarrow 3.5\%}} - 50 \cdot v_{3.5\%}^{2n} = 969.52 \Rightarrow v_{3.5}^{2n} = 0.31047$

Thus: $2n = \dfrac{\ln(0.31047)}{\ln(\underbrace{v}_{\frac{1}{1.035}})} = 34 \Rightarrow n = 17$

Exam FM Question 207:

Deposit 1000 in each of Bank X and Bank Y. Bank X is associated with simple interest at an annual rate of 10% for the first 5-year and 7% thereafter. Bank Y's interest at an annual rate of 5% compounded quarterly. The interest credited in the 8th year by Bank Y exceeds the interest credited in the 8th year by Bank X by N

Question: What is the absolute value of N?



Exam FM Question 207

$1000(1+10\%\times5+7\%\times3)-1000(1+10\%\times5+7\%\times2)$

$1000(1+\frac{5\%}{4})^{32}-1000(1+\frac{5\%}{4})^{28}$

Solve. $N=\Big[\underbrace{1000(1+\frac{5\%}{4})^{32}-1000(1+\frac{5\%}{4})^{28}}_{72.13820538}\Big]-\Big[1000(1+10\%\times5+7\%\times3)-1000(1+10\%\times5+7\%\times2)\Big]$

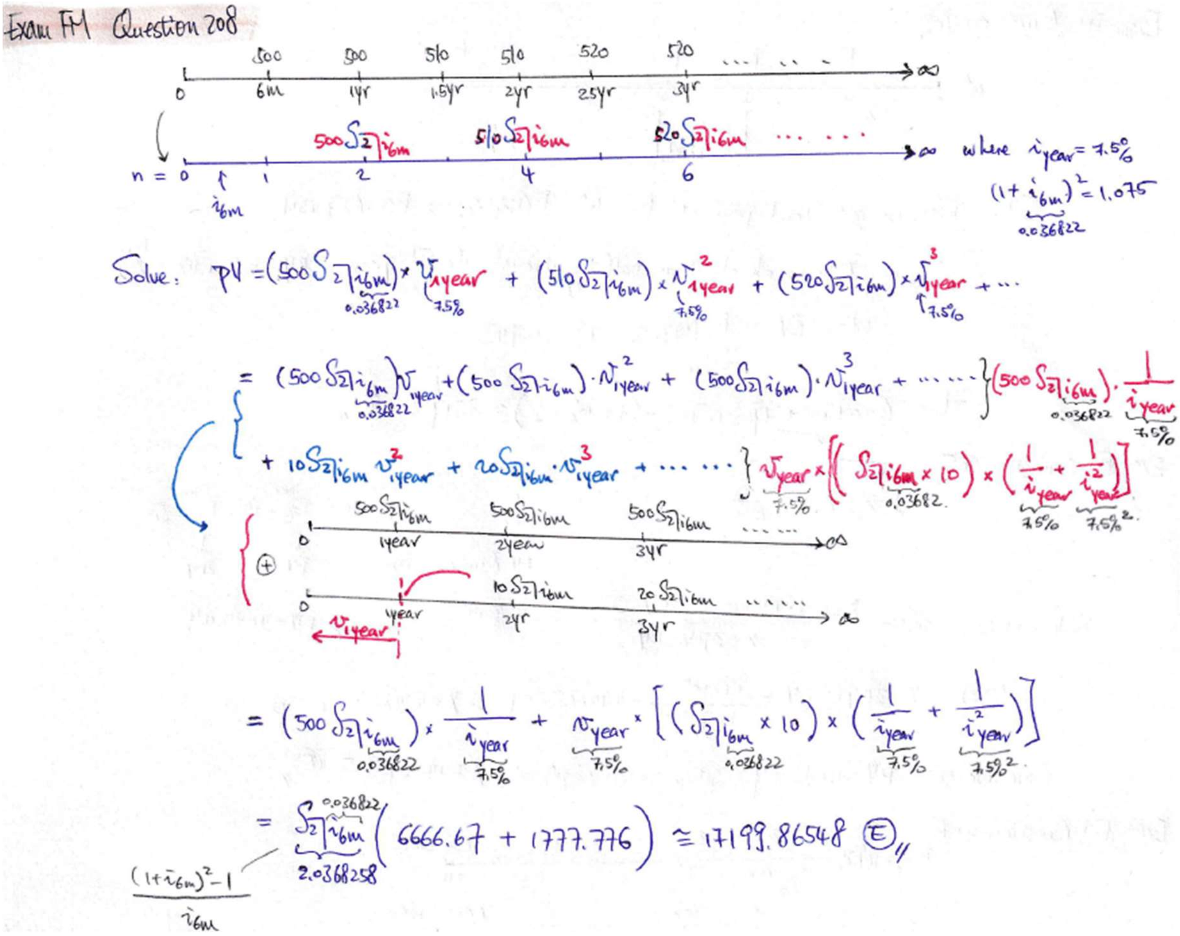
$=72.13820538-1000\times7\%$

$\simeq 2.138$ Ⓑ〃

Exam FM Question 208:

Scholarship fund pays a sum of the money twice a year. Scholarship pays out 500 at the end of 6 months and another 500 at the end of 1 year. Every year thereafter, the 2 semi-annual payments will be increase by 10. For example, in year two, both payments will be 510 and in year three both payments will be 520. The scholarship fund earns an annual rate of 7.5%

Question: What is the fund balance needed today to provide this scholarship



Exam FM Question 208

where $i_{year}=7.5\%$

$(1+\underbrace{i_{6m}}_{0.036822})^2=1.075$

Solve. $PV=(500S_{\overline{2}|\underbrace{i_{6m}}_{0.036822}})\times \underset{7.5\%}{v_{1year}}+(510S_{\overline{2}|i_{6m}})\times \underset{7.5\%}{v^2_{1year}}+(520S_{\overline{2}|i_{6m}})\times \underset{7.5\%}{v^3_{1year}}+\cdots$

$=(500S_{\overline{2}|\underbrace{i_{6m}}_{0.036822}})v_{1year}+(500S_{\overline{2}|i_{6m}})\cdot v^2_{1year}+(500S_{\overline{2}|i_{6m}})\cdot v^3_{1year}+\cdots\cdots\}(500S_{\overline{2}|\underbrace{i_{6m}}_{0.036822}})\cdot\frac{1}{\underbrace{i_{year}}_{7.5\%}}$

$+10S_{\overline{2}|i_{6m}}v^2_{1year}+20S_{\overline{2}|i_{6m}}\cdot v^3_{1year}+\cdots\cdots\}\ \underbrace{v_{year}}_{7.5\%}\times\Big[\Big(S_{\overline{2}|\underbrace{i_{6m}}_{0.03682}}\times10\Big)\times\Big(\frac{1}{\underbrace{i_{year}}_{7.5\%}}+\frac{1}{\underbrace{i^2_{year}}_{7.5\%^2}}\Big)\Big]$

$=(500S_{\overline{2}|\underbrace{i_{6m}}_{0.036822}})\times\frac{1}{\underbrace{i_{year}}_{7.5\%}}+\underbrace{v_{year}}_{7.5\%}\times\Big[\Big(S_{\overline{2}|\underbrace{i_{6m}}_{0.036822}}\times10\Big)\times\Big(\frac{1}{\underbrace{i_{year}}_{7.5\%}}+\frac{1}{\underbrace{i^2_{year}}_{7.5\%^2}}\Big)\Big]$

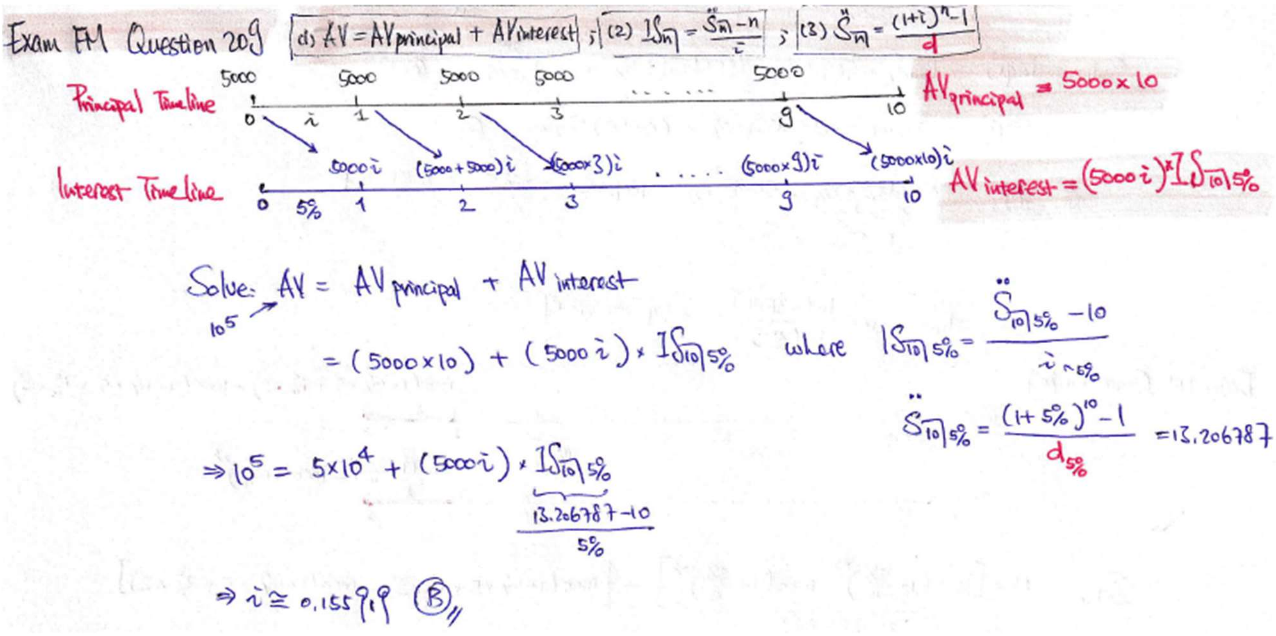
$=\underbrace{S_{\overline{2}|\overset{0.036822}{i_{6m}}}}_{2.0368258}\Big(6666.67+1777.776\Big)\simeq 17199.86548$ Ⓔ〃

$\frac{(1+i_{6m})^2-1}{i_{6m}}$

Exam FM Question 209:

Sam deposits 5000 at the beginning of each year for 10-year. An annual rate of i. All interest rate reinvested at an annual rate 5%. Sam has 100,000 at the end of 10-year

Question: What is i?



Exam FM Question 209 $\boxed{(1)\ AV = AV_{principal} + AV_{interest}}$; $\boxed{(2)\ I\ddot{s}_{\overline{n}|} = \frac{\ddot{s}_{\overline{n}|} - n}{i}}$; $\boxed{(3)\ \ddot{s}_{\overline{n}|} = \frac{(1+i)^n - 1}{d}}$

Principal Timeline $AV_{principal} = 5000 \times 10$

Interest Timeline $AV_{interest} = (5000i) \times I\ddot{s}_{\overline{10}|5\%}$

Solve: $\overset{10^5 \nearrow}{AV} = AV_{principal} + AV_{interest}$

$= (5000 \times 10) + (5000i) \times I\ddot{s}_{\overline{10}|5\%}$ where $I\ddot{s}_{\overline{10}|5\%} = \frac{\ddot{s}_{\overline{10}|5\%} - 10}{i_{=5\%}}$

$\ddot{s}_{\overline{10}|5\%} = \frac{(1+5\%)^{10} - 1}{d_{5\%}} = 13,206787$

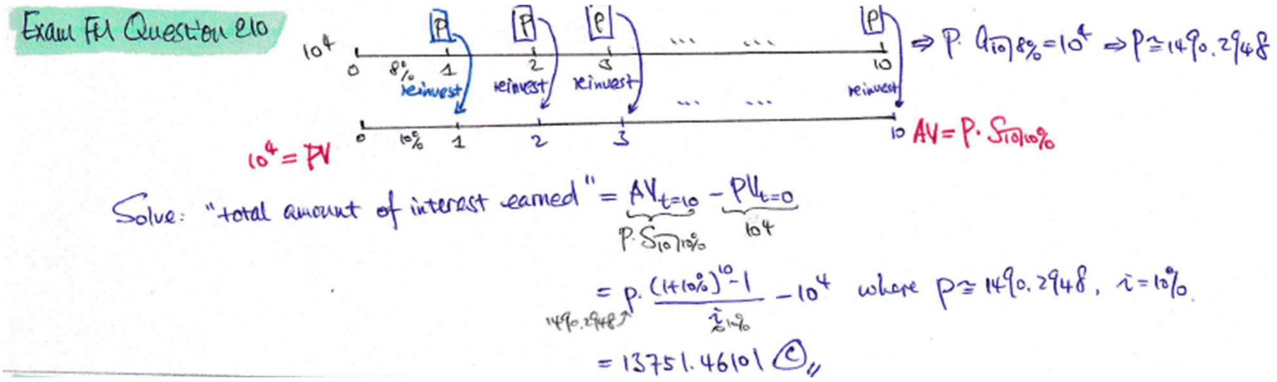
$\Rightarrow 10^5 = 5 \times 10^4 + (5000i) \times \underbrace{I\ddot{s}_{\overline{10}|5\%}}_{\frac{13.206787 - 10}{5\%}}$

$\Rightarrow i \cong 0,155919$ Ⓑ//

Exam FM Question 210:

Loan of 10,000 repaid at an annual rate of 8% with level payments at the end of year for 10-year. The lender immediately deposits each payment into an account earning an annual rate 10%

Question: What is the total amount of interest earned?



Exam FM Question 210

$\Rightarrow P \cdot a_{\overline{10}|8\%} = 10^4 \Rightarrow P \cong 1490.2948$

$AV = P \cdot s_{\overline{10}|10\%}$

$10^4 = PV$

Solve: "total amount of interest earned" $= \underbrace{AV_{t=10}}_{P \cdot s_{\overline{10}|10\%}} - \underbrace{PV_{t=0}}_{10^4}$

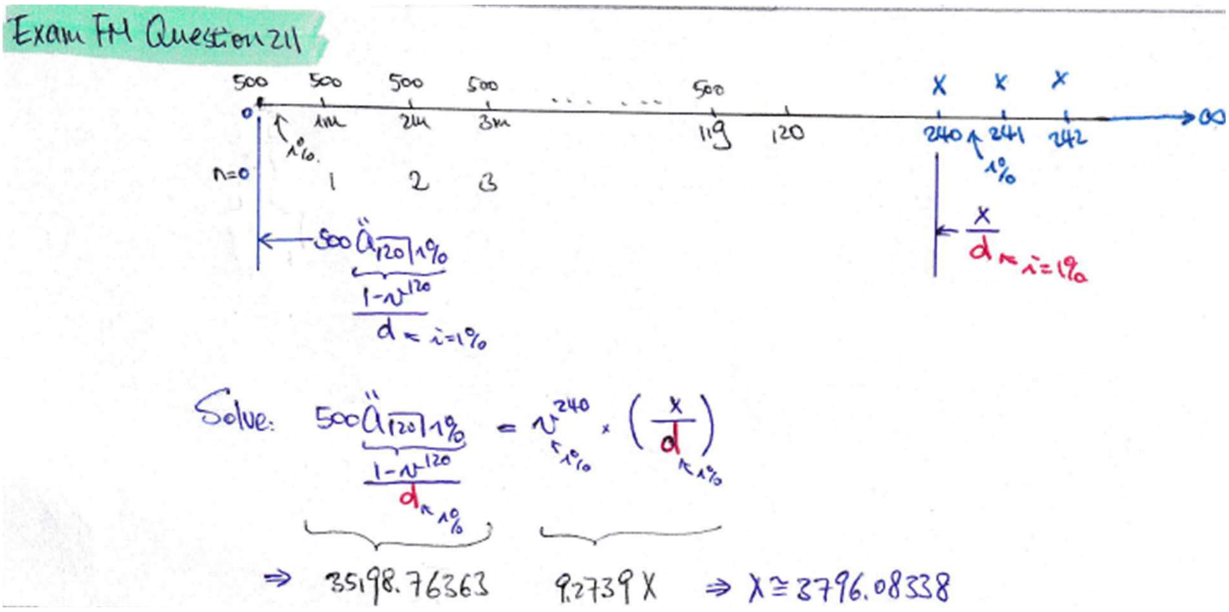
$= \overset{}{\underset{1490.2948 \nearrow}{P}} \cdot \frac{(1+10\%)^{10} - 1}{i_{10\%}} - 10^4$ where $P \cong 1490.2948$, $i = 10\%$

$= 13751.46101$ Ⓒ//

Exam FM Question 211:

A fund accumulates at an annual rate of 12% convertible monthly. Beginning today, the worker will deposit 500 monthly for 10-year. No deposit or withdrawals made for subsequent 10-year. Exactly 20 years from today, monthly payments of X will be made to this charity

Question: What is X?



Exam FM Question211

$-500\underbrace{\ddot{a}_{\overline{120}|1\%}}_{\frac{1-v^{120}}{d} \leftarrow i=1\%}$

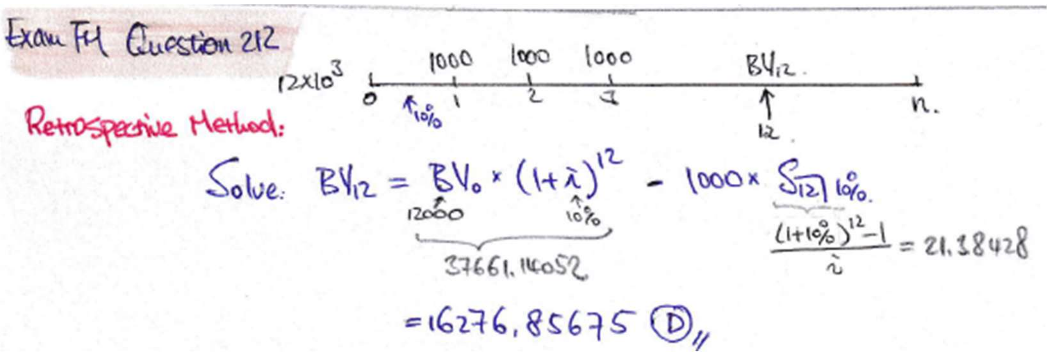
$\frac{X}{d} \leftarrow i=1\%$

Solve: $\underbrace{500\underbrace{\ddot{a}_{\overline{120}|1\%}}_{\frac{1-v^{120}}{d \leftarrow 1\%}}}_{\Rightarrow 35198.76363} = \underbrace{\underset{\nwarrow 1\%}{v^{240}} \times \left(\frac{X}{\underset{\nwarrow 1\%}{d}}\right)}_{9.2739X} \Rightarrow X \cong 3796.08338$

Exam FM Question 212

Payments of 1000 at the end of each year paid on loan of 12,000. An annual rate of 10%

Question: What is the outstanding loan balance immediately after the 12th payment?



Exam FM Question 212

Retrospective Method:

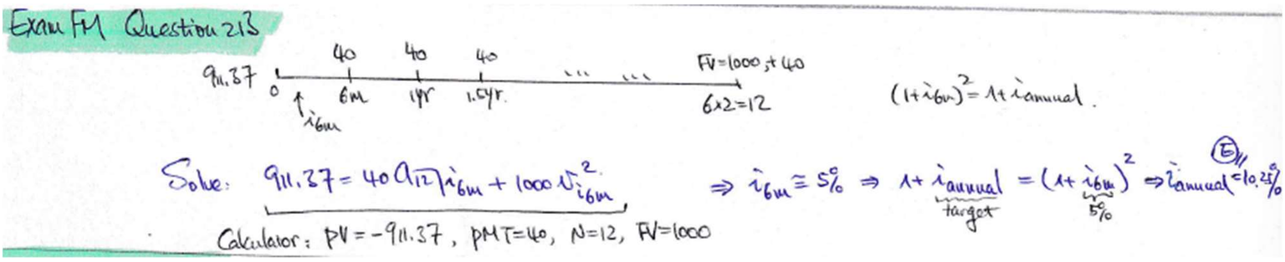
Solve: $BV_{12} = \underbrace{\underset{12000}{BV_0} \times (1+\underset{10\%}{i})^{12}}_{37661.14052} - 1000\times \underbrace{s_{\overline{12}|10\%}}_{\frac{(1+10\%)^{12}-1}{i} = 21.38428}$

$= 16276.85675$ Ⓓ//

Exam FM Question 213

A 6-year 1000 face amount bond has an annual coupon rate of 8% semi-annually. The bond currently sells for 911.37

Question: What is the annual yield rate?



Exam FM Question 213

$(1+i_{6m})^2 = 1+i_{annual}$.

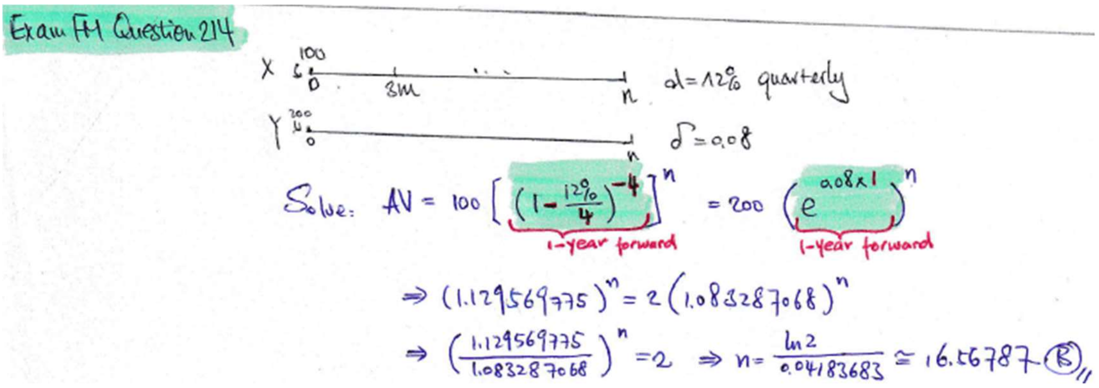
Solve: $\underbrace{911.37 = 40a_{\overline{12}|i_{6m}} + 1000v^{2}_{i_{6m}}}$ $\Rightarrow i_{6m} \cong 5\% \Rightarrow 1+\underbrace{i_{annual}}_{target} = (1+\underbrace{i_{6m}}_{5\%})^2 \Rightarrow i_{annual} = 10.25\%$ Ⓔ//

Calculator: PV=-911.37, PMT=40, N=12, FV=1000

Exam FM Question 214

Deposit 100 into Fund X, which has an annual rate of discount 12% convertible quarterly. At the same time, Sarah deposits 200 into Fund Y which earns an annual force of interest of 0.08. After n years. The balance of Fund X = balance Fund Y

Question: What is n



Exam FM Question 214

$d = 12\%$ quarterly

$\delta = 0.08$

Solve: $AV = 100\left[\underbrace{\left(1-\frac{12\%}{4}\right)^{-4}}_{\text{1-year forward}}\right]^{n} = 200\left(\underbrace{e^{0.08\times 1}}_{\text{1-year forward}}\right)^{n}$

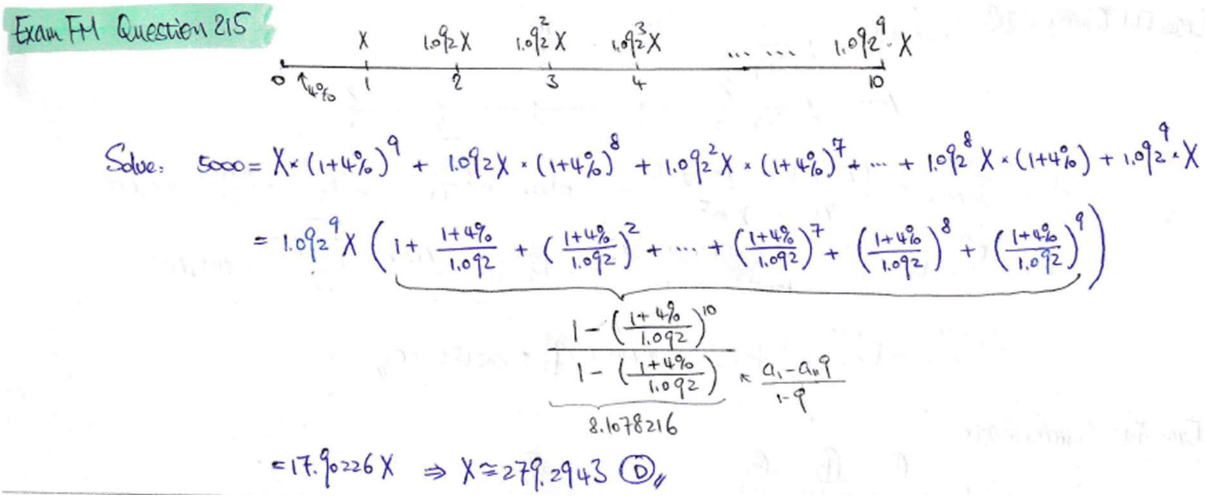
$\Rightarrow (1.129569775)^n = 2(1.083287068)^n$

$\Rightarrow \left(\frac{1.129569775}{1.083287068}\right)^n = 2 \Rightarrow n = \frac{\ln 2}{0.04183683} \cong 16.56787$ Ⓑ

Exam FM Question 215

Deposit at the end of each year for 10-year. An annual rate of 4%. The first deposit is X, each subsequent deposit is 9.2% greater than the previous one. The AV of the fund immediately after the 10th deposit is 5000.

Question: What is X?



Exam FM Question 215

Solve: $5000 = X\times(1+4\%)^9 + 1.092X\times(1+4\%)^8 + 1.092^2X\times(1+4\%)^7 + \cdots + 1.092^8X\times(1+4\%) + 1.092^9\times X$

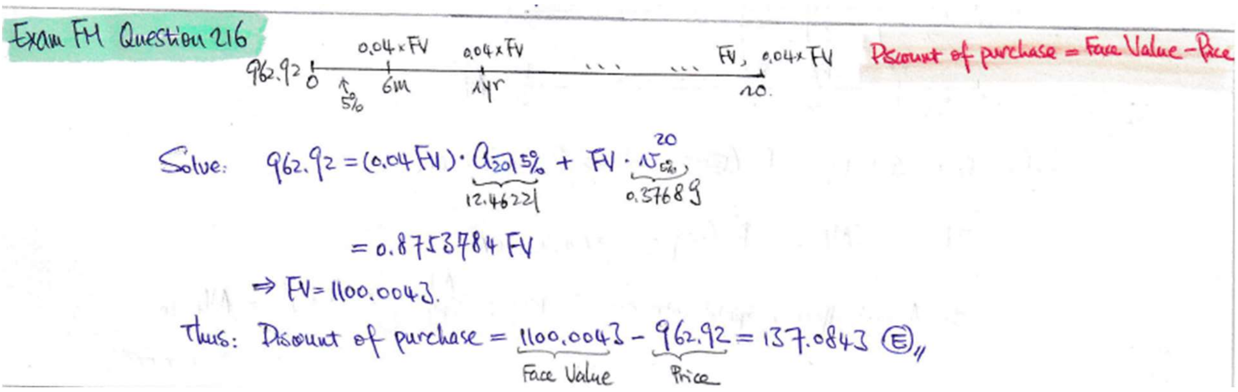
$= 1.092^9X\Big(\underbrace{1+\frac{1+4\%}{1.092}+\left(\frac{1+4\%}{1.092}\right)^2+\cdots+\left(\frac{1+4\%}{1.092}\right)^7+\left(\frac{1+4\%}{1.092}\right)^8+\left(\frac{1+4\%}{1.092}\right)^9}_{\underbrace{\frac{1-\left(\frac{1+4\%}{1.092}\right)^{10}}{1-\left(\frac{1+4\%}{1.092}\right)}}_{8.1078216}}\Big)$ $\frac{a_1-a_n q}{1-q}$

$= 17.90226X \Rightarrow X \approx 279.2943$ Ⓓ

Exam FM Question 216

Price of a ten-year bond is 962.92. An annual coupon rate of 8% paid semiannually. The annual yield rate is 10% convertible semiannually.

Question: What is the discount of this purchase?



Exam FM Question 216

Discount of purchase = Face Value − Price

Solve: $962.92 = (0.04FV)\cdot\underbrace{a_{\overline{20|}5\%}}_{12.46221} + FV\cdot\underbrace{v^{20}_{5\%}}_{0.37689}$

$= 0.8753784\,FV$

$\Rightarrow FV = 1100.0043.$

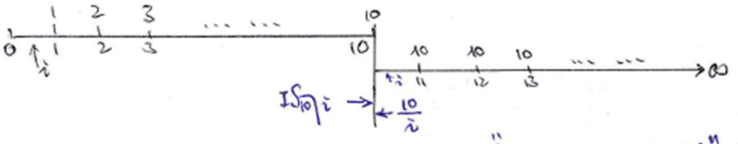
Thus: Discount of purchase $= \underbrace{1100.0043}_{\text{Face Value}} - \underbrace{962.92}_{\text{Price}} = 137.0843$ Ⓔ

Exam FM Question 217

Deposit into an account at the end of each year for ten years. The deposit in year one is 1, year two is 2, so on so forth. An annual rate of i. Immediately following the final deposit, the entire account balance is used to purchase a perpetuity-immediate with level payments if 10 at an annual rate of i.

Question: What is the price of this perpetuity?

Exam FM Question 217 $I\ddot{s}_{\overline{n}|} = \frac{\ddot{s}-n}{i}$, $\ddot{s} = \frac{(1+i)^n-1}{\boxed{d}}$



Solve: $I\ddot{s}_{\overline{10}|i} = \frac{10}{i}$ where $I\ddot{s}_{\overline{10}|i} = \frac{\ddot{s}-10}{i}$, $\ddot{s} = \frac{(1+i)^n-1}{d}$

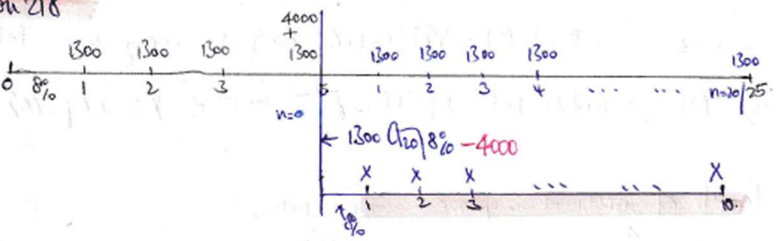
$\Rightarrow \frac{\ddot{s}-10}{x} = \frac{10}{x} \Rightarrow \ddot{s}_{\overline{10}|} = 20$ BAII Plus gives $i = 12.3\%$ $\Rightarrow \frac{10}{i} \cong 81.30$ Ⓓ//

Exam FM Question 218

25-year loan repaid with payments of 1300 at the end of each year. An annual rate of 8%. Pays an additional 4000 at the time of the 5th payment and will repay the remaining balance with a payment of X at the end of each of the subsequent ten years.

Question: What is X?

Exam FM Question 218



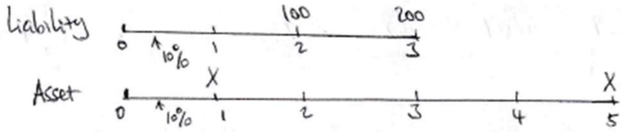
Solve. $PV_{t=5} = \underbrace{1300\, a_{\overline{20}|}8\% - 4000}_{12763-4000} = X \underbrace{a_{\overline{10}|}8\%}_{6.71} \Rightarrow X \simeq 1305.96$ Ⓒ//

Exam FM Question 219

A liability cash flow of 100 at the end of year 2 and a second liability cash flow of 200 at the end of year 3. Asset cash flows of X at the end of years 1 and 5. An annual rate of 10%

Question: What is the absolute value of the difference between the Macaulay durations of the asset and liability cash flows?

Exam FM Question 219



Solve: $D^{mac}_{Asset} = \frac{(Xv)\times 1 + (Xv^5)\times 5}{Xv + Xv^5}$ where $v = \frac{1}{1.1}$ $\Rightarrow D^{mac}_{Asset} = \frac{4.0137}{1.53} = 2.6233$

$D^{mac}_{Liability} = \frac{(100v^2)\times 2 + (200v^3)\times 3}{100v^2 + 200v^3} \Rightarrow D^{mac}_{Liability} = \frac{616.0781367}{232.9075883} = 2.6451612 9$

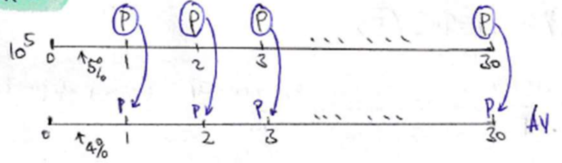
$\Rightarrow |D^{mac}_{Asset} - D^{mac}_{Liability}| = |2.6233 - 2.6451612 9| \cong .02186$ Ⓒ//

Exam FM Question 220

A 100,000 loan repaid with level payments at the end of each year for 30 years based on an annual rate of 5%. The bank reinvests the loan payments at an annual rate of 4%.

Question: What is the bank's annual yield rate over the 30-year period?

Exam FM Question 220



Solve: First: get P: $P \cdot a_{\overline{30|}5\%} = 10^5 \Rightarrow P \cong 6505.1435$

Then: $AV_{t=30} = P \cdot a_{\overline{30|}4\%} = 364840.5683$

$\Rightarrow$ $\underbrace{\text{"Annual effective yield rate over 30-year"}}_{i}$: $\underset{\uparrow \atop 10^5}{PV_{t=0}} \times (1+i)^{30} = \underset{\uparrow \atop 364840.5683}{\underbrace{AV_{t=30}}}$

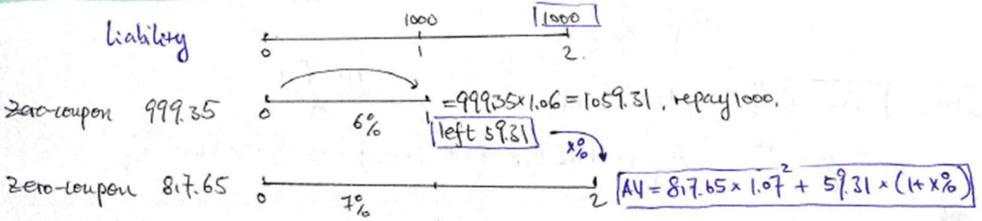
$\Rightarrow (1+i)^{30} \cong 3.6484 \Rightarrow r = 4.40872\%$ Ⓓ//

Exam FM Question 221

2-year immediate annuity with annual payments of 1000 for a price of 1817. The 1817 was reinvested in two zero-coupon bonds. The first is a 1-year bond with an annual rate of 6%. The second is a 2-year bondwith an annual rate of 7%. 999.35 is invested in the first bond and 817.65 is invested in the second bond. 2 bonds are held to maturity. As long as the effective one-year reinvestment rate is at least X% one year from now, the principal and interest earned will be sufficient to make the two annuity payments.

Question: What is X?

Exam FM Question 221



Solve: First, at t=1, first Bond's AV = 999.35×1.06 = 1059.31, repay 1000, left 59.31

At t=2. $AV_{t=2} = 817.65 \times 1.07^2 + 59.31(1+x\%) = 1000 \Rightarrow X \cong 7.69266\%$ Ⓔ//

Exam FM Question 222

Give 4- and 5-year spot rate. Question: what is the one-year annual rate for the 5th year?

Exam FM Question 222

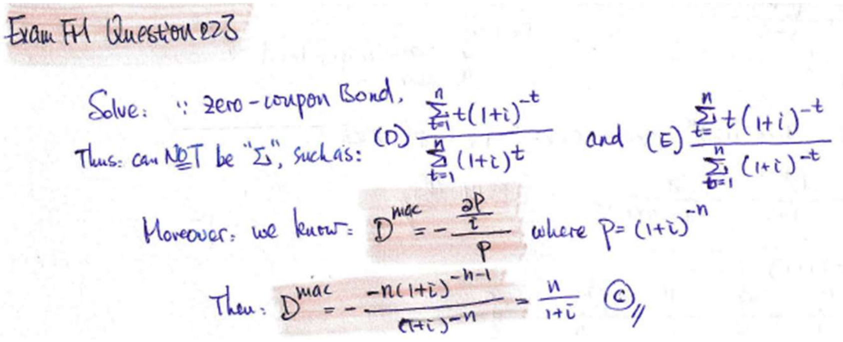
Length of investment in years	Spot rate
4	9.00%
5	9.50%

Solve: $(1+\underbrace{S_4}_{9\%})^4 \cdot (1+\overset{\text{one-year rate} \uparrow \quad \downarrow \text{4 years from today}}{{}_1f_4}) = (1+\underbrace{S_5}_{9.5\%})^5$

$\Rightarrow {}_1f_4 \cong 11.523\%$ Ⓔ//

Exam FM Question 223

Question: Determine the modified duration for a zero-coupon bond that is currently priced at an annual yield rate i with an n-year maturity?



Exam FM Question 223

Solve: ∵ zero-coupon Bond,

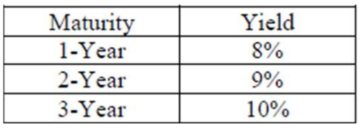
Thus: can NOT be "Σ", such as: (D) $\frac{\sum_{t=1}^{n} t(1+i)^{-t}}{\sum_{t=1}^{n}(1+i)^{t}}$ and (E) $\frac{\sum_{t=1}^{n} t(1+i)^{-t}}{\sum_{t=1}^{n}(1+i)^{-t}}$

Moreover: we know: $D^{mac} = -\frac{\frac{\partial P}{i}}{P}$ where $P = (1+i)^{-n}$

Then: $D^{mac} = -\frac{-n(1+i)^{-n-1}}{(1+i)^{-n}} = \frac{n}{1+i}$ ⓒ//

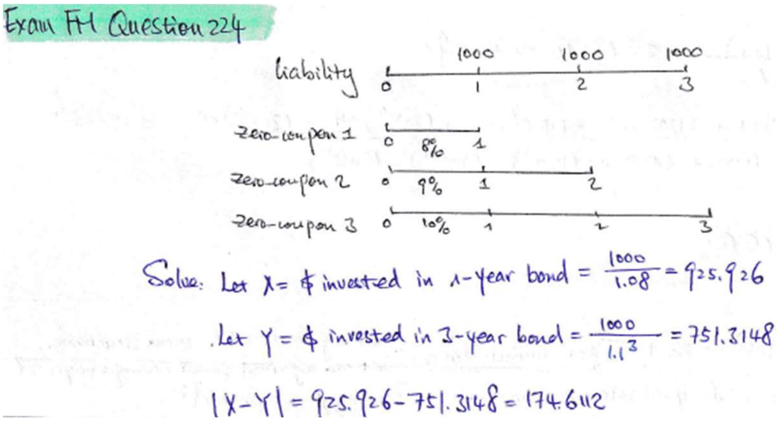
Exam FM Question 224

Liabilities of 1000 due at the end of each of the next three years. Exactly match the cash flows. The following zero-coupon bonds are available:



Maturity	Yield
1-Year	8%
2-Year	9%
3-Year	10%

Question: how much more will be invested in the 1-year bond than in the 3-year bond?



Exam FM Question 224

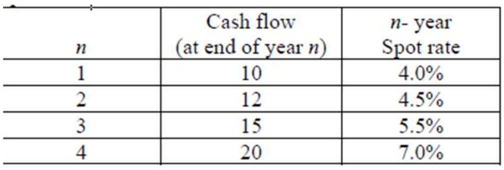
Solve: Let X = \$ invested in 1-year bond = $\frac{1000}{1.08} = 925.926$

Let Y = \$ invested in 3-year bond = $\frac{1000}{1.1^3} = 751.3148$

$|X-Y| = 925.926 - 751.3148 = 174.6112$

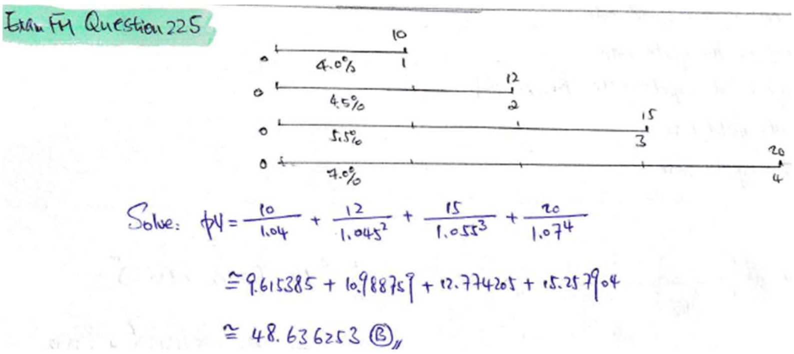
Exam FM Question 225

Give the particular investment and the spot rates.



n	Cash flow (at end of year n)	n- year Spot rate
1	10	4.0%
2	12	4.5%
3	15	5.5%
4	20	7.0%

Question: What is PV of this investment at the beginning of year one?



Exam FM Question 225

Solve: $PV = \frac{10}{1.04} + \frac{12}{1.045^2} + \frac{15}{1.055^3} + \frac{20}{1.07^4}$

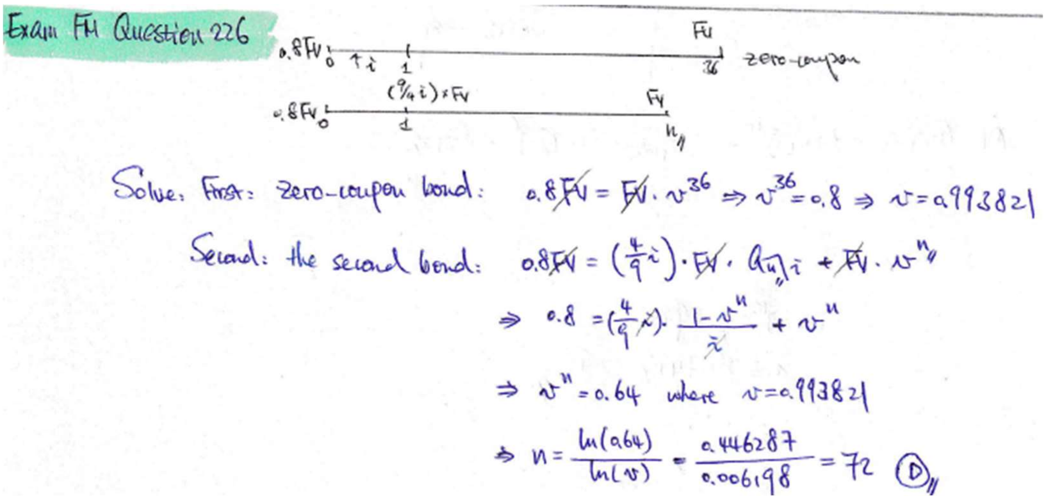
$\cong 9.615385 + 10.988759 + 12.774205 + 15.257904$

$\cong 48.636253$ Ⓑ//

Exam FM Question 226

A 36-year zero-coupon bond is 80% of its face value. A second bond, with the same price, same face value, same annual yield rate, offers annual coupons with the coupon rate equal to 4/9 of the annual yield rate.

Question: Number of years until maturity for the second bond?



Solve, First: zero-coupon bond: $0.8\cancel{FV} = \cancel{FV}\cdot v^{36} \Rightarrow v^{36}=0.8 \Rightarrow v=0.993821$

Second: the second bond: $0.8\cancel{FV} = \left(\frac{4}{9}i\right)\cdot\cancel{FV}\cdot a_{\overline{n}|i} + \cancel{FV}\cdot v^{n}$

$\Rightarrow 0.8 = \left(\frac{4}{9}\cancel{i}\right)\cdot\frac{1-v^{n}}{\cancel{i}} + v^{n}$

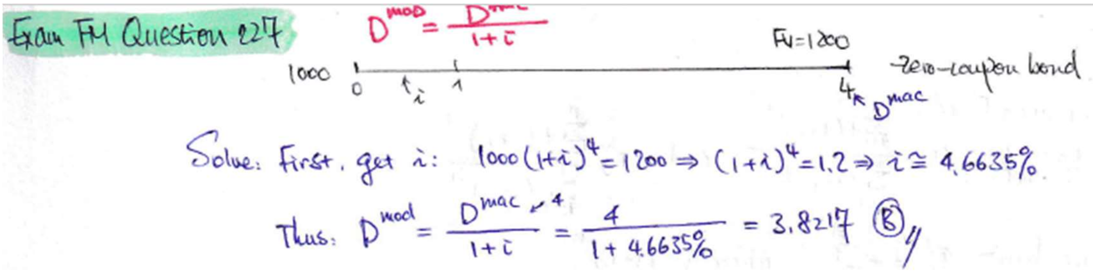
$\Rightarrow v^{n}=0.64$ where $v=0.993821$

$\Rightarrow n=\frac{\ln(0.64)}{\ln(v)} = \frac{0.446287}{0.006198} = 72$ Ⓓ//

Exam FM Question 227

A 1200 face amount zero-coupon bond with a price of 1000. The Macaulay duration is 4 years and the modified duration is d years

Question: W hat is d?



$D^{MOD} = \frac{D}{1+i}$

Solve: First, get i: $1000(1+i)^{4}=1200 \Rightarrow (1+i)^{4}=1.2 \Rightarrow i\cong 4.6635\%$

Thus: $D^{mod} = \frac{\overset{4}{D^{mac}}}{1+i} = \frac{4}{1+4.6635\%} = 3.8217$ Ⓑ//

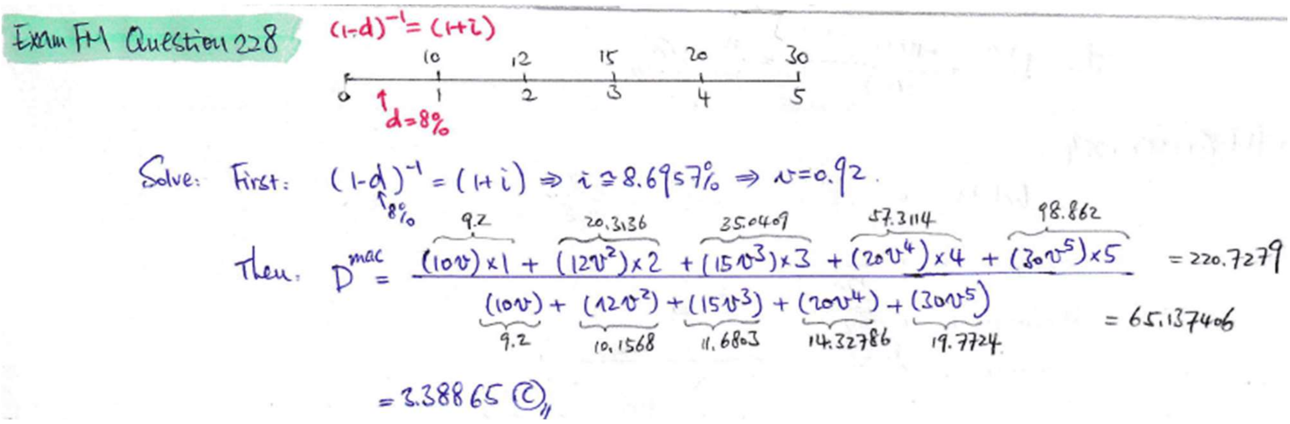
Exam FM Question 228

Liability has the following CFs

End of year	Cash Outflow
1	10
2	12
3	15
4	20
5	30

An annual rate is 8%

Question: What is the Macaulay duration?



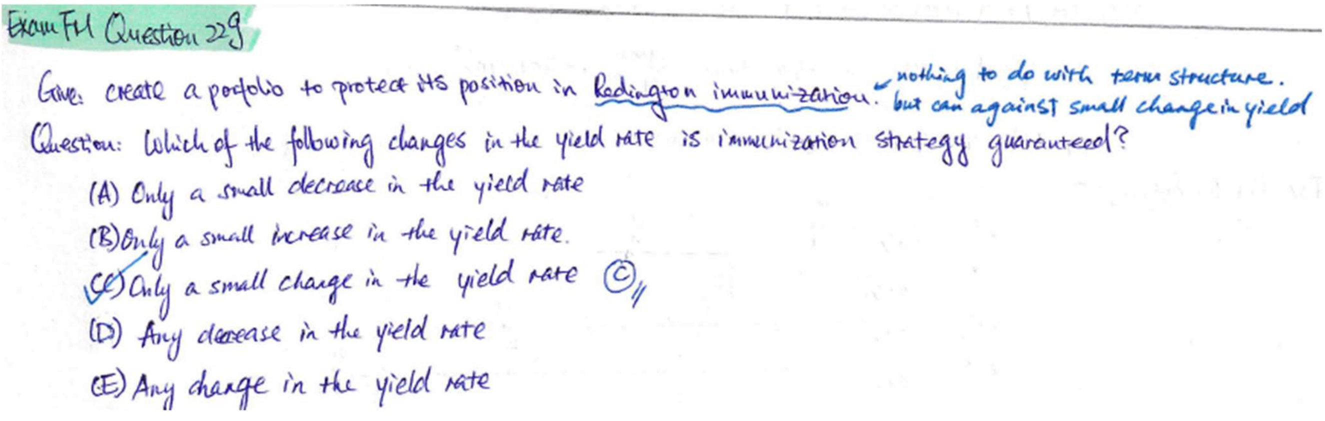
$(1-d)^{-1}=(1+i)$

Solve: First: $(1-\underset{8\%}{d})^{-1}=(1+i) \Rightarrow i\cong 8.6957\% \Rightarrow v=0.92.$

Then: $D^{mac} = \frac{\overbrace{(10v)\times 1}^{9.2} + \overbrace{(12v^{2})\times 2}^{20.3136} + \overbrace{(15v^{3})\times 3}^{35.0409} + \overbrace{(20v^{4})\times 4}^{57.3114} + \overbrace{(30v^{5})\times 5}^{98.862}}{\underbrace{(10v)}_{9.2} + \underbrace{(12v^{2})}_{10.1568} + \underbrace{(15v^{3})}_{11.6803} + \underbrace{(20v^{4})}_{14.32786} + \underbrace{(30v^{5})}_{19.7724}}$ $=220.7279$ $=65.137406$

$= 3.38865$ Ⓒ//

Exam FM Question 229



Exam FM Question 229

Give: create a portfolio to protect its position in Redington immunization. ← nothing to do with term structure. but can against small change in yield

Question: Which of the following changes in the yield rate is immunization strategy guaranteed?

(A) Only a small decrease in the yield rate

(B) Only a small increase in the yield rate.

[✓ (C)] Only a small change in the yield rate ⓒ//

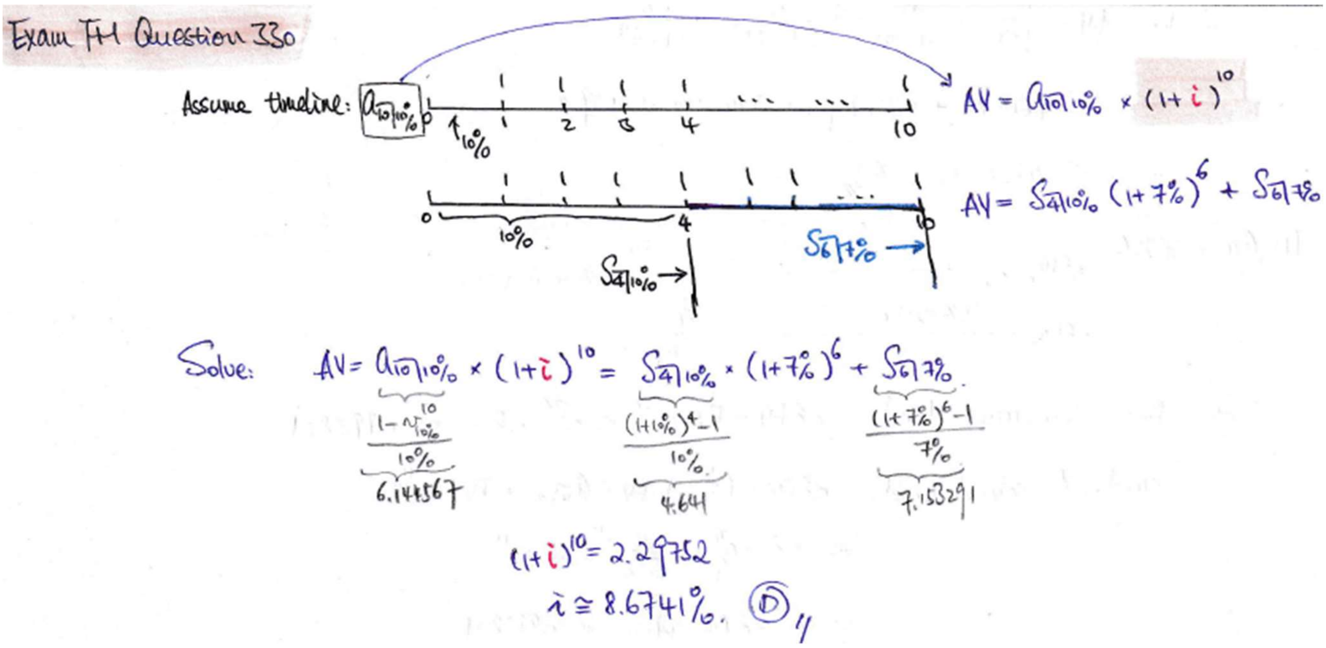
(D) Any decrease in the yield rate

(E) Any change in the yield rate

Exam FM Question 230

A loan repaid by level end-of-year payments for ten years, an annual rate of 10%. The bank invests these payments at an annual rate of 10% for the first 4-year and 7% for the next 6-year.

Question: What is the annual yield rate on this loan over the ten-year period?



Exam FM Question 330

Assume timeline: $\boxed{a_{\overline{10}|10\%}}$

$AV = a_{\overline{10}|10\%} \times (1+i)^{10}$

$AV = s_{\overline{4}|10\%}(1+7\%)^{6} + s_{\overline{6}|7\%}$

Solve: $AV = \underbrace{\underbrace{a_{\overline{10}|10\%}}_{\frac{1-v_{10\%}^{10}}{10\%}}}_{6.144567} \times (1+i)^{10} = \underbrace{\underbrace{s_{\overline{4}|10\%}}_{\frac{(1+10\%)^{4}-1}{10\%}}}_{4.641} \times (1+7\%)^{6} + \underbrace{\underbrace{s_{\overline{6}|7\%}}_{\frac{(1+7\%)^{6}-1}{7\%}}}_{7.153291}$

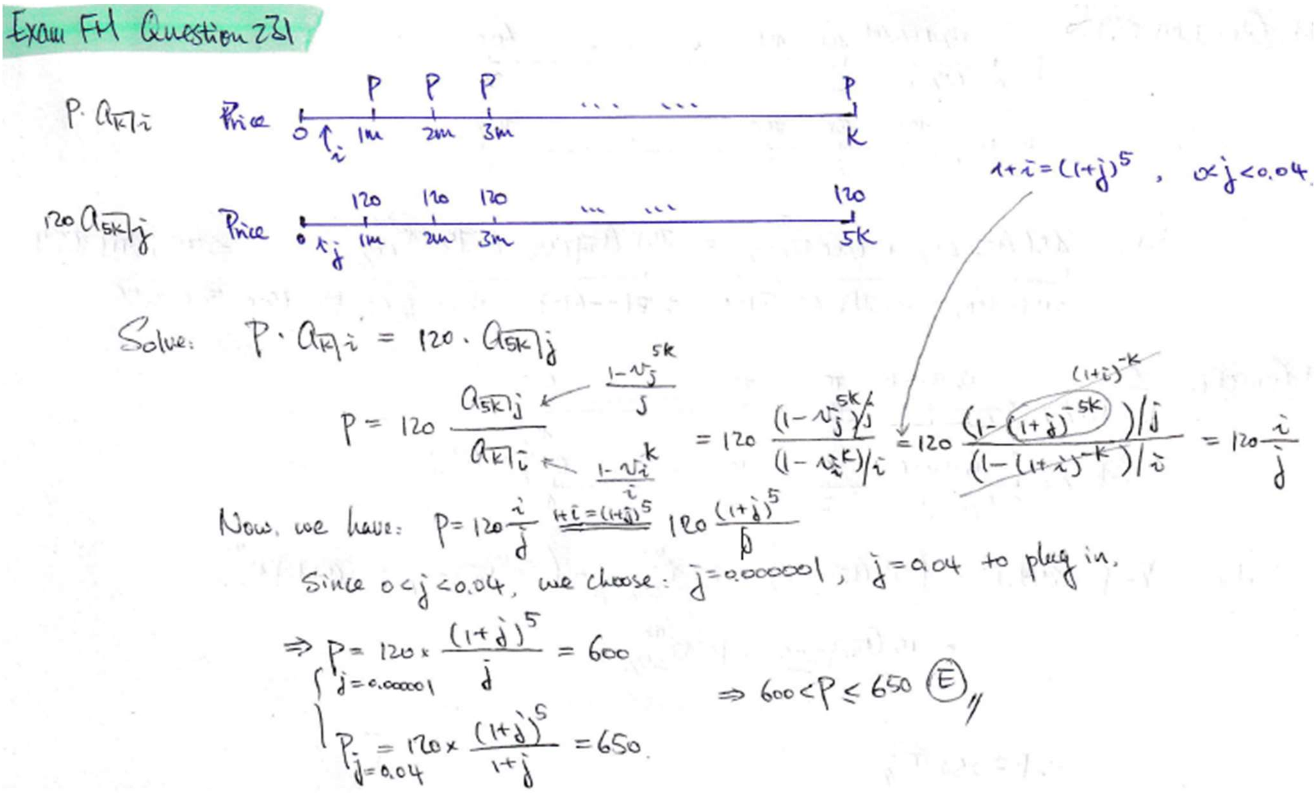
$(1+i)^{10} = 2.29752$

$i \cong 8.6741\%$. ⓓ//

Exam FM Question 231

2 loans. Each loan is for the same amount and is repaid with level end-of month payments. he first is charged a monthly rate of i and makes payments of P for k months. he second is charged a monthly rate of j and makes payments of 120 for 5k months. 0 < j < 0.04 and $i=(1+j)^5-1$

Question: Which statement of P is true?



Exam FM Question 231

$P\cdot a_{\overline{k}|i}$ Price

P P P … … P

0 $\uparrow i$ 1m 2m 3m k

$120\,a_{\overline{5k}|j}$ Price

120 120 120 … … 120

0 $\uparrow j$ 1m 2m 3m 5k

$1+i=(1+j)^5$, $0<j<0.04$

Solve: $P\cdot a_{\overline{k}|i} = 120\cdot a_{\overline{5k}|j}$

$a_{\overline{5k}|j} \leftarrow \frac{1-v_j^{5k}}{j}$, $a_{\overline{k}|i} \leftarrow \frac{1-v_i^{k}}{i}$

$$P = 120\,\frac{a_{\overline{5k}|j}}{a_{\overline{k}|i}} = 120\,\frac{(1-v_j^{5k})/j}{(1-v_i^{k})/i} = 120\,\frac{\cancel{(1-\overset{(1+i)^{-k}}{(1+j)^{-5k}})/j}}{\cancel{(1-(1+i)^{-k})/i}} = 120\,\frac{i}{j}$$

Now, we have: $P = 120\,\frac{i}{j} \overset{1+i=(1+j)^5}{=\!=} 120\,\frac{(1+j)^5}{j}$

Since $0<j<0.04$, we choose: $j=0.000001$, $j=0.04$ to plug in.

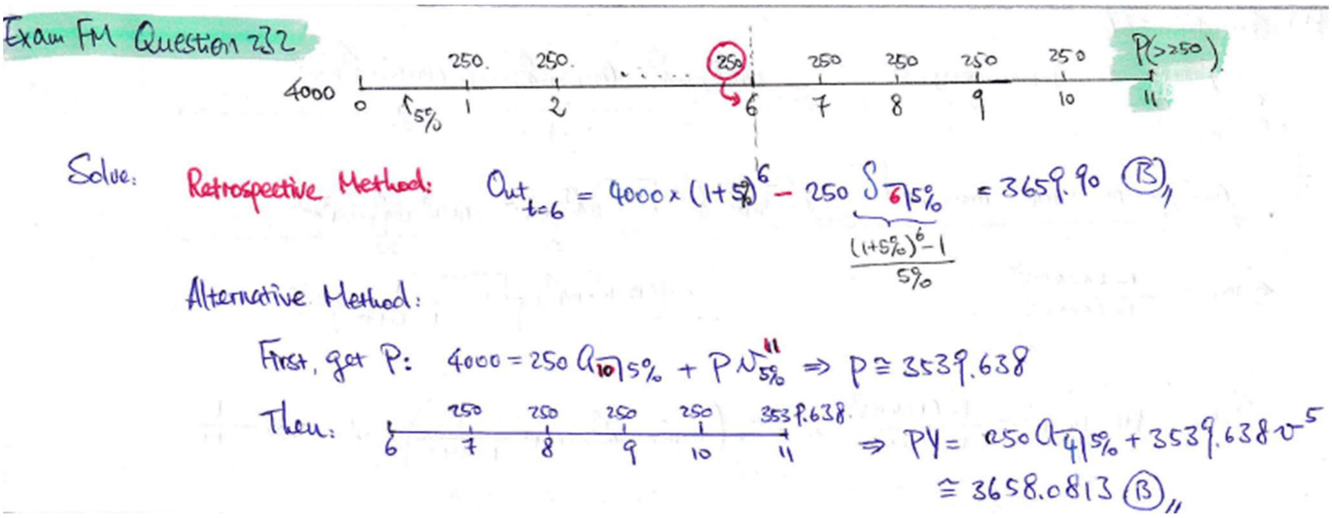
$$\Rightarrow \begin{cases} P_{j=0.000001} = 120\times\frac{(1+j)^5}{j} = 600 \\ P_{j=0.04} = 120\times\frac{(1+j)^5}{1+j} = 650. \end{cases}$$

$\Rightarrow 600<P\leq 650$ Ⓔ //

Exam FM Question 232

A loan of 4000 with an annual rate of 5%. Repaid by payments of 250 at the end of each year for ten years along with a final balloon payment at the end of the 11th year.

Question: What is outstanding loan balance at the beginning of the 7th year?



Exam FM Question 232

4000 0 $\uparrow 5\%$

250. 250. …… 250 250 250 250 250 P(>250)

1 2 6 7 8 9 10 11

Solve: Retrospective Method: $Out_{t=6} = 4000\times(1+5\%)^6 - 250\,\underbrace{s_{\overline{6}|5\%}}_{\frac{(1+5\%)^6-1}{5\%}} = 3659.90$ Ⓑ //

Alternative Method:

First, get P: $4000 = 250\,a_{\overline{10}|5\%} + P\,v_{5\%}^{11} \Rightarrow P \doteq 3539.638$

Then:

250 250 250 250 3539.638

6 7 8 9 10 11

$\Rightarrow PV = 250\,a_{\overline{4}|5\%} + 3539.638\,v^{5}$

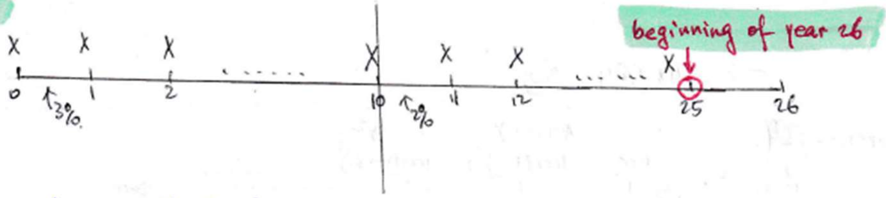
$\doteq 3658.0813$ Ⓑ //

Exam FM Question 233

A saving account with an annual rate of 3% for each of the first 10-year and an annual rate of 2% for each year thereafter. The investor deposits an amount X at the beginning of each year, starting with year 1, the account balance just after the deposit in the beginning of year 26 is 100,000.

Question: Which of the following expression is correct?

Exam FM Question233.



Solve: First, we observe all 5 choices:

(1). Left hand-side: "the total discounted periods should be 25" ⇒ can only be Ⓐ or Ⓓ

(2) Right hand-side: the difference between Ⓐ & Ⓓ is the 2nd term:

Since, we are standing at t=0, thus, must have $(1.03)^{-10}$

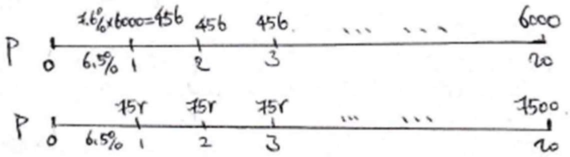
⇒ Ⓓ//

Exam FM Question 234

Two 20-year par-value bonds with annual coupons. Each bond sells for the same price and provides an annual effective yield rate of 6.5%. The first bond has a redemption value of 6000 , a annual coupon of 7.6%. The second bond has a redemption value of 7500, an annual coupon of r%.

Question: What is r?

Exam FM Question 234:



Solve: $$\underbrace{456\,a_{\overline{20}|6.5\%} + 6000\,v^{20}_{6.5\%}}_{PMT=456,\ n=20,\ I/Y=6.5,\ FV=6000\ \Rightarrow\ PV=-6727.22} = 75r\,\underbrace{a_{\overline{20}|6.5\%}}_{n=20,\ I/Y=6.5,\ FV=7500} + 7500\,v^{20}_{6.5\%}$$

⇒ $75r = 417.3654$

⇒ $r = 5.56$

Exam FM Question 235

Provide the following information about Bond X and Bond Y:

i) Both bonds are 20-year bonds.

ii) Both bonds have face amount 1500.

iii) Both bonds have an annual rate of 7% compounded semiannually.

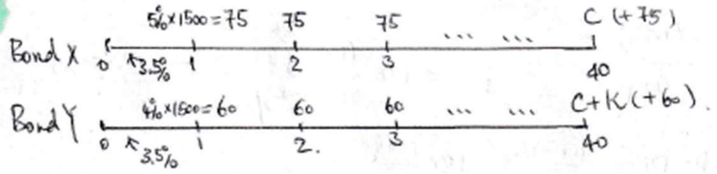
iv) Bond X has an annual coupon rate of 10% paid semiannually and a redemption value C.

v) Bond Y has an annual coupon rate of 8% paid semiannually and a redemption value C+K.

vi) The price of Bond X exceeds the price of Bond Y by 257.18.

Question: What is K?

Exam FM Question 235:



Solve:

$$X-Y=257.18=\left(75\,a_{\overline{40}|3.5\%}+\cancel{C}\,v^{40}_{3.5\%}\right)-\left(60\,a_{\overline{40}|3.5\%}+(\cancel{C}+K)\,v^{40}_{3.5\%}\right)$$

$$=\underbrace{15\,a_{\overline{40}|3.5\%}}_{320.3261}-K\,\underbrace{v^{40}_{3.5\%}}_{0.252572}$$

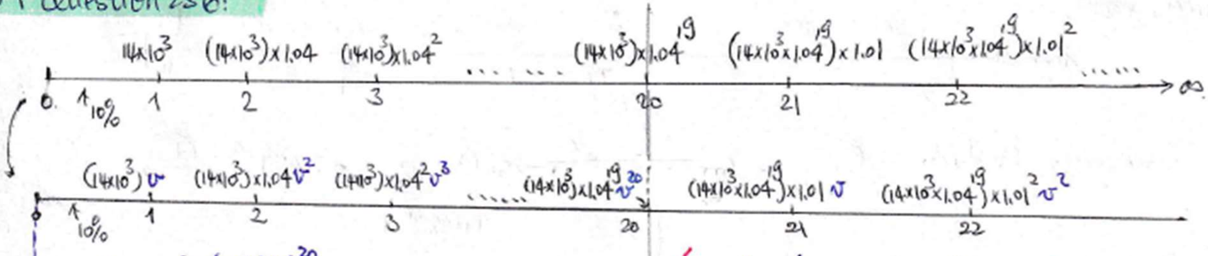
$$\Rightarrow K\fallingdotseq 250\ \textcircled{E}$$

Exam FM Question 236

A perpetuity-immediate with annual payments. Priced at X. An annual rate of 10%. The amount of the first payment is 14,000. Each payment, from the 2nd through the 20th, is 4% larger than the previous one. The 21st payment and each subsequent payment will be 1% larger than the previous one.

Question: What is X?

Exam FM Question 236:



$$\leftarrow 14\times10^3 v\,\frac{1-(1.04v)^{20}}{1-(1.04v)}\quad \wedge\ \frac{1-a_n q}{1-q}$$

$$\leftarrow\left((14\times10^3\times1.04^{19})\times1.01v\,\frac{1}{1-1.01v}\right)v^{20}$$

Solve:

$$PV=14\times10^3 v\,\frac{1-(1.04v)^{20}}{1-(1.04v)}+\underbrace{v^{20}}_{0.148644}\times\underbrace{\left(14\times10^3\times1.04^{19}\times1.01v\,\frac{1}{\underbrace{1-1.01v}_{0.081818}}\right)}_{49202.6728}\quad\text{where } v=\frac{1}{1.1}$$

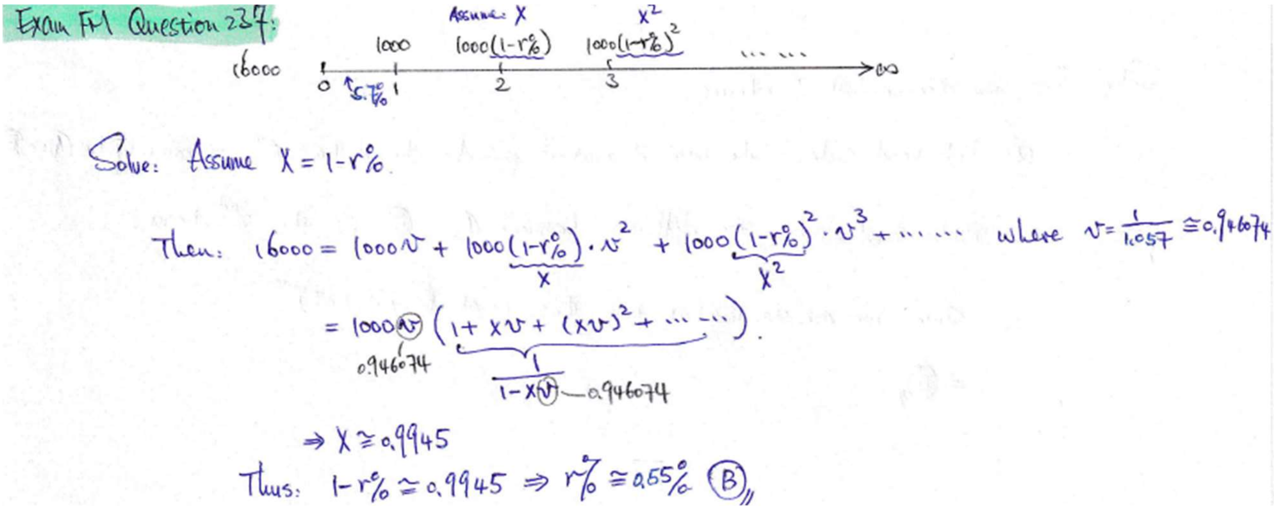
$$\underbrace{14\times10^3\times\frac{1}{1.1}\times\frac{0.674304}{0.054545}}_{157338.9112}$$

$$=206541.584\ \textcircled{c}$$

Exam FM Question 237

An annual perpetuity-immediate with price 16,000. The first payment is 1,000 and the payments decrease by r% each year. The annual yield rate is 5.7%.

Question: What is r?



Solve: Assume $X = 1-r\%$.

Then: $16000 = 1000v + 1000\underbrace{(1-r\%)}_{X}\cdot v^2 + 1000\underbrace{(1-r\%)^2}_{X^2}\cdot v^3 + \cdots\cdots$ where $v = \frac{1}{1.057} \cong 0.946074$

$= 1000\underset{0.946074}{\boxed{v}}\left(\underbrace{1 + Xv + (Xv)^2 + \cdots\cdots}_{\frac{1}{1-X\underset{0.946074}{\boxed{v}}}}\right).$

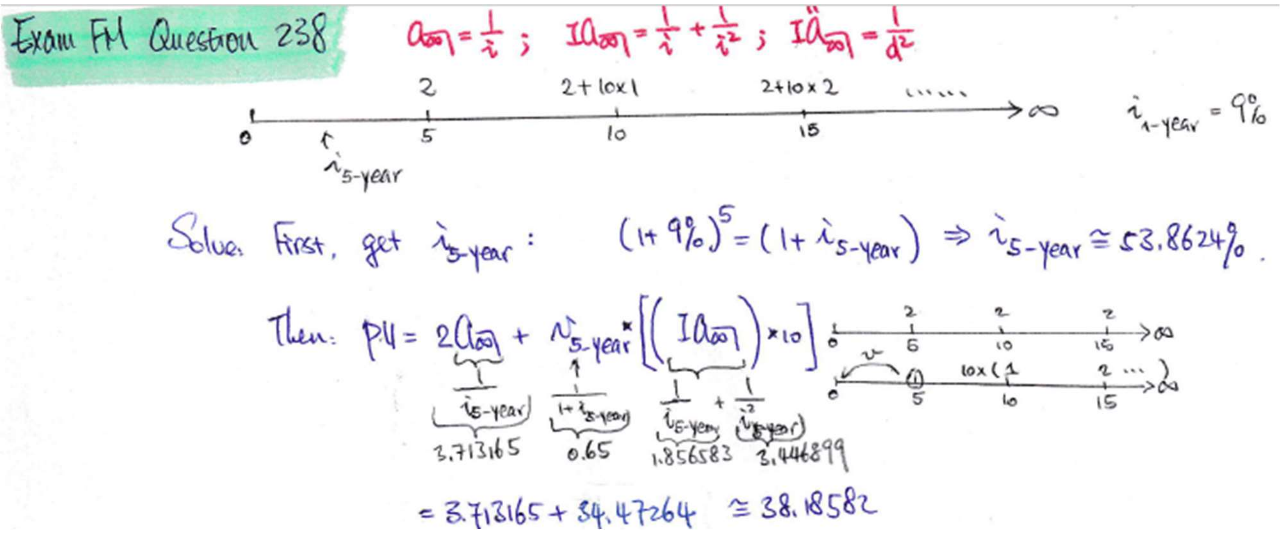
$\Rightarrow X \cong 0.9945$

Thus: $1-r\% \cong 0.9945 \Rightarrow r\% \cong 0.55\%$ Ⓑ//

Exam FM Question 238

A perpetuity makes payments every 5 years with a first payment of 2 to be paid 5 years from now. Each subsequent payment is 10 more than the previous one. The annual rate is 9%.

Question: What is the PV of the perpetuity?



$a_{\overline{\infty|}} = \frac{1}{i}$; $Ia_{\overline{\infty|}} = \frac{1}{i} + \frac{1}{i^2}$; $I\ddot{a}_{\overline{\infty|}} = \frac{1}{d^2}$

Solve: First, get $i_{5\text{-year}}$: $(1+9\%)^5 = (1+i_{5\text{-year}}) \Rightarrow i_{5\text{-year}} \cong 53.8624\%$.

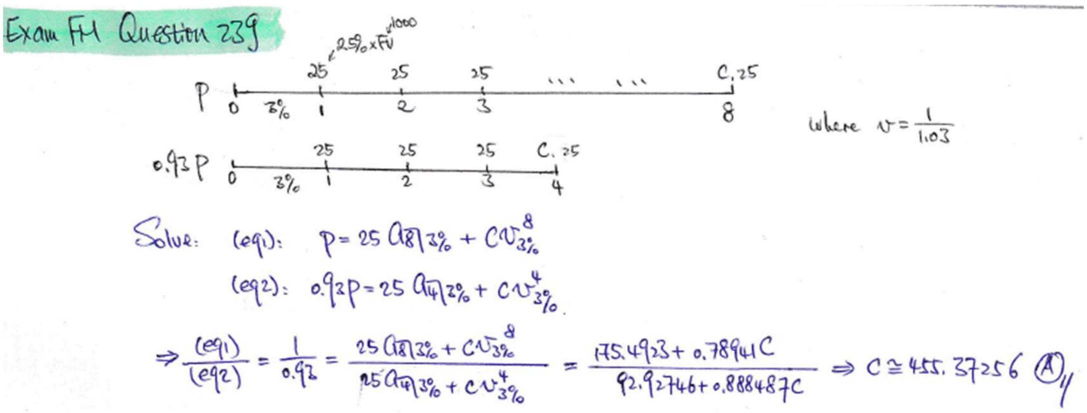
Then: $PV = 2\underbrace{a_{\overline{\infty|}}}_{\underbrace{\frac{1}{i_{5\text{-year}}}}_{3.713165}} + \underset{\underbrace{\frac{1}{1+i_{5\text{-year}}}}_{0.65}}{v_{5\text{-year}}} * \left[\left(\underbrace{Ia_{\overline{\infty|}}}_{\underbrace{\frac{1}{i_{5\text{-year}}}}_{1.856583} + \underbrace{\frac{1}{i^2_{5\text{-year}}}}_{3.446899}}\right)\times 10\right]$

$= 3.713165 + 34.47264 \cong 38.18582$

Exam FM Question 239

A four-year 1000 face amount bond, an annual coupon rate of 5% paid semiannually, redemption value of C. An annual yield rate of 6% convertible semiannually. If the term of the bond had been two years, the purchase price would have been 7% less.

Question: What is C?



Exam FM Question 239

where $v = \frac{1}{1.03}$

Solve: (eq1): $P = 25\, a_{\overline{8}|3\%} + C v^{8}_{3\%}$

(eq2): $0.93P = 25\, a_{\overline{4}|3\%} + C v^{4}_{3\%}$.

$$\Rightarrow \frac{(eq1)}{(eq2)} = \frac{1}{0.93} = \frac{25\, a_{\overline{8}|3\%} + C v^{8}_{3\%}}{25\, a_{\overline{4}|3\%} + C v^{4}_{3\%}} = \frac{175.4923 + 0.78941C}{92.92746 + 0.888487C} \Rightarrow C \cong 455.37256 \text{ Ⓐ}$$

Exam FM Question 240

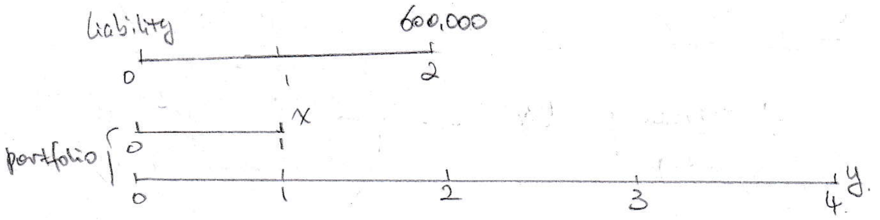
Give: ① Fully Immunized Portfolio

i) Liability: single payment of 600,000 due in 2 years

ii) Portfolio: 1-year zero-coupon bond maturing for x

and 4-year zero-coupon bond maturing for y

② annual effective rate is 4.6%



Solve: matching PV: $x \cdot (1+1.046) + y \cdot v^2 = 600.000$ eq(1) where $v = \frac{1}{1.046}$

matching D^{mac}: $\frac{(xv)\times 1 + (yv^4)\times 4}{xv + yv^4} = 2$ eq(2)

$\Rightarrow y = 656455 - 1.144x \xrightarrow{\text{plug in eq(2)}} x = 382741.21$ Ⓓ//

Exam FM Question 241

Give: ① An investor pays 4000 today for a 3-year investment that returns cash flows of 1400 at the end of each year.

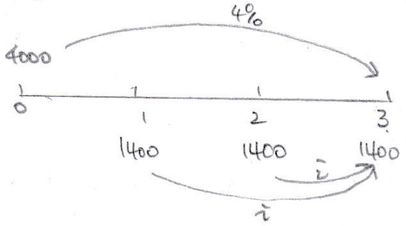
② The cash flow can be reinvested at the positive annual effective interest rate of i

③ Using 4%, the NPV of this investment is 0

NPV = 0 ⇔ 2 investments are equal (NOT only: PV is the same, AV is also the same)

Thus, this Question becomes: AV of 4000 at t=3 using 4% $\xleftrightarrow{\text{same as}}$ AV of 1400(t=1,2,3) at t=3 using i

Solve: $4000(1+4\%)^3 = 1400(1+i)^2 + 1400(1+i) + 1400$



$1400i^2 + 4200i + 1400\times 3 - 4499.46 = 0$

using $i = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\Rightarrow i = 0.06968 \cong 6.968\%$ Ⓔ

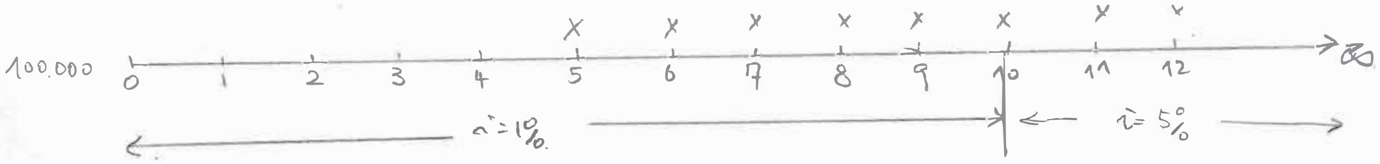
Exam FM Question 242

Give: ① Perpetuity X with the first payment occurring 5 years from today

② Perpetuity's PV = 100,000

③ annual rate 1% for the first 10-year, 5% thereafter

Question: what is X



$$\text{Solve: } 100{,}000 = \underbrace{v^4}_{\left(\frac{1}{1+1\%}\right)^4}\left(X \cdot a_{\overline{6}|1\%}\right) + \underbrace{v^{10}}_{\left(\frac{1}{1+1\%}\right)^{10}} \cdot \frac{X}{i} \quad i \text{ — } 5\% \quad \Rightarrow 23.6689X = 100{,}000 \Rightarrow X = 4224.95 \text{ Ⓑ}$$

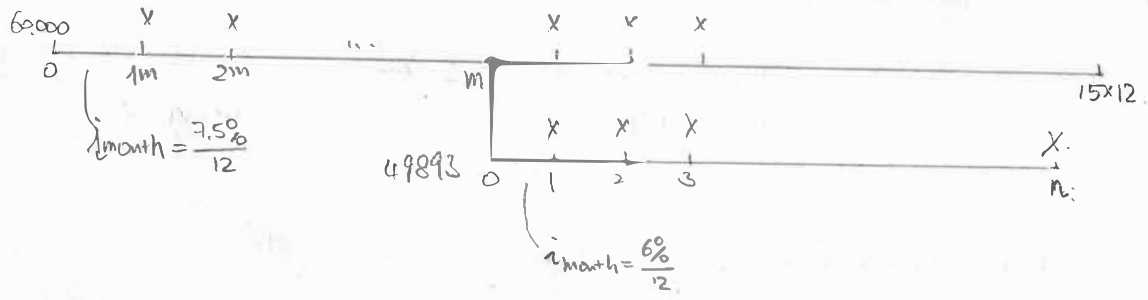
[redacted]

Exam FM Question 243

Give: ① Loan: 15-year, 60,000, repaid with X at the end of each month, annual rate 7.5% convert monthly

② when Loan balance is 49893, refinance with 6% convert monthly, Payment remains at X, final payment is a drop payment

Question: "Total number of payment" including the smaller one



Solve: Total number = m + n.

First: $X \cdot a_{\overline{15\times12}|\frac{7.5}{12}\%} = 60{,}000 \Rightarrow X = 556.21$

Then: $556.21 \cdot a_{\overline{t}|\frac{7.5\%}{12}} = 49893 \Rightarrow t \doteq 131.99 = 132 \Rightarrow$ Thus: $\underline{\underline{m}} = \underbrace{15\times12}_{180} - \underbrace{t}_{132} = \underline{\underline{48}}$

Second: $49893 = 556.21 \cdot a_{\overline{n}|\frac{6\%}{12}} \Rightarrow n \doteq 119.32$ drop payment $\} \Rightarrow n = 120$

$m + n = 48 + 120 = 168.$

Ⓓ

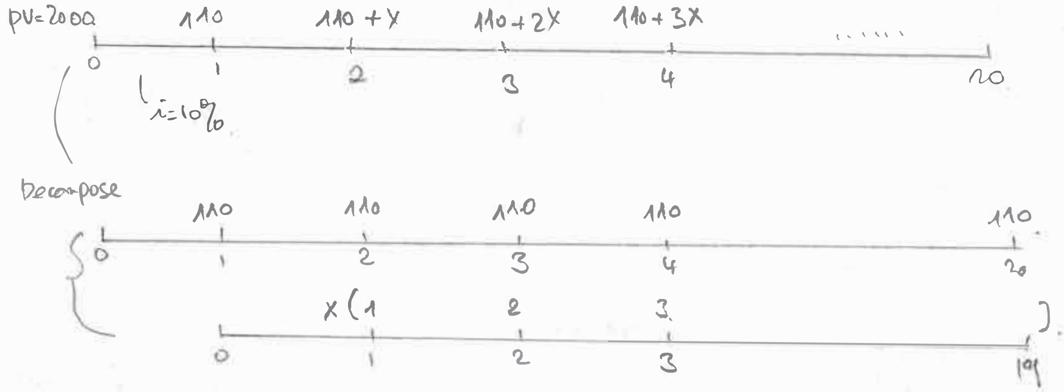
Exam FM Question 244

$Ia_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{i}$; $\ddot{a}_{\overline{n}|} = (1+i)a_{\overline{n}|}$

Give:
① Greg: 20-year increasing annuity-immediate , PV = 2000
② First payment 110, succeeding payment = X + previous payment
③ effective rate: 10%

Question: what is X

Solve.



$2000 = 110\, a_{\overline{20}|10\%} + \underbrace{v}_{\frac{1}{1.1}} \left(Ia_{\overline{19}|10\%} \right)$ where: $Ia_{\overline{19}|10\%} = \frac{\ddot{a}_{\overline{19}|10\%} - 19 \cdot v^{19}}{i}$

$i = 10\%$

$\ddot{a}_{\overline{19}|10\%} = (1+i)\, a_{\overline{19}|10\%}$

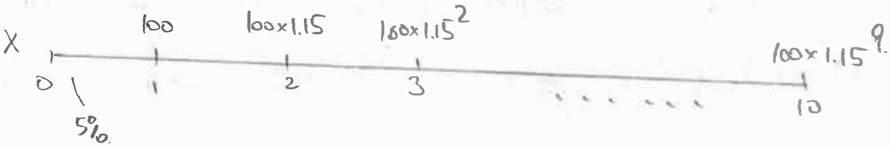
$\Rightarrow 2000 = 936.49 + 55.44X$

$\Rightarrow X \cong 19.183$ Ⓒ //

Exam FM Question 245

Give: ① 10-year annual-immediate , payment 100, increase by 15% each year.
② $i = 5\%$

Question: what is PV = X ?



Solve:

$$X = 100v + (100 \times 1.15)v^2 + (100 \times 1.15^2)v^3 + \cdots + (100 \times 1.15^9)v^{10}$$

$$= 100v \left(1 + 1.15v + (1.15v)^2 + \cdots + (1.15v)^9\right)$$

$$= 100v \cdot \frac{1 - (1.15v)^9 \cdot (1.15v)}{1 - 1.15v} \quad \leftarrow \frac{a_1 - a_n q}{1-q}$$

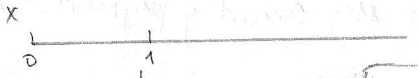
where $v = \frac{1}{1.05}$

$\Rightarrow X \cong 1483.62$ Ⓔ //

Exam FM Question 246

Give: ① A loan of X: repaid with level payments of R, for n-year

(i) interest paid in year 1 = 797.50.



Interest repaid = $\boxed{X \cdot i = 797.50}$ eq(1)

(ii) Principal repaid in (n-4) is 865

	Principaal Repaid
t=1	v^n
t=2	v^{n-1}
t=3	v^{n-2}
⋮	⋮
t=t	$v^{n-(t-1)}$

$\Rightarrow 865 = R \cdot v^{n-[(n-4)-1]} \Rightarrow \boxed{R \cdot v^5 = 865}$ eq(2)

(iii) Principal Outstanding at (n-1) is 1144.50.

Outstanding = $\boxed{v \cdot R = 1144.50}$ eq(3)

Question: what is X

Solve: From eq(2) and eq(3), we know: $v^4 = \frac{865}{1144.50} \Rightarrow i = 0.0725$

Plug into eq(1) we get X = 10999 (D)//

Exam FM Question 247. Simple discount: $PV(1 - i\% \times t)^{-1} = FV$

Give:
① $i\%$: $0 < i\% < 10\%$
② $AV = Y$ at $t=3$

Assume: $Y=100$, $i=5\%$.

$i \times t$ ↑ ↑ Y.

(i) Simple interest of $i\%$ is Q ← PV ⇔ $\boxed{Q(1 + 5\% \times 3) = 100}$ eq(1) ⇒ Q = 86.9565

(ii) compond interest of $i\%$ is R ⇔ $\boxed{R(1+5\%)^3 = 100}$ eq(2) ⇒ R = 86.3837.

(iii) Simple discount of $i\%$ is S ⇔ $\boxed{S(1 - 5\% \times 3)^{\boxed{-1}} = 100.}$ eq(3) ⇒ S = 85

(iv) compound discount of $i\%$ is T ⇔ $\boxed{T(1-5\%)^{-3} = 100}$ eq(4) ⇒ T = 85.7375.

Determine:

(A) Q < R < S < T
(B) R < Q < S < T
[✓ (C)] S < T < R < Q
(D) T < S < Q < R
(E) T < S < R < Q

⇒ S < T < R < Q. (C)//

Exam FM Question 248

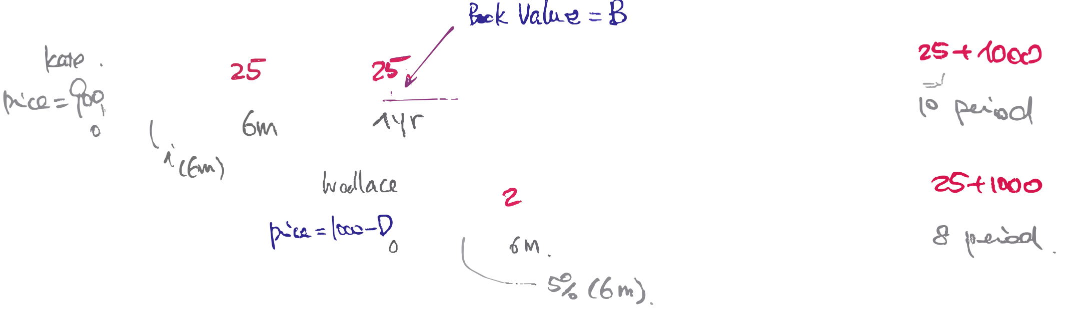
Give

① kate: 5-Year, 1000 FV, bought today with discount 100, annual coupon rate 5% semi-annually — 2.5% 6m

② 1-year later: Wallace bought 4-year bond, FV 1000, annual coupon rate 5% semi-annually
Discount on the price is "D", yield 10% semi-annually

③ Book Value: kate at t=1 is B

Question: what is B−D



Solve. kate:

$$900 = 25\,a_{\overline{10|}\,i\%_{6m}} + 1000\,v^{10}_{i\%_{6m}} \Rightarrow i_{6m} \doteq 3.72\%$$

Then: $B = 25\,a_{\overline{8|}\,3.72\%} + 1000\,v^{8}_{3.72\%} \Rightarrow B \doteq 916.90$

Wallace:

$$1000-D = 25\,a_{\overline{8|}\,5\%} + 1000\,v^{8}_{5\%} \Rightarrow 1000-D \cong 838.42 \Rightarrow D = 161.58$$

$\Rightarrow B-D \cong 755.$ Ⓒ

Exam FM Question 249

Attention to "Method 2"

Give. Liability: 1000 at $t=2$, 300 at $t=4$
Asset: X at $t=1$, Y at $t=3$

Question: $\frac{Y}{X}$, so that, Asset & Liability match eachother's PV and duration, and $i=5\%$

Solve: Method 1:

PV
$$PV_{\text{liability}} = \overbrace{1000v^2 + 300v^4}^{1153.84} \quad \text{where } v = \frac{1}{1.05}$$
$$PV_{\text{asset}} = Xv + Yv^3$$
$\Rightarrow 0.95X + 0.86Y = 1153.84$

D^{mac}
$$D_{\text{liability}} = \frac{\overbrace{(1000v^2)\times 2 + (300v^4)\times 4}^{1814.06+987.25}}{1153.84}$$
$$D_{\text{Asset}} = \frac{(0.95X)\times 1 + (Yv^3)\times 3}{1153.84}$$
$\Rightarrow 0.95X + 2.59Y = 1814.06+987.25$

$\Rightarrow \begin{cases} X = 352.4 \\ Y = 952.29 \end{cases} \Rightarrow \frac{Y}{X} \cong 2.7$ (B

Method 2

$PV_{\text{liability}} = PV_{\text{Asset}} \Rightarrow 0.95X + 0.86Y = 1153.84$ ①

$PV'_{\text{Liability}} = PV'_{\text{Asset}} \Leftrightarrow$
$$\begin{cases} 1000(1+i)^{-2\,\prime} + 300(1+i)^{-4\,\prime} = -2000(1+i)^{-3} - 1200(1+i)^{-5} \\ X(1+i)^{-1\,\prime} + Y(1+i)^{-3\,\prime} = -X(1+i)^{-2} - 3Y(1-i)^{-4} \end{cases} \Rightarrow -2667.91 = -0.907X - 2.47$$
same
where $i=5\%$ ②

$$① \,\&\, ② \Rightarrow \begin{cases} X = 346.61 \\ Y = 952.57 \end{cases} \Rightarrow \frac{Y}{X} = 2.75 \text{ Ⓑ}$$

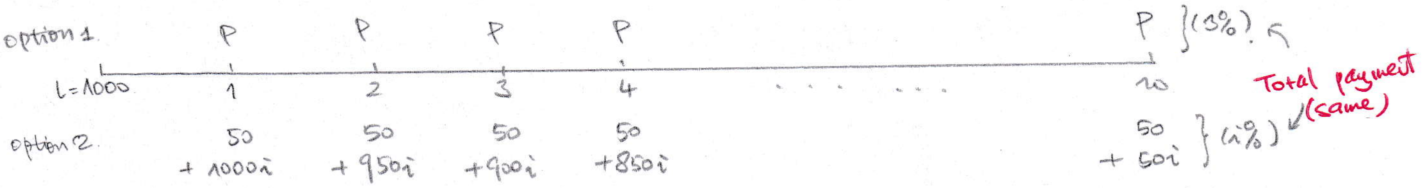
Exam FM Question 250.

Give: (1) Homeower borrows 1000, repaid with payment at the end of each year for 20 years

(2) Two repaid options → (a) equal payment annually, based on annual effective interest rate 3%
→ (b) each year: 50 + interest on unpaid balance with rate i

(3) two "total payment" are the same

Question: what is "i"?



Solve: $$\boxed{\underbrace{P\cdot 20}_{\text{total payment under Option I}} = \underbrace{50\times 20 + i\,(1000+950+900+850+\cdots+50)}_{\text{total payment under option II}}}$$

where $$\begin{cases} P\cdot a_{\overline{20|}3\%} = 1000 \Rightarrow P = 67.215. \\ i\,(\underbrace{1000+950+900+850+\cdots+50}) = i\cdot \dfrac{n(\overset{1000}{a_1}+\overset{50}{a_n})}{2} \end{cases} \quad \left(n = \dfrac{\overset{a_1}{1000}-\overset{a_n}{50}}{\underbrace{50}_{\text{公差}}} + 1\right)$$

$$\Leftrightarrow \quad 67.215\times 20 = 50\times 20 + i\cdot\frac{20\times(1000+50)}{2} \Rightarrow i = 3.279\%. \quad ⓒ //$$

Exam FM Question 251

Give (1) debt: amortized using 120 level payment at the end of each month

(2) annual effective interest rate 8% (no compounding information $\Leftrightarrow$ $1+8\% = (1+i_{month})^{12}$

(3) Principal Amount in 6th payment: 600

Question: Principal Amount in 24th payment?

Solve: Recall. Loan

	Level Payment	Interest	Outstanding	Principal Repaid
$t=0$			$a_{\overline{n|}}$ (all future CFs) of 1	level payment − Interest
$t=1$	1	$a_{\overline{n|}}\cdot i = 1-v^n$	$a_{\overline{n-1|}}$ (all future CFs) of 1	$1-(1-v^n) = v^n$
$t=2$	1	$a_{\overline{n-1|}}\cdot i = 1-v^{n-1}$	$a_{\overline{n-2|}}$	v^{n-1}
⋮	⋮	⋮	⋮	⋮
$t=t$	1	$1-v^{n-(t-1)}$	$a_{\overline{n-t|}}$	$v^{n-(t-1)}$.

Solve: $P\cdot v^{\overset{120}{n}-5} = 600$

where $(1+i_{month})^{12} = 1+8\% \Rightarrow i_{month} = 0.006434$

Thus $\Rightarrow P \cong 1255.23$.

Question: $P\cdot v^{\overset{120}{n}-23} = P\cdot v^{97} \cong 673.83$

Ⓑ //

Exam FM. Question 252.

Give
① Fund X: force of Intere $\delta_t = \frac{2}{1+2t}$ $(0 \le t \le 20)$

② Fund Y: annual effective rate of i

③ Amt of 1 invested in each fund at time 0. After 20 years. AV in both funds are the same.

Question: value of fund Y after 5-year

Solve.
$$\underbrace{e^{\int_0^{20} \frac{2}{1+2t}\,dt}}_{AV_{t=20} \text{ of Fund X}} = \underbrace{(1+i)^{20}}_{AV_{t=20} \text{ of fund Y}} \Rightarrow \underbrace{e^{\ln(1+2t)\big|_0^{20}}}_{\underbrace{e^{\ln 41 - \ln 1}}_{41}} = (1+i)^{20} \Rightarrow i = 20.4\%$$

Thus, Question $= (1+i)^5 \cong 2.53$ Ⓑ //

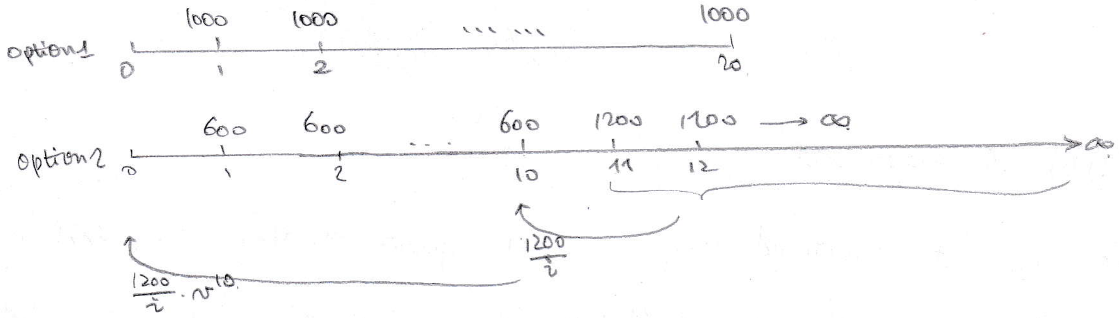
Exam FM Question 253

Give. ⓐ 2 annuities-immediate
(i) Annuity-immediate: annual payment of 1000 for 20 years.
(ii) perpetuity: 1st 10 payments, 600 (annual)
Beginning in year 11. 1200 (forever)

ⓑ annual rate i

ⓒ both options has present value X

Question: X



Solve:
$$1000\, a_{\overline{20|}i} = 600\, a_{\overline{10|}i} + \frac{1200}{i} \cdot v^{10}$$

$$1000 \cdot \frac{1-v^{20}}{\cancel{i}} = 600 \cdot \frac{1-v^{10}}{\cancel{i}} + \frac{1200}{\cancel{i}} \cdot v^{10}$$

$$10v^{20} + 6v^{10} - 4 = 0$$

$$(10v^{10} - 4)(v^{10} + 1) = 0 \Rightarrow v^{10} = \frac{4}{10} \Rightarrow v \cong 0.9124 \Rightarrow i \cong 9.59\%$$

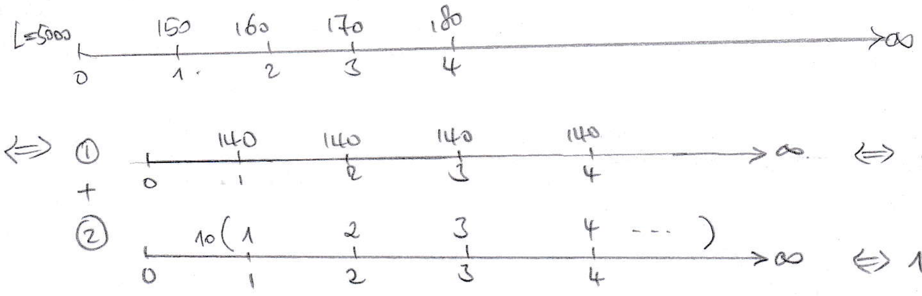
Thus: $X = 1000\, a_{\overline{20|}9.59\%} \Rightarrow X \cong 8757.35$ Ⓑ

Exam FM Question 254

Give: ① Perpetuity-immediate, price at 5000, annual effective rate i

② Perpetuity: 150 in 1st year, increase by 10 each year after

Question: what is i?



$\Leftrightarrow \frac{140}{i}$

$\Leftrightarrow 10\cdot\left(\underbrace{\frac{1}{i}+\frac{1}{i^2}}_{\text{increasing annuity-immediate}}\right)$

Solve: $5000 = \frac{140}{i} + 10\left(\frac{1}{i}+\frac{1}{i^2}\right)$

$\Rightarrow 5000i^2 - 150i - 10 = 0 \Rightarrow i = \frac{150 \pm \sqrt{150^2+4\times5000\times10}}{5000\times2} \Rightarrow i \cong 6.217\%$ Ⓑ //

Exam FM Question 255

Give: ① Erin borrows X at 5% (annual), principal and interest paid in lump sum at end of 20 years.

② If loan is repaid with level payment $^{(P)}$ at the end of first 10 years, interest will be 1000 less

$X = P\cdot a_{\overline{10}|5\%}$

Question: what is X

$$\underbrace{\text{Total Interest in lump sum}}_{X\cdot(1+5\%)^{20}-X} - 1000 = \underbrace{\text{Interest in level payment}}_{10\cdot P - X} \quad \text{where } P = \frac{X}{a_{\overline{10}|5\%}}$$

$$\Leftrightarrow \quad 1.653X - 1000 = \underbrace{10\cdot\frac{X}{\underbrace{a_{\overline{10}|5\%}}_{7.72}}}_{0.29X} - X \Rightarrow X \cong 733.675 \text{ Ⓓ} //$$

Exam FM Question 256

Give: ① 20-year loan of 500, 5% annual rate, repaid with level payment at the end of each year

Question: Installment in which the principal and interest portion are most near to each other

"interest repaid" = "principal repaid"

	Outstanding	Level Payment	Interest repaid	Principal repaid
$t=0$	$P\cdot a_{\overline{20}\mid}$			
$t=1$	$P\cdot a_{\overline{19}\mid}$	P	$P\cdot a_{\overline{20}\mid}\cdot i$	$P-Pa_{\overline{20}\mid}\cdot i$
⋮				
t	$P\cdot a_{\overline{20-t}\mid}$	P	$P\cdot a_{\overline{20-(t-1)}\mid}\cdot i$	$P-Pa_{\overline{20-(t-1)}\mid}\cdot i$

← most near →

Solve: $\underbrace{P\cdot\frac{1-v^{20-(t-1)}}{i}\cdot i}_{\text{interest repaid}} = \underbrace{P - P\cdot\frac{1-v^{20-(t-1)}}{i}\cdot i}_{\text{principal repaid}}$

$1 - v^{20-(t-1)} = v^{20-(t-1)}$

$\Rightarrow 2\cdot v^{20-t+1} = 1$ with $i = 5\%$

$\Rightarrow (21-t)\ln(1.05) = \ln 2 \Rightarrow t \cong 6.79$ Ⓑ //

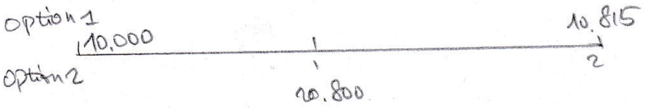
Exam FM Question 257

Give: ① PV of 10,000 now and 10.815 two-year from now

same PV

② 20.800 one-year from now at either of two different rate x and y ⟺ x and y are the solution of eq(1)

Question: $|x-y|$.



Solve: $10.000 + 10.815 v^2 = 20,800 v$

$\Rightarrow 10.815 v^2 - 20.800 v + 10.000 = 0$

$\Rightarrow v = \dfrac{20.800 \pm \sqrt{20.800^2 - 4 \times 10.815 \times \overset{432600000}{10.000}}}{2 \times 10.815} \Rightarrow v_1 = 0.97087$ or $v_2 = 0.95238$.

$\Rightarrow i_1 = 0.03$ or $i_2 = 0.05$

$\Rightarrow |X-Y| = 0.02$.

Exam FM Question 258

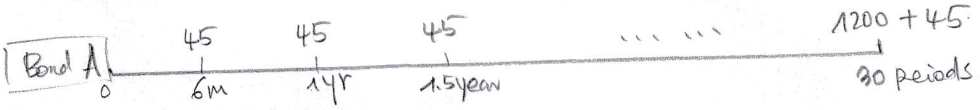
Give: ① Bond A: 15-year, 1000 FV, annual coupon rate 9% semiannually. redeemed at 1200, bought to yield = 8.4% convertible semiannually

② Bond B: n-year, 1000 FV, annual coupon rate 8% seminally. redeemed at 1376.69, bought to yield = 8.4% convertible smianually.

Question: what is n?

$\text{Price} = 45 a_{\overline{30}|4.2\%} + 1200 v_{4.2\%}$

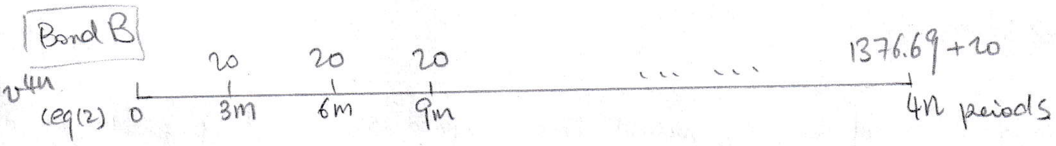
$= 1108.85$ (eq(1))



$i_{6m} = \frac{8.4\%}{2}$

$\text{Price} = 20 \cdot a_{\overline{4n}|2.1\%} + 1376.69 v^{4n}$

(eq(2))



$i_{3m} = \frac{8.4\%}{4}$

$= 2.1\%$

Solve: $eq(1) = eq(2) \Rightarrow 1108.85 = 20 \cdot a_{\overline{4n}|2.1\%} + 1376.69 v^{4n}$

$\Rightarrow v^{4n} = 0.3687$

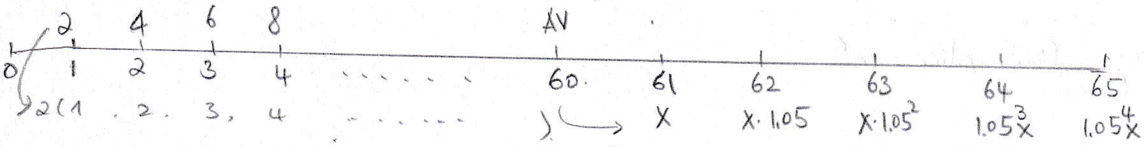
$\Rightarrow 4n \cdot \underbrace{\ln(v)}_{0.02078} = \underbrace{\ln(0.3687)}_{0.9976}$

$\Rightarrow 4n = 48$

$\Rightarrow n = 12$ Ⓐ//

Exam FM Question 259

Give:



① Marilyn. deposit 2 at the end of 1st year, & increase by 2 thereafter, for 60-year, with i

② Use AV at year 60, to purchase a 5-year annuity-immediate paying i, with X 1st payment increasing by 5%.

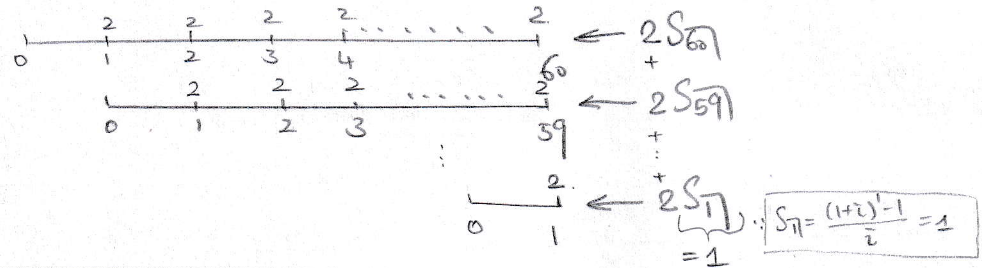
Question: which one is correct?

Ⓐ $$\frac{2(\ddot{s}_{\overline{60|}}-60)}{i} = \frac{X(1-(1.05v)^5)}{i-0.05}$$ wrong (left side: $\ddot{s}_{\overline{60|}}$).

Solve $2\cdot Is_{\overline{n|}} = 2\cdot\frac{\ddot{s}_{60}-n}{i}$ 等比数列 $\frac{Xv-1.05^4Xv^5(1.05v)}{1-1.05v}\;\frac{/v}{/v} = \frac{X(1-(1.05v)^5)}{\underbrace{\frac{1}{v}}_{1+i}-1.05} = \frac{X(1-(1.05v)^5)}{i-0.05}$

Ⓑ $$2(Ia)_{\overline{60|}} = Xv^{60}\left(\overset{v}{1}+1.05v^{2}+\dots+(1.05v)^{\overset{5}{4}}\right)$$ worg (right side; should multiple by "v").

Ⓒ ✓ correct $$\underbrace{2\sum_{t=0}^{59} s_{\overline{60-t|}}} = X\cdot(v+1.05v^2+\dots+1.05^4v^5)$$



$\because s_{\overline{1|}} = \frac{(1+i)^1-1}{i} = 1$

Exam FM Question 260.

Give: A company purchases
- 15 1-year zero-coupon bond: price=961.54
- 20 2-year zero-coupon bond: price=966.14
- 30 3-year zero-coupon bond: price=878.41

Question: Macaulay Duration of this Portfolio.

Solve: $$\boxed{D_{\text{portfolio}} = \underbrace{\text{weight}_1}_{\frac{\text{Value}_1}{\text{Total Value}}} \times D^{mac}_{\text{Asset}_1} + \underbrace{\text{weight}_2}_{\frac{\text{Value}_2}{\text{total Value}}} \times D^{mac}_{\text{Asset}_2} + \underbrace{\text{weight}_3}_{\frac{\text{Value}_3}{\text{total value}}} \times D^{mac}_{\text{Asset}_3}}.$$

$\text{Value}_1 = 15\times961.54 = 14423.1$

$\text{value}_2 = 20\times966.14 = 19322.8$

$\text{value}_3 = 30\times878.61 = 26358.3.$

$\text{Total} = 1+2+3 = 60104.2$

$$= \frac{14423.1}{60104.2}\times1 + \frac{19322.8}{60104.2}\times2 + \frac{26358.3}{60104.2}\times3$$

$$= 0.24\times1 + 0.32\times2 + 0.44\times3$$

$$= 2.2 \quad Ⓓ\,//$$