SOA and CAS: Exam FM[1]
Written Solutions: 119-170

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This document only provides written solutions to official example problems 119-170. For official sample questions, check out the official websites of Society of Actuaries and the Casualty Actuarial Society.

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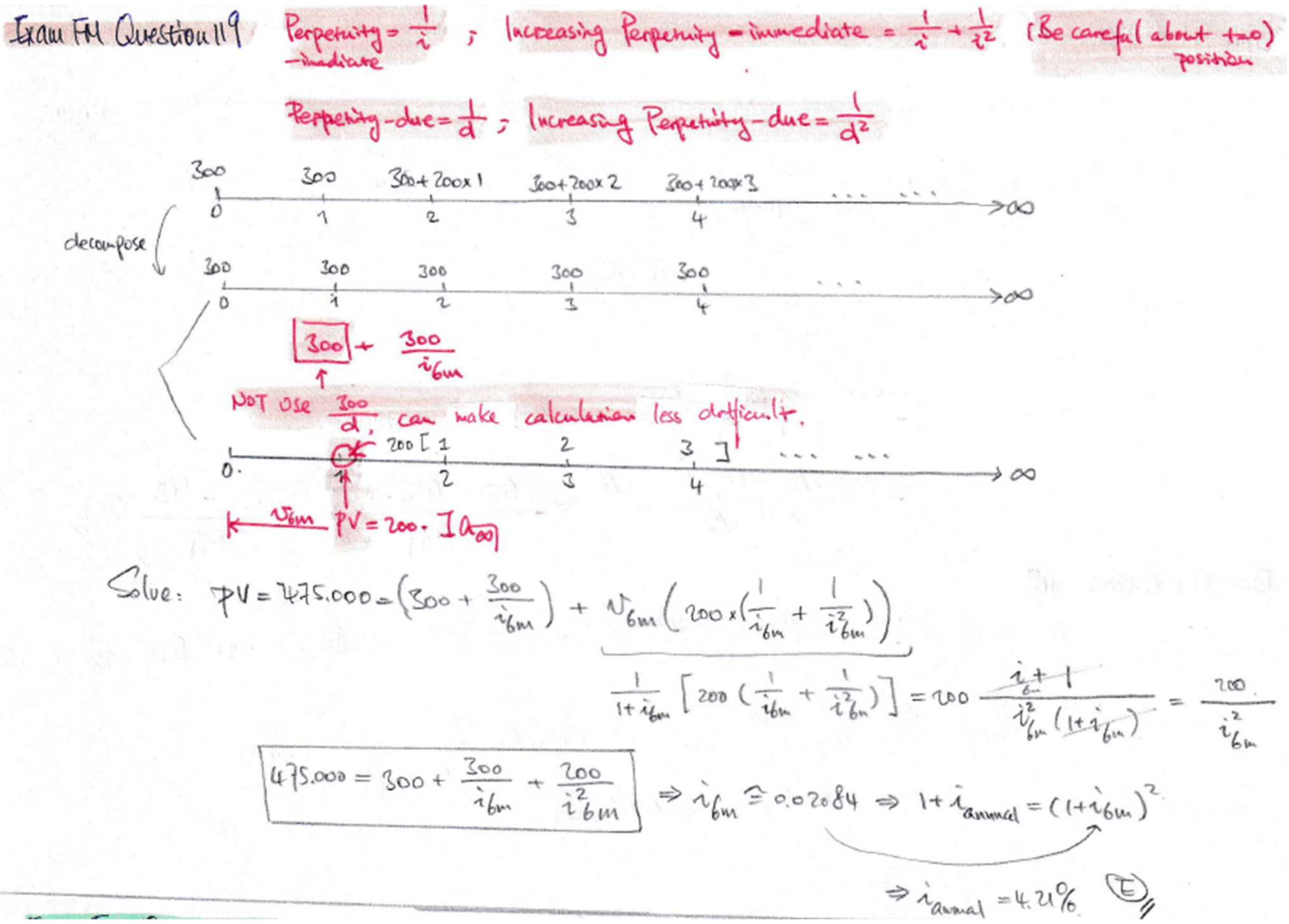
[2] Email: liyifinhub@outlook.com The written solutions were drafted when I was preparing for the exam. Please email me if you find any errors.

Exam FM Question 119:

A perpetuity-due with semi-annual payments consists of 2 level payments of 300, followed by a series of increasing payments. Beginning with the 3rd payment, each payment is 200 larger than the pervious one. Using an annual rate of I, the PV of perpetuity is 475,000

Question: What is i?



Exam FM Question119

Perpetuity-immediate $= \frac{1}{i}$; Increasing Perpetuity-immediate $= \frac{1}{i} + \frac{1}{i^2}$ (Be careful about two position)

Perpetuity-due $= \frac{1}{d}$; Increasing Perpetuity-due $= \frac{1}{d^2}$

$\boxed{300} + \frac{300}{i_{6m}}$

NOT use $\frac{300}{d}$, can make calculation less difficult.

$PV = 200 \cdot (Ia)_{\overline{\infty}|}$

Solve: $PV = 475.000 = \left(300 + \frac{300}{i_{6m}}\right) + \underbrace{v_{6m}\left(200 \times \left(\frac{1}{i_{6m}} + \frac{1}{i^2_{6m}}\right)\right)}$

$\frac{1}{1+i_{6m}}\left[200\left(\frac{1}{i_{6m}} + \frac{1}{i^2_{6m}}\right)\right] = 200 \frac{\cancel{i_{6m}+1}}{i^2_{6m}\cancel{(1+i_{6m})}} = \frac{200}{i^2_{6m}}$

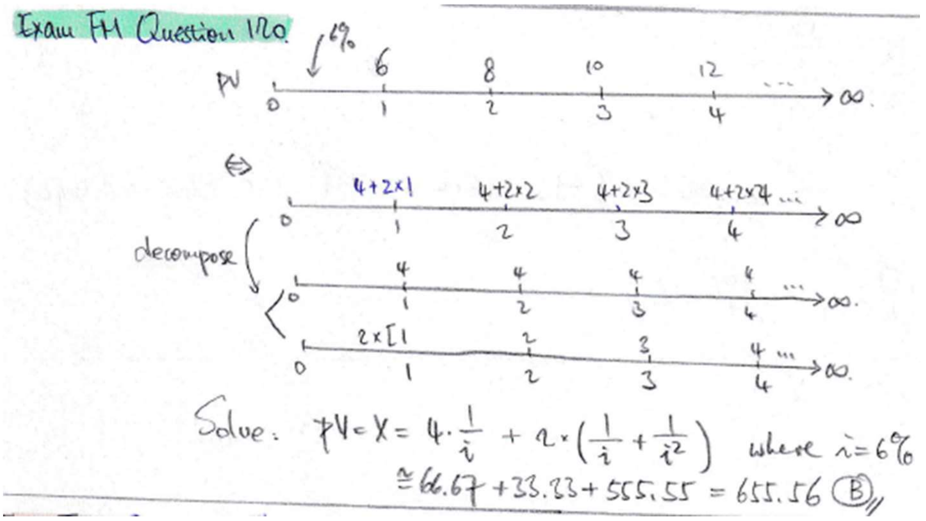
$\boxed{475.000 = 300 + \frac{300}{i_{6m}} + \frac{200}{i^2_{6m}}} \Rightarrow i_{6m} \cong 0.02084 \Rightarrow 1 + i_{annual} = (1+i_{6m})^2$

$\Rightarrow i_{annual} = 4.21\%$ Ⓔ//

Exam FM Question 120:

At an annual rate of 6%, the PV of a perpetuity immediate with successive annual payments of 6, 8, 10, 12,⋯, is equal to X

Question: What is X?



Exam FM Question 120

Solve: $PV = X = 4 \cdot \frac{1}{i} + 2 \times \left(\frac{1}{i} + \frac{1}{i^2}\right)$ where $i = 6\%$

$\cong 66.67 + 33.33 + 555.55 = 655.56$ Ⓑ//

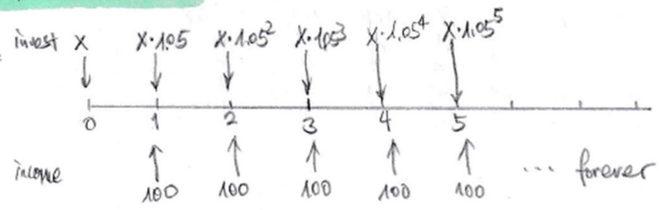
Exam FM Question 121:

Give: A project requires an investment of X today. Additional investments are required at the beginning of each of the next 5 years, with each year's investment 5% greater than the previous one. The investment is expected to produce an income of 100 per year at the end of year forever. At an annual rate 10.25%, the project has a net present value of zero.

Question: What is X?

Exam FM Question121

Give:



① NPV of this project = 0

② $i_{annual} = 10.25\%$.

Question: X

Solve: $\boxed{PV_{invest}} = X + X\cdot 1.05v + X\cdot 1.05^2 v^2 + X\cdot 1.05^3 v^3 + X\cdot 1.05^4 v^4 + X\cdot 1.05^5 v^5$

$= X\left[1 + (1.05v) + (1.05v)^2 + \cdots + (1.05v)^5\right]$

$= X \cdot \underbrace{\frac{1 - (1.05v)^5\cdot(1.05v)}{1 - 1.05v}}_{\frac{0.253784}{0.04765} \cong \boxed{5.3260}} \leftarrow \frac{a_1 - a_n q}{1-q}$ where $v = \frac{1}{1+0.1025} \cong 0.9070$

$PV_{income} = \boxed{\text{Perpetuity- Immediate}} = \boxed{100\cdot\frac{1}{i}} = 100\cdot\frac{1}{0.1025} = 975.6098$

$\because NPV = 0$

$\therefore \boxed{PV_{income} = PV_{invest}} \Rightarrow 5.3260X = 975.6098 \Rightarrow X = 183.1787$ Ⓐ //

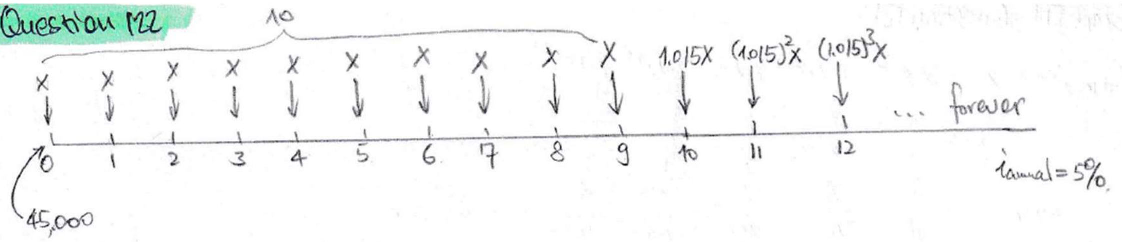
Exam FM Question 122:

Give: A perpetuity-due with annual payments consists of 10 level payments of X followed by a series of increasing payments. Beginning with the 11th payment, each payment is 1.5% larger than the preceding payment. Using an annual effective interest rate of 5%, the PV of the perpetuity is 45,000

Question: What is X?

Exam FM Question 122

Give:



Question: X

$$\text{Solve: } \underset{\overset{\nearrow}{45{,}000}}{PV_{investment}} = X\cdot\underbrace{\ddot{a}_{\overline{10}|5\%}}_{\underbrace{\overset{5\%}{(1+i)}\,\underbrace{a_{\overline{10}|5\%}}_{7.7217}}_{8.1078}} + v^9\Big(\underbrace{1.015X\cdot v + 1.015^2X\cdot v^2 + 1.015^3X\cdot v^3 + \dots}_{1.015X\cdot v\left(\underbrace{1+(1.015v)+(1.015v)^2+\dots}_{\frac{1}{1-1.015v}}\right)}\Big)$$

$$\text{where } v=\frac{1}{1+i}=\frac{1}{1.05}$$

$$= 8.1078\cdot X + v^9\cdot \underline{\underline{29X}}$$

$$= 8.1078X + 18.6936X$$

$$= 26.8014X$$

$$\Rightarrow 26.8014X = 45{,}000 \Rightarrow X \cong 1679.01 \text{ Ⓐ}_{//}$$

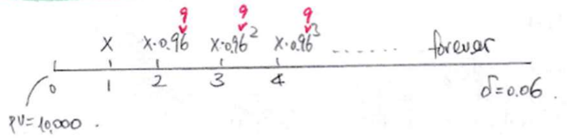
Exam FM Question 123:

Give: A perpetuity-immediate provides annual payments that decrease by 0.4% each year. The price of the perpetuity is 10,000 at an annual force of interest of 0.06.

Question: What is the amount of the perpetuity's 1st payment?

Exam FM Question 123

Give:



Question: X

Solve: ① 1-year: $(1+i)=e^{\int_0^1 \delta_t \underset{1}{dt}}=e^{\frac{0.06\times 1}{t}}=1.061836 \Rightarrow i_{annual}=0.061836$

② $PV = Xv + X\cdot 0.\overset{9}{\vee}96v^2 + X\cdot 0.\overset{9}{\vee}96^2 v^3 + X\cdot 0.\overset{9}{\vee}96^3 v^4 + \dots$

$= Xv\,(\underbrace{1+0.\overset{9}{\vee}96v+(0.\overset{9}{\vee}96v)^2+(0.\overset{9}{\vee}96v)^3+\dots}_{\frac{1}{1-0.\overset{9}{\vee}96v}})$ where $v=\frac{1}{1+i_{annual}}=0.941765$

$= 15.18925X$

$\because$ PV = 10,000

$\therefore 15.18925X = 10,000 \Rightarrow X \cong 658.36$ Ⓓ//

Exam FM Question 124:

Give: Approval to start no more than 2 projects in a year. 2 different projects were recommended, each of which requires an investment of 800 to be made at the beginning of the year. The CFs for each of the 3 projects are as follows:

End of year	Project A	Project B	Project C
1	500	500	500
2	500	300	250
3	-175	-175	-175
4	100	150	200
5	0	200	200

The company uses an annual effective interest rate of 10% to discount the CFs

Question: Which combination of projects the company should select?

Exam FM Question 124

Give: Project A. B. C's CFs. ; and $i_{annual} = 10\%$

Question: which project or combination to select

Solve: → As long as "NPV>0", one can choose. Ⓓ//

$$NPV_A = -800 + \underbrace{500v}_{454.5454} + \underbrace{500v^2}_{413.2231} - \underbrace{175v^3}_{131.48} + \underbrace{100v^4}_{68.3013} \quad \text{where } v = \frac{1}{1+10\%} = 0.909090$$

$$= -800 + 804.5898$$

$$= 4.5898 > 0 \qquad (\checkmark)$$

$$NPV_B = -800 + \underbrace{500v}_{454.5454} + \underbrace{300v^2}_{247.9333} - \underbrace{175v^3}_{131.48} + \underbrace{150v^4}_{102.452} + \underbrace{200v^5}_{124.1843}$$

$$= -2.365 < 0. \qquad (X)$$

$$NPV_C = -800 + \underbrace{500v}_{454.5454} + \underbrace{250v^2}_{206.6116} - \underbrace{175v^3}_{131.48} + \underbrace{200v^4}_{136.6027} + \underbrace{200v^5}_{124.1843}$$

$$= -9.536 \qquad (X)$$

Exam FM Question 125

Give: (1) Loan: \$100,000, repay at the end of year, for 20-year, $i_{annual} = 10\%$

(2) Repay: First 10 payments = amount of interest due ⟺ no principal repaid in first 10 periods.

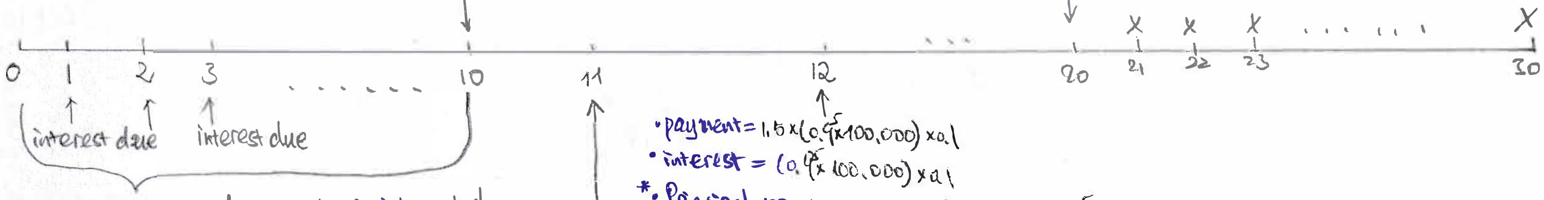
Second 10 payments = 150% of the amount of interest due ⟺ at the end of 20-year. Principal Outstanding $= 0.95^{10} \cdot 100000$

Third 10 payments = X ⟺ $X \cdot a_{\overline{10|}10\%} = \text{Outstanding}_{20} \leftarrow 0.95^{10} \cdot 100000$

Question: X

$\text{Outstanding}_{t=10} \overset{\text{Still}}{=} 100000.$

$\text{Outstanding}_{t=20} = 0.95^{10} \times 100,000$



Even if you donot know what's interest due, you should know: no principal is repaid

- payment = $1.5 \times (0.95^5 \times 100,000) \times 0.1$
- interest = $(0.95^5 \times 100,000) \times 0.1$

*• Principal repay = payment − interest $= (0.95^5 \times 100,000) \times 0.1 \times (1.5-1) = 0.05 \times (0.95^5 \times 100,000)$

$\Rightarrow \boxed{\text{Outstanding}_{12}} = \text{Outstanding}_{11} - {}^{*}\text{Principal repay} = \boxed{\underbrace{0.95^2}_{(1-0.05)\times0.95} \times 100,000}$

- payment = 150% interest $= 1.5 \times (100,000) \times 0.1$
- interest $= (100,000) \times 0.1$

*• principal repay = payment − interest $= \underbrace{(1.5-1)}_{0.5} \times (100,000) \times 0.1 = 0.05 \times (100,000)$

$\Rightarrow \boxed{\text{Outstanding}_{11}} = \text{Outstanding}_{10} - {}^{*}\text{Principal repay} = 100,000 - 0.5 \times (100,000) = \boxed{0.95 \times (100,00)}$

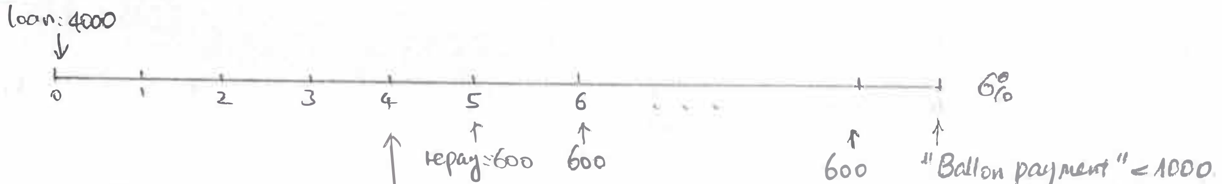
以0.95x(100,000)为基值,往后做

$\Rightarrow \text{Outstanding}_{20} = 0.95^{10} \times 100,000$

$\because \text{Outstanding}_{20} = X \cdot \underbrace{a_{\overline{10|}10\%}}_{6.14457}$

$\Rightarrow X \doteq 9744.16$ Ⓓ//

Exam FM Question 126 (Similar to Q129)



Question: Ballon Payment

$4000 = v^4 \cdot [600 \cdot a_{\overline{n|}6\%}]$ where $i = 6\%$

$\Rightarrow a_{\overline{n|}6\%} = 8.4165$

$\Rightarrow n \doteq 12.07$

First. take integal ⇒ n = 12 ⟺ meaning: 11 regular = 600

last ballon payment: X

Again: ⇒ $4000 = v^4 [600\, a_{\overline{11|}6\%} + P v^{12}]$ where $i = 6\%$

$\Rightarrow P = 639.43.$

↑向上取整

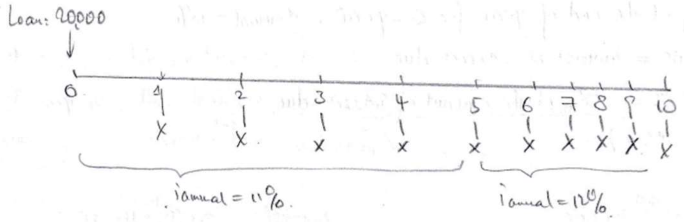
Exam FM Question 127:

Give: A loan of 20,000 is repaid by a payment of X at the end of year for 10-year. The loan has an annual effective rate of 11% for the first 5 years and 12% thereafter.

Question: What is X?

Exam FM Question 127

Give:



Question: X

Solve: $PV = 20{,}000 = X \cdot \underbrace{\underbrace{a_{\overline{5}|11\%}}_{3.6959}}_{3.6959X} + \underbrace{\overset{\frac{1}{1+11\%}}{v}{}^{5} \cdot \big(X \cdot \underbrace{a_{\overline{5}|12\%}}_{3.6048}\big)}_{2.1392X}$

$\boxed{20{,}000 = 5.8352X}$

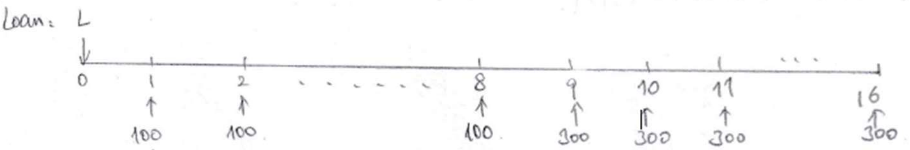
$\Rightarrow X = 3427.49$ Ⓒ//

Exam FM Question 128:

Give: A 6-year loan of L is repaid with a payment at the end of each year. 100 for the first 8 years. 300 for the final 8 years. An annual rate of I, such that 1/(1+i)^8>0.3. The outstanding principal is L+25 after the 1st payment of 100 is made.

Question: What is the outstanding balance immediately after the 8th annual payment of 100 has been made.

Exam FM Question 128



i where $\frac{1}{(1-i^8)} > 0.3$

interest $= L \cdot i$

principal repaid $= 100 - Li$

Outstanding $= L - (100 - Li)$ } same

$\text{Outstanding}_{t=1} = L + 25$

Solve. $L = 100 \cdot a_{\overline{8}|i} + v^8 \cdot (300 \cdot a_{\overline{8}|i}) \iff L = 100 \cdot \frac{1-v^8}{i} + v^8 \cdot 300\, \frac{1-v^8}{i} \Rightarrow iL = 100 + 200v^8 - 300v^{16}$

$L + 25 = L - (100 - Li) \Rightarrow Li = 125$

$\Rightarrow 100 + 200v^8 - 300v^{16} = 125 \Rightarrow v^8 = \frac{-200 \pm \sqrt{200^2 - 4 \times (-300) \times (-25)}}{2 \times (-300)} = 0.5$

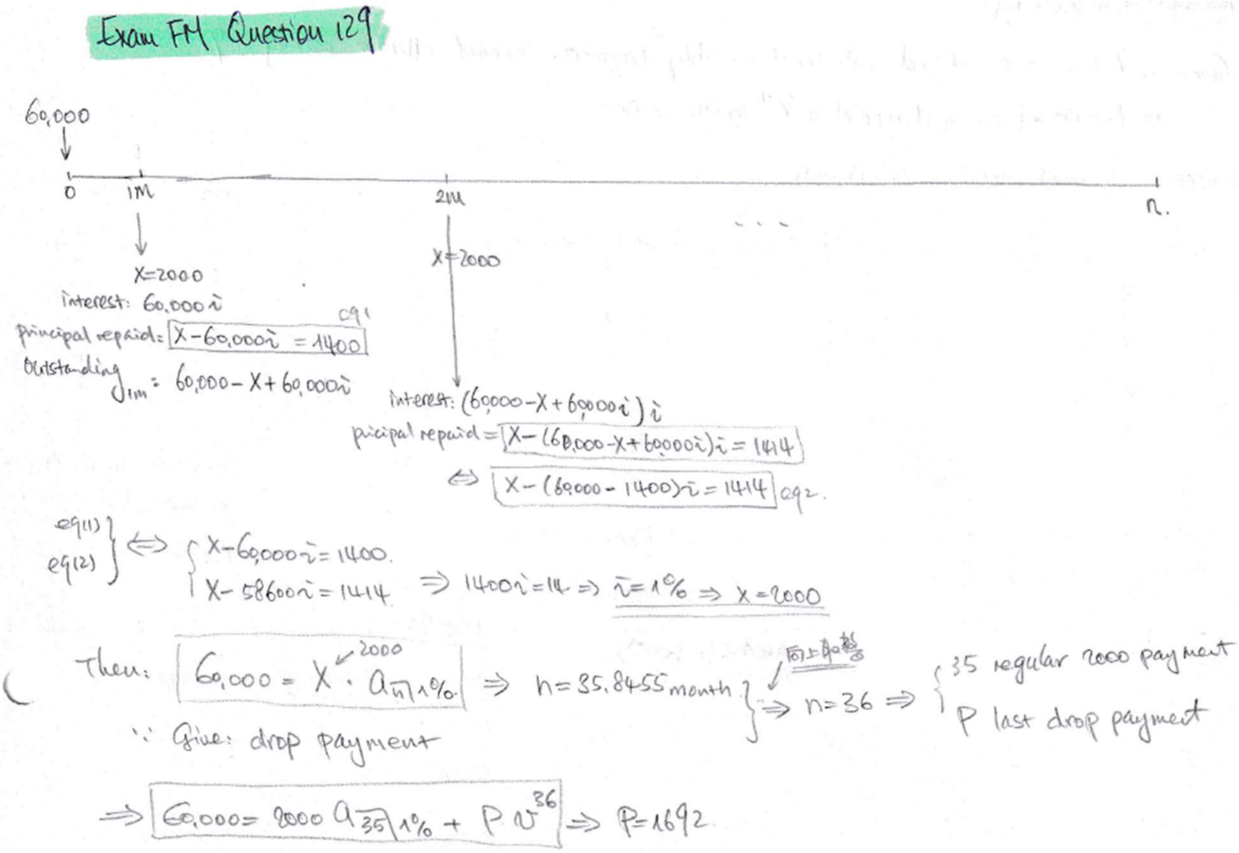
Question: Outstanding after 8th payment of 100.

$\text{Outstanding}_{t=8} = 300 \cdot a_{\overline{8}|i} = 300\, \frac{1-\overset{\swarrow 0.5}{v^8}}{\underset{\nwarrow 2^{1/8}-1}{i}} = 1657.$ Ⓐ//

Exam FM Question 129:

Give: A loan of 60,000 with a nominal annual rate compounded monthly. The loan is repaid with level payments, with a final drop payment. All payments are made at the end of month. The principal portion of the payment is 1,400 for the 1st month and 1,414 for the 2nd month.

Question: What is the drop payment?



Exam FM Question 129

interest: $60{,}000i$

principal repaid: $\boxed{X-60{,}000i=1400}$ eq1

Outstanding_{1m}: $60{,}000-X+60{,}000i$

interest: $(60000-X+60000i)i$

pricipal repaid $=\boxed{X-(60{,}000-X+60000i)i=1414}$

$\Leftrightarrow \boxed{X-(60000-1400)i=1414}$ eq2.

$$\left.\begin{matrix}eq(1)\\eq(2)\end{matrix}\right\}\Leftrightarrow\begin{cases}X-60{,}000i=1400.\\X-58600i=1414\end{cases}\Rightarrow 1400i=14\Rightarrow \underline{i=1\%\Rightarrow X=2000}$$

Then: $\boxed{60{,}000=\overset{2000}{X}\cdot a_{\overline{n}|1\%}}\Rightarrow n=35.8455\text{month}$ 向上取整 $\Rightarrow n=36\Rightarrow\begin{cases}35\text{ regular 2000 payment}\\P\text{ last drop payment}\end{cases}$

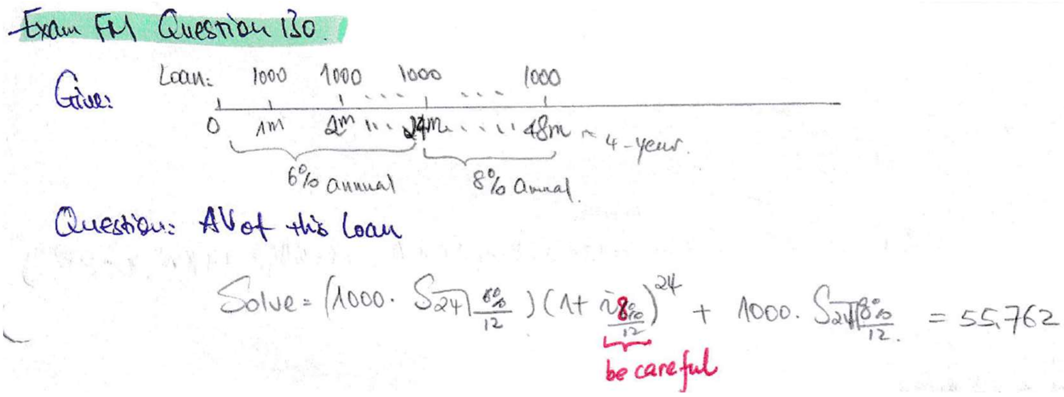
∵ Give: drop payment

$$\Rightarrow\boxed{60{,}000=2000\,a_{\overline{35}|1\%}+P\,v^{36}}\Rightarrow P=1692.$$

Exam FM Question 130:

Give: Borrow money for tuition. 1,000 at the end of each month for 4 years. No payments are made to repay the loan until the end of 5 years. Interest rate 6% convertible monthly for the first 2 years, 8% convertible money for the following 2 years.

Question: What is the loan balance at the 4 fours immediately following the receipt of the final 1,000?



Exam FM Question 130.

Give:

Question: AV of this Loan

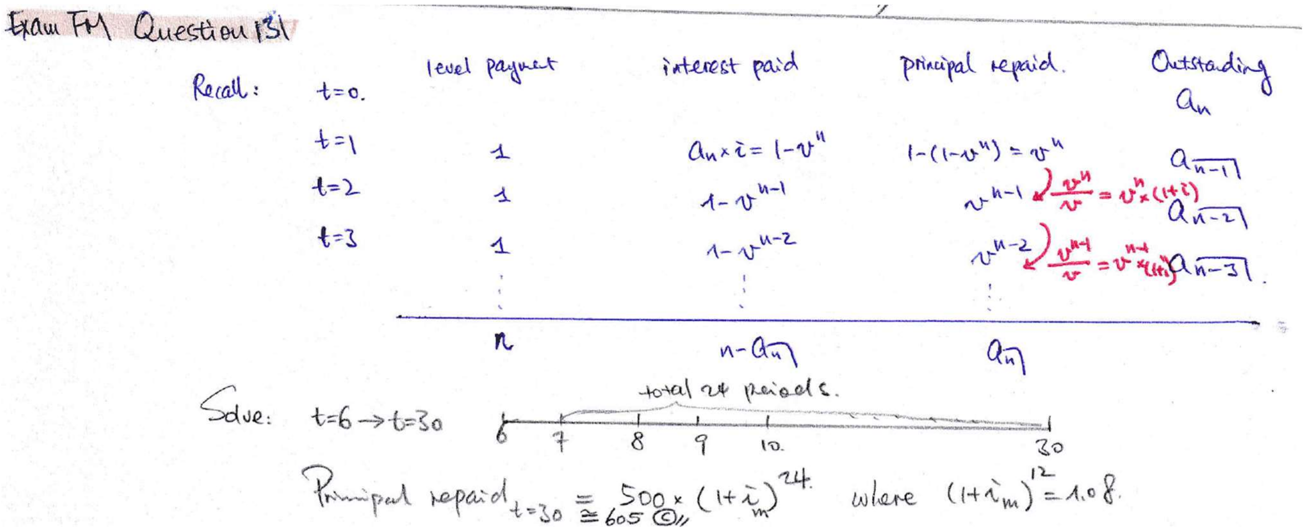
$$\text{Solve}=\left(1000\cdot s_{\overline{24}|\frac{6\%}{12}}\right)\left(1+\underbrace{i_{\frac{8\%}{12}}}_{\text{be careful}}\right)^{24}+1000\cdot s_{\overline{24}|\frac{8\%}{12}}=55{,}762$$

SS

Exam FM Question 131:

A loan is amortized with level monthly payments at annual rate of 10%. The amount of principal repaid in the 6th month is 500

Question: What is the principal repaid in the 30th month?



Exam FM Question 131

Recall:

		level payment	interest paid	principal repaid.	Outstanding a_n
	t=0.				
	t=1	1	$a_n \times i = 1-v^n$	$1-(1-v^n) = v^n$	$a_{\overline{n-1}\rceil}$
	t=2	1	$1-v^{n-1}$	v^{n-1} $\frac{v^n}{v} = v^n \times (1+i)$	$a_{\overline{n-2}\rceil}$
	t=3	1	$1-v^{n-2}$	v^{n-2} $\frac{v^{n-1}}{v} = v^{n-1} \times (1+i)$	$a_{\overline{n-3}\rceil}$.
	⋮	⋮	⋮	⋮	
		n	$n-a_{\overline{n}\rceil}$	$a_{\overline{n}\rceil}$	

Solve: t=6 → t=30

total 24 periods.

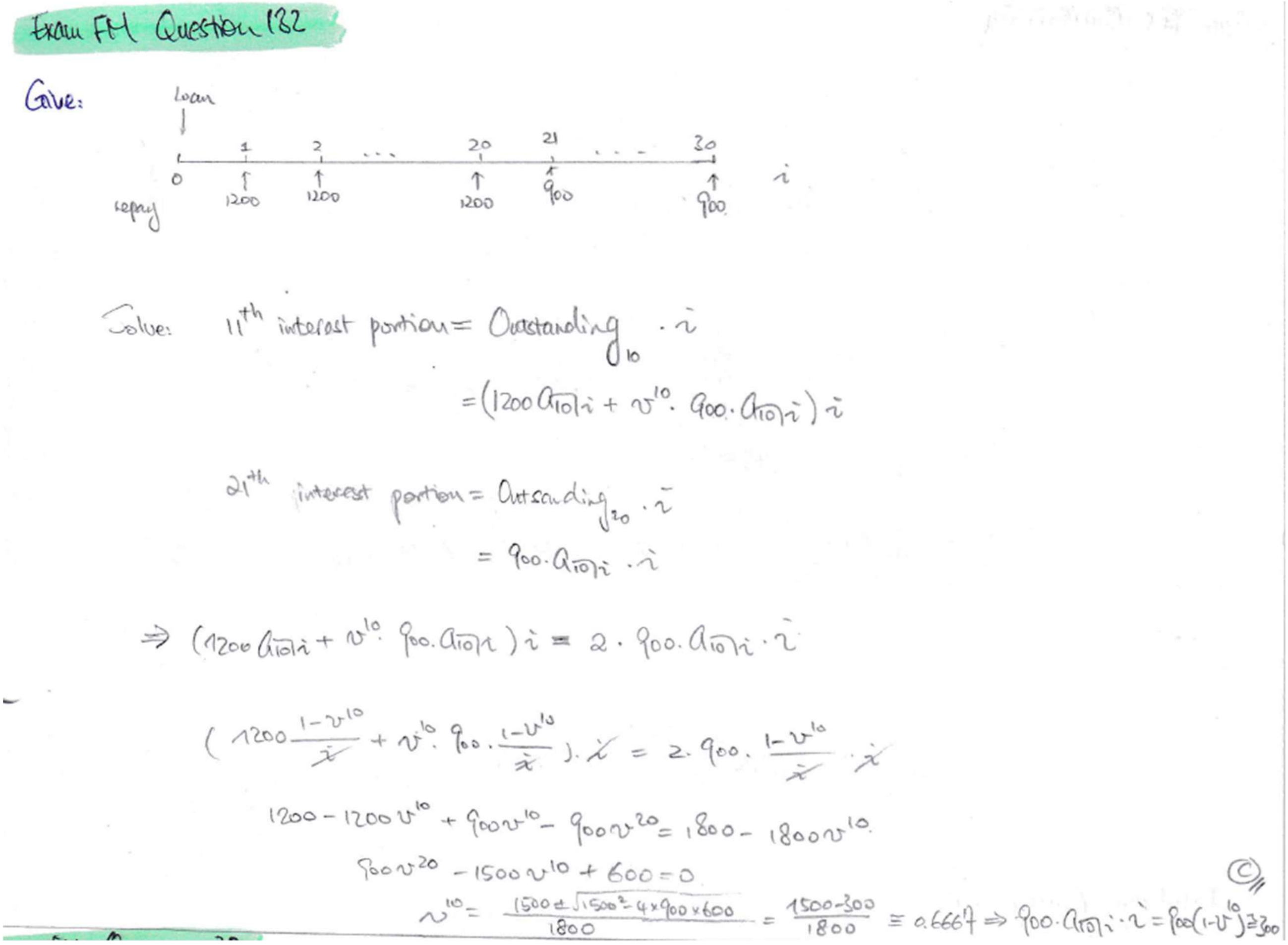
6 7 8 9 10. 30

$$\text{Principal repaid}_{t=30} = 500 \times (1+i_m)^{24} \cong 605 ☺ \quad \text{where } (1+i_m)^{12} = 1.08.$$

Exam FM Question 132:

Give: 30-year loan with payment at the end of each year. Each of the first 20 payments 1,200 and each of the last 10 payments is 900. Annual rate i. The interest portion of the 11^{th} payment is 2 times the interest portion of the 21th payment.

Question: What is the interest portion of the 21th payment?



Exam FM Question 132

Give:

Solve: 11^{th} interest portion = $\text{Outstanding}_{10} \cdot i$

$$= \left(1200\, a_{\overline{10}|i} + v^{10} \cdot 900 \cdot a_{\overline{10}|i}\right) i$$

21^{th} interest portion = $\text{Outstanding}_{20} \cdot i$

$$= 900 \cdot a_{\overline{10}|i} \cdot i$$

$$\Rightarrow \left(1200\, a_{\overline{10}|i} + v^{10} \cdot 900 \cdot a_{\overline{10}|i}\right) i = 2 \cdot 900 \cdot a_{\overline{10}|i} \cdot i$$

$$\left(1200 \frac{1-v^{10}}{\cancel{i}} + v^{10} \cdot 900 \cdot \frac{1-v^{10}}{\cancel{i}}\right) \cdot \cancel{i} = 2 \cdot 900 \cdot \frac{1-v^{10}}{\cancel{i}} \cdot \cancel{i}$$

$$1200 - 1200v^{10} + 900v^{10} - 900v^{20} = 1800 - 1800v^{10}$$

$$900v^{20} - 1500v^{10} + 600 = 0$$

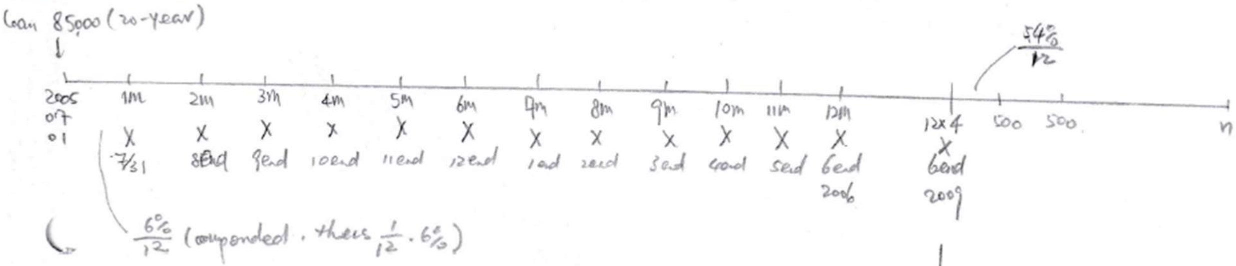
$$v^{10} = \frac{1500 \pm \sqrt{1500^2 - 4\times900\times600}}{1800} = \frac{1500-300}{1800} \cong 0.6667 \Rightarrow 900 \cdot a_{\overline{10}|i} \cdot i = 900(1-v^{10}) \cong 300$$

Exam FM Question 133:

Give: 20-year loan of 85,000 on July 1, 2005 at an annual rate of 6% compounded monthly. The loan was to be paid by level monthly payments at the end of each month with first payment on July 31, 2005. Right after the regular monthly payment on June 30, 2009, the loan was refinanced at a new rate 5.4% compounded monthly, and the remaining balance will be paid with monthly payments beginning July 31, 2009. The amount of each payment is 500 except the final drop payment.

Question: What is the last payment?

Exam FM Question 133



$$\leftarrow 500\, a_{\overline{n}|\frac{5.4\%}{12}}$$

$$\text{Outstanding} = \underbrace{85{,}000\left(1+\frac{6\%}{12}\right)^{12\times4}}_{107991.5787} - \underbrace{X\cdot \underbrace{s_{\overline{12\times4}|\frac{6\%}{12}}}_{54.0978}}_{32943.94} \longrightarrow$$

Solve. First: $85{,}000 = X\cdot a_{\overline{240}|\frac{6\%}{12}} \Rightarrow X \cong 608.97$

Second: $\text{Outstanding}_{48} = 107991.5787 - 32943.94 = 75047.6414$

$\because \text{Outstanding}_{48} = 500\, a_{\overline{n}|\frac{5.4\%}{12}} \Rightarrow n \cong 250.62$

$\because$ Last Payment is "drop" $\Rightarrow \boxed{n=251}$, instead of Ballon 250

$\Leftrightarrow$ 250 normal payment \$500, 251th drop payment

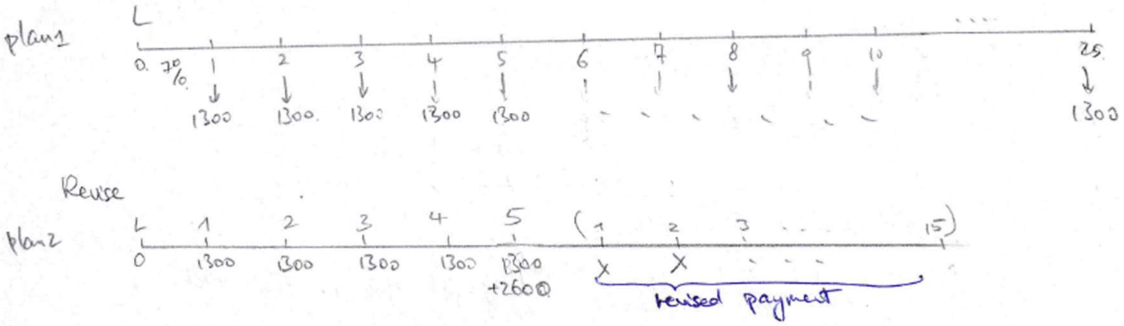
$\therefore$ Last period = 2009 June + 251 = May 2030

Exam FM Question 134:

Give: 25-year loan repaid with annual payments of 1,300 at an annual rate of 7%. Borrower pays an additional 2,600 at the time of 5th payment and wants to repay the remaining balance over 15 years.

Question: what is the revised annual payment?

Exam FM Question 134



Solve: $L = 1300\, a_{\overline{25}|7\%} \Rightarrow L = 15149.658$ (plan 1)

(plan2): $$\underbrace{L\cdot(1+\underset{7\%}{i})^5 - \underbrace{1300\, s_{\overline{5}|7\%}}_{7475.96} - 2600} = X\cdot \underbrace{a_{\overline{15}|7\%}}_{9.1} \Rightarrow X = 1227$$

Exam FM Question 135:

A 1000-par 30-year bond, annual coupon rate of 7% paid semiannually. Bond is callable immediately following the payment of any of the 20th through the 59th coupons.

i) If the bond is called before payment of the 40th coupon, the redemption value is 1,250.

ii) If the bond is called immediately after the payment of the 40th through the 59th coupons, the redemption value is 1,125.

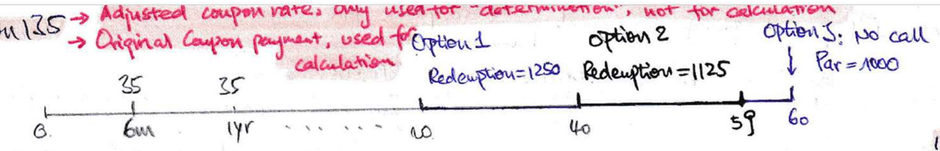
iii) If the bond is not called, it will be redeemed at par.

An annual nominal yield rate at least 5% convertible semiannually.

Question: What is n?

Exam FM Question 135 → Adjusted coupon rate, only used for "determination", not for calculation

→ Original Coupon payment, used for calculation



Solve: Option 1: Adjusted Coupon rate$_1$ = $\frac{35}{1250}$ = 2.8% > Yield$_{6m}$ = 2.5% } premium case ⇒ call at n=20: $\boxed{\text{price}_1}$ = $35a_{\overline{20}|2.5\%} + 1250v^{20}_{2.5\%}$ (1308.46)

Option 2: Adjusted Coupon rate$_2$ = $\frac{35}{1125}$ = 3.11% > Yield$_{6m}$ = 2.5% } premium case ⇒ call at n=40: $\boxed{\text{price}_2}$ = $35a_{\overline{40}|2.5\%} + 1125v^{40}_{2.5\%}$ (1297.58)

option 3: (No call) $\boxed{\text{Price}_3}$ = $35a_{\overline{60}|2.5\%} + 1000v^{60}_{2.5\%}$ (1309.08)

⇒ we know: option 2's price is the lowest of all. Thus n=40 Ⓒ

Exam FM Question 136:

20-year bond:

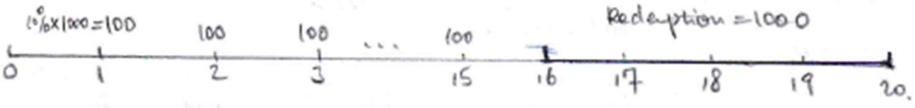
i) Par value is 1000.

ii) Annual coupon rate is 10%.

iii) Callable at par on any of the last five coupon dates.

Question: what is the maximum purchase price with an annual yield rate of at least 5%?

Exam FM Question136



Solve: ∵ Coupon rate =10% > 5% = yield rate ⇒ Premium Case

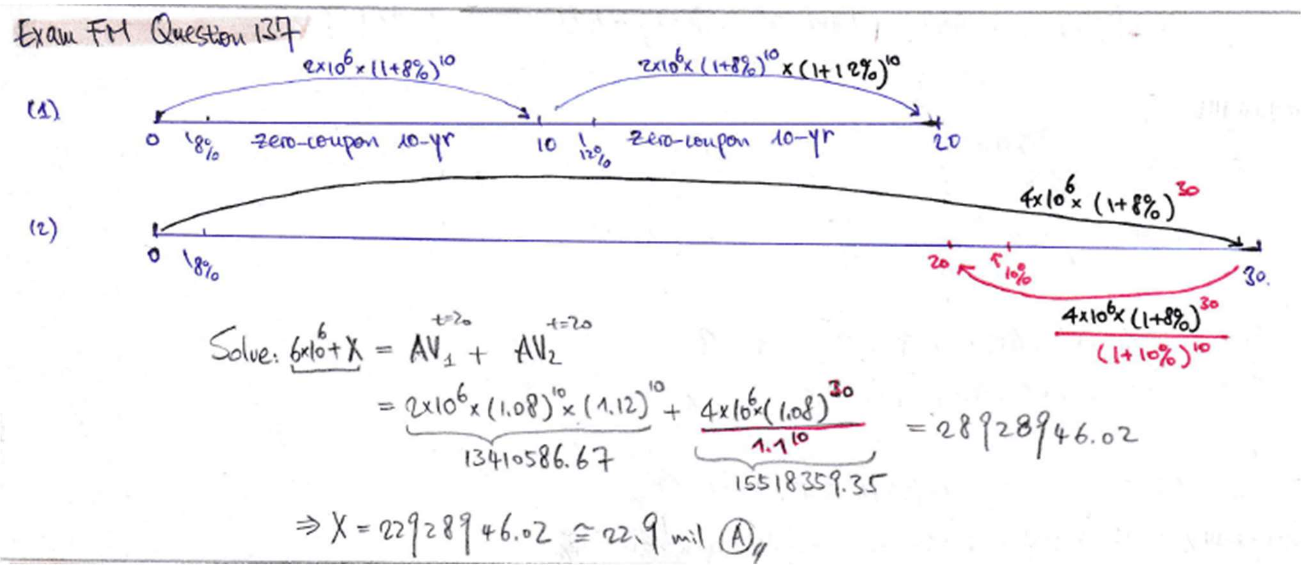
∴ Lowest Yield ⇔ Max price ⇔ call at earlist n=16

$$\text{Price} = \underbrace{100\,a_{\overline{16}|5\%} + 1000\,v^{16}_{5\%}}_{\text{calculator: PMT=100; n=16; I/Y=5; FV=1000} \Rightarrow \text{PV}} = 1542 \text{ Ⓑ}$$

Exam FM Question 137:

Invests 2 million in a 10-year zero-coupon bond and 4 million in a 30-year zero-coupon bond. Annual yield rate for both bonds is 8%. When the 10-year bond matures, the company reinvests in another 10-year zero-coupon bond. The bond annual yield rate at that time is 12%. After 20 years from the initial investment, the 30-year bond is sold to yield an annual rate of 10%. The maturity of the second 10-year bond and the sale of the 30-year bond result in a gain of X on the initial investment of 6 million.

Question: What is X?



Solve: $\underline{6\times10^6+X} = \overset{t=20}{AV_1} + \overset{t=20}{AV_2}$

$= \underbrace{2\times10^6\times(1.08)^{10}\times(1.12)^{10}}_{13410586.67} + \underbrace{\frac{4\times10^6\times(1.08)^{30}}{1.1^{10}}}_{15518359.35} = 28928946.02$

$\Rightarrow X = 22928946.02 \cong 22.9$ mil Ⓐ//

Exam FM Question 138:

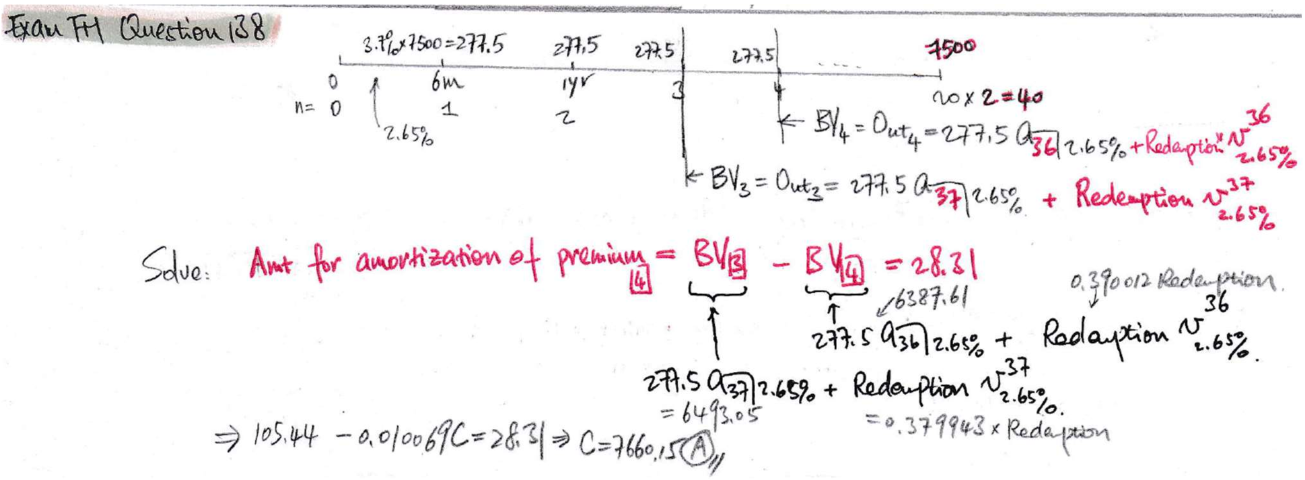
A 20-year bond with face amount 7500:

i) An annual coupon rate of 7.4% paid semiannually.

ii) Annual yield rate 5.3% convertible semiannually.

iii) The amount for amortization of premium in the 4th coupon payment is 28.31.

Question: what is the redemption value of the bond?



$\leftarrow BV_4 = Out_4 = 277.5\,a_{\overline{36}|2.65\%} + Redaption \times v^{36}_{2.65\%}$

$\leftarrow BV_3 = Out_3 = 277.5\,a_{\overline{37}|2.65\%} + Redemption\ v^{37}_{2.65\%}$

Solve: Amt for amortization of $premium_{\boxed{4}} = \underbrace{BV_{\boxed{3}}} - \underbrace{BV_{\boxed{4}}} = 28.31$

BV_3: $\underbrace{277.5\,a_{\overline{37}|2.65\%}}_{=6493.05} + \underbrace{Redaption\ v^{37}_{2.65\%}}_{=0.379943\times Redaption}$.

BV_4: $\underbrace{277.5\,a_{\overline{36}|2.65\%}}_{6387.61} + \underbrace{Redaption\ v^{36}_{2.65\%}}_{0.390012\ Redemption}$.

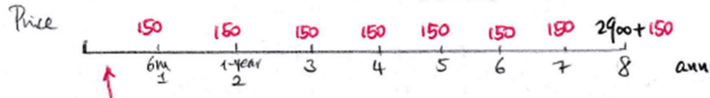
$\Rightarrow 105.44 - 0.010069C = 28.31 \Rightarrow C = 7660.15$ Ⓐ//

Exam FM Question 139:

8-year callable bond, 10% annual coupon rate payable semiannually. A face value of 3,000. Redemption value of 2,800. The purchase price assumes the bond is called at the end of the 4th year for 2,900 and provides an annual yield of 10.0%. bond is called for 2,960 after the 1st coupon payment. Annual effective yield rate is i.

Question: What is i?

Exam FM Question 139 Frequency = Semi-annual { coupon⇔yield⇔6m | n→2n



FV = 3000
annual coupon = 10% } Annual coupon payment = 300

⇒ semi-annual coupon = 150.

r_{6m}: $(1+r_{6m})^2 = 1.10 \Leftrightarrow r_m = 4.88\%$

Solve: First: get $Price_{t=0} = \underbrace{150\, a_{\overline{8}|}4.88\% + 2900\, v_{6m}^{8}}_{PMT=150,\ n=8,\ I/Y=4.88,\ FV=2900} = 2955.075$

Second: $2955.075 = \underbrace{150\, a_{\overline{1}|}i_{6m} + 2960\, v_{6m}^{1}}_{PMT=150,\ N=1,\ FV=2960,\ PV=-2955.075} \Rightarrow i_{6m} \cong 5.24\% \Rightarrow (1+i_{annual}) = (1+i_{6m})^2$

$\Rightarrow i_{annual} = 10.75\%$ ©//

Exam FM Question 140:

20-year bond. Face amount 1,000

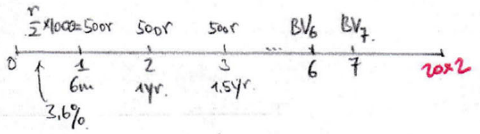
i) An annual coupon rate of r payable semiannually. Redeemable at par.

ii) Annual yield rate convertible semiannually is 7.2%.

iii) The amount for accumulation of discount in the 7th coupon payment is 4.36.

Question: what is r?

Exam FM Question 140.



→ The Amount for accumulation of discount in the 7th coupon = $BV_7 - BV_6$

Solve. ∵ $BV_6 - BV_7 = 4.36$ where $BV_6 = (500r \cdot a_{\overline{34}|}3.6\% + 1000\, v^{34}) = 9716r + 300.4473$

$BV_7 = (500r \cdot a_{\overline{33}|}3.6\% + 1000\, v^{33}) = 9565.79r + 311.26$

∴ $-(9716r + 300.4473) + (9565.79r + 311.26) = 4.36 \Rightarrow r = 0.0429$ ©//

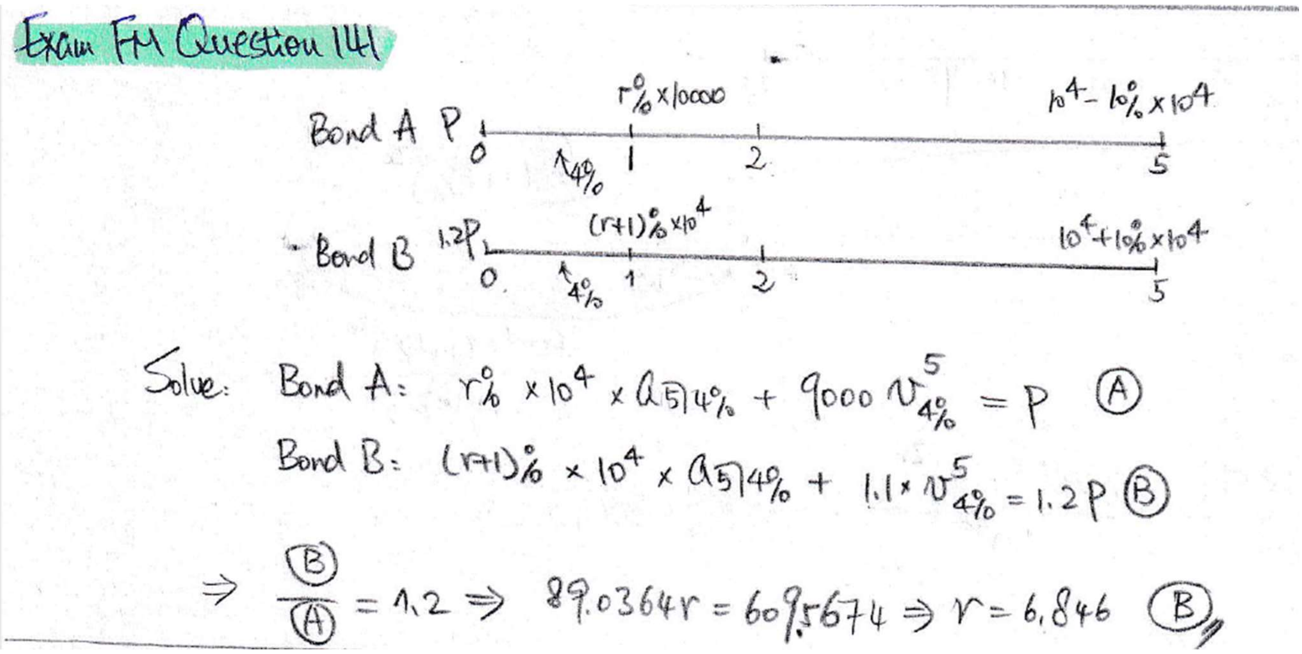
Exam FM Question 141:

Bond A and Bond B are two annual coupon, 5-year, 10,000 par value bonds. An annual effective rate of 4%.

i) Bond A has an annual coupon rate of r%, redemption value 10% below par, price of P.

ii) Bond B has an annual coupon rate of (r+1)%, redemption value 10% above par, price of 1.2P.

Question: What is r%?



Exam FM Question 141

Solve: Bond A: $r\% \times 10^4 \times a_{\overline{5}|4\%} + 9000\, v_{4\%}^{5} = P$ Ⓐ

Bond B: $(r+1)\% \times 10^4 \times a_{\overline{5}|4\%} + 1.1 \times v_{4\%}^{5} = 1.2P$ Ⓑ

$\Rightarrow \frac{Ⓑ}{Ⓐ} = 1.2 \Rightarrow 89.0364r = 609.5674 \Rightarrow r = 6.846$ Ⓑ//

Exam FM Question 142:

n-year bond, where n > 10:

i) 8% semiannual coupons, face amount 1,000

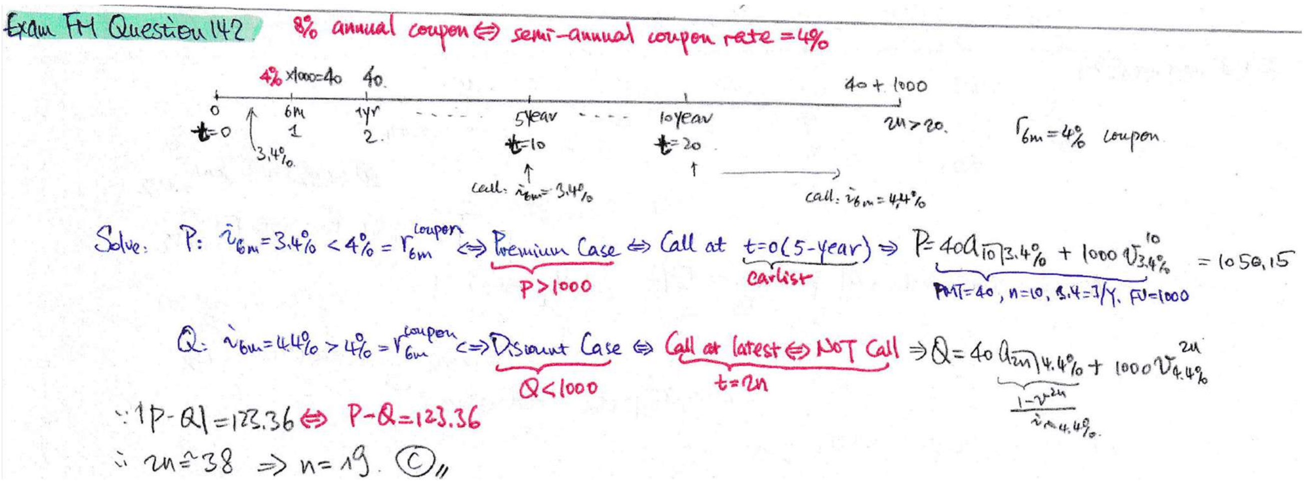
ii) redeemable at par.

iii) Callable at par 5 years after issue or 10 years after issue.

iv) P is the price to guarantee a yield of 6.8% convertible semiannually, Q is the price to guarantee a yield of 8.8% convertible semiannually.

v) |P – Q| = 123.36.

Question: What is n?



Exam FM Question 142 8% annual coupon ⇔ semi-annual coupon rate = 4%

Solve: P: $i_{6m} = 3.4\% < 4\% = r_{6m}^{\text{coupon}} \Leftrightarrow \underbrace{\text{Premium Case}}_{P>1000} \Leftrightarrow \text{Call at } \underbrace{t=0(5\text{-year})}_{\text{earlist}} \Rightarrow P = \underbrace{40a_{\overline{10}|3.4\%} + 1000\, v_{3.4\%}^{10}}_{PMT=40,\ n=10,\ 3.4=I/Y,\ FV=1000} = 1050.15$

Q: $i_{6m} = 4.4\% > 4\% = r_{6m}^{\text{coupon}} \Leftrightarrow \underbrace{\text{Disount Case}}_{Q<1000} \Leftrightarrow \underbrace{\text{Call at latest} \Leftrightarrow \text{NOT Call}}_{t=2n} \Rightarrow Q = 40\underbrace{a_{\overline{2n}|4.4\%}}_{\frac{1-v^{2n}}{i_{4.4\%}}} + 1000\, v_{4.4\%}^{2n}$

$\because |P-Q| = 123.36 \Leftrightarrow P-Q = 123.36$

$\therefore 2n \doteq 38 \Rightarrow n = 19.$ Ⓒ//

Exam FM Question 143:

4-year contract which requires to deposit 500 into a fund that earns an annual rate of 5.0%. The insurer expects that 100 in claims will be paid at the end of each year, for the next 4 years. At the end of the 4th year, the insurer is required to return 75% of the remaining fund balance to the insured.

To issue this policy, the insurer incurs 100 in expenses today. It also collects a fee of 125 at the end of 2 years.

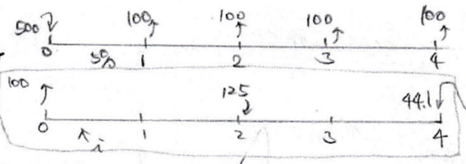
Question: What is the yield rate?

Exam FM Question 143

↓ inflow, ↑ outflow

2 timelines: Get AV_{4}

Get i



$AV_4 = 500(1+5\%)^4 - 100\underbrace{S_{\overline{4|}5\%}}_{\frac{(1+5\%)^4-1}{5\%}} \cong 176.74$

$\Rightarrow$ Remaing ↓ $= 0.25 \cdot AV_4 = 44.1$

Solve: $\underbrace{-100(1+i)^4}_{\text{AV of outflow}} + \underbrace{125(1+i)^2 + 44.1}_{\text{AV of inflow}} = 0$ Assume $(1+i)^2 = X$

$-100X^2 + 125X + 44.1 = 0 \Rightarrow X = \frac{-125 \pm \sqrt{\underbrace{125^2+4\times100\times44.1}_{17640}}}{-200} = \frac{-307.38}{-200} = 1.5369$

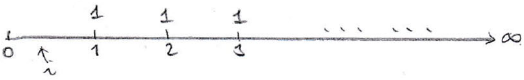
$\Leftrightarrow (1+i)^2 = 1.5369 \Leftrightarrow 1+i = 1.2397 \Leftrightarrow i \cong 23.97\%$ Ⓑ //

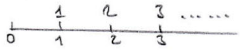
Exam FM Question 144:

A perpetuity-immediate with annual payments of 1, annual effective rate of i. Macaulay duration of 17.6 years.

Question: what is the Macaulay duration using an annual rate of 2i instead of i?

Exam FM Question 144





$\frac{1}{i} + \frac{1}{i^2} = (Ia)_{\overline{\infty|}}$

Solve: $D^{mac} = 17.6 = \frac{(v)\times1 + (v^2)\times2 + (v^3)\times3 + (v^4)\times4 + \cdots\cdots}{v + v^2 + v^3 + v^4 + \cdots\cdots}$ ↙ ↖ $\frac{1}{i} = a_{\overline{\infty|}}$

$= \frac{\frac{1}{i} + \frac{1}{i^2}}{\frac{1}{i}} \Rightarrow i \cong 6.0241\%$

Then: $\frac{\frac{1}{2i} + \frac{1}{(2i)^2}}{\frac{1}{2i}} = \frac{2i+1}{2i} = \frac{1.120482}{0.120482} \simeq 9.299$ Ⓑ //

Exam FM Question 145:

2 bonds with same annual yield rate i, the modified durations are a years and b years, with 0 < a < b.

One of these two bonds has a Macaulay duration of c years, with a < c < b.

Question: which of the following is an expression for the Macaulay duration of the other bond?

Exam FM Question 145

Solve: give: $D_A^{mod} = a$, $D_B^{mod} = b$ ↙ a

we also know $D^{mac} = (1+i)D^{mod}$, and $c > a$

Then: we have: Bond A: $D_A^{mac} = c$; $D_A^{mod} = a$.

$\Rightarrow D_B^{mac} = \underbrace{(1+i)}_{\frac{c}{a}} \times \underbrace{D_B^{mod}}_{b} = \frac{cb}{a}$ Ⓐ //

Exam FM Question 146

Give: (1) Old: 20-year Bond, $i^{old} = 10\%$, $D_{mac}^{old} = 11$

(2) New: $i^{new} = 10.25\%$

Question: Percentage Price Change $= \dfrac{\text{Bond Price}^{new} - \text{Bond Price}^{old}}{\text{Bond Price}^{old}}$, using first order "Macaulay" Approxi-

Solve: ∵ First order "Macaulay" Approximation: $\text{Bond Price}^{new} = \text{Bond Price}^{old} \cdot \left(\dfrac{1+i^{old}}{1+i^{new}}\right)^{D_{mac}}$

$= \text{Bond Price}^{old} \cdot \left(\dfrac{1+0.1}{1+0.1025}\right)^{11}$

$\Rightarrow \dfrac{\text{Bond Price}^{new}}{\text{Bond Price}^{old}} = \left(\dfrac{1.1}{1.1025}\right)^{11} \cong 0.9753.$

Thus: Percentage Price chang $= \dfrac{\text{Bond Price}^{new}}{\text{Bond Price}^{old}} - 1 = -0.02466 \cong -2.47\%$ Ⓑ//

Exam FM Question 147:

Exam FM Question 147

Determine which of the following statements regarding asset-liability management techniques is true.

(A) Redington immunization requires that the convexity of the liabilities is greater than the convexity of the asset (wrong: $C^{Asset} > C^{Liability}$)

(B) ✓ An advantage of the Redington immunization technique over the cash-flow matching technique is that the portfoli manager has more investment choices available. "CFs matching" is stricter than "Redington".

(C) Both Redington immunization & full immunization are based on the assumption that the yield curve has higher yields for longer term investment. (wrong: Not related to term structure)

(D) A full immunized portfolio ensures that the present value of assets will exceed the present value of liabilities with non-parallel shifts in the yield curves (wrong: PV of Asset = PV of liability)

(E) ✗ A cash-flow matched portfolio requires ~~less~~ more rebalancing than a Redington immunized portfolio, but ~~more~~ less rebalancing than a fully immunized portfolio. CFs matching: Stricter than Redington

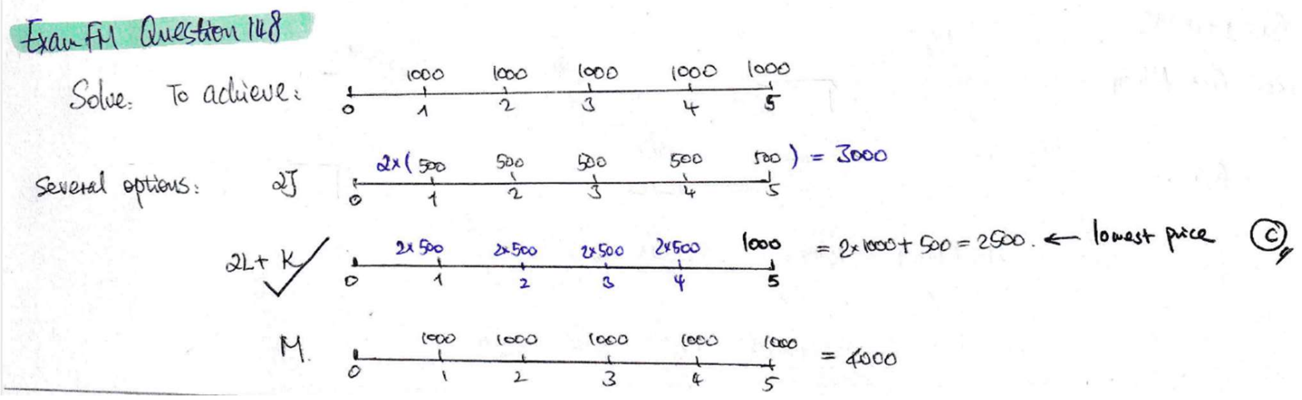
Exam FM Question 148:

Liabilities: 1000 at the end of each of next five years. Investments available are:

Investment	Price	Cash Flows
J	1500	500 at the end of each year for 5 years
K	500	1000 at the end of year 5
L	1000	500 at the end of each year for 4 years
M	4000	1000 at the end of each year for 5 years

We need fully immunized.

Question: what is the lowest price?



Exam FM Question 148

Solve: To achieve:

	1000	1000	1000	1000	1000
0	1	2	3	4	5

Several options:

2J

	2×(500	500	500	500	500)	= 3000
0	1	2	3	4	5	

2L+ K ✓

	2×500	2×500	2×500	2×500	1000	= 2×1000+ 500 = 2500. ← lowest price Ⓒ
0	1	2	3	4	5	

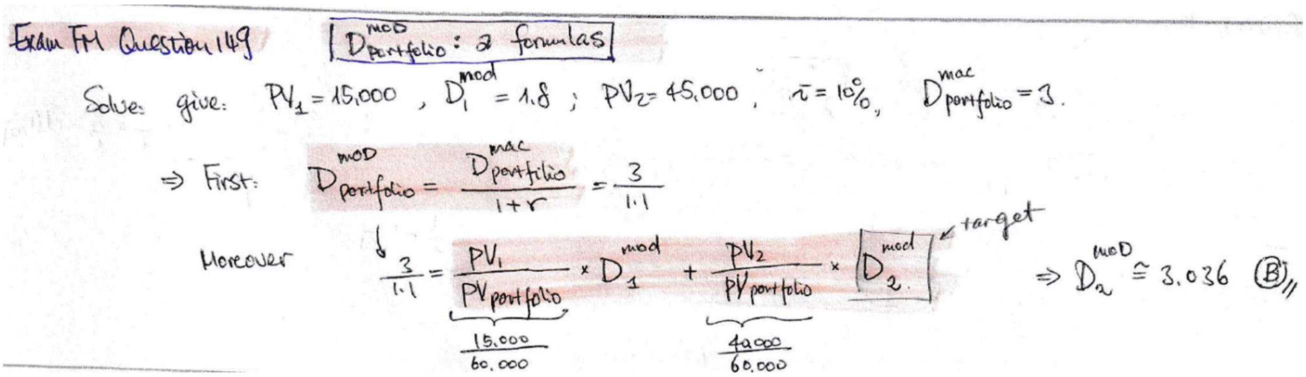
M.

	1000	1000	1000	1000	1000	= 4000
0	1	2	3	4	5	

Exam FM Question 149:

Loan 79,860 due three years from now. The company invests 15,000 in a bond with modified duration 1.80 and 45,000 in a bond with modified duration D^{mod}. Annual yield rate is 10%.

Question: what is D^{mod}?



Exam FM Question 149 $\boxed{D_{portfolio}^{MOD} : 2 \text{ formulas}}$

Solve: give: $PV_1 = 15{,}000$, $D_1^{mod} = 1.8$; $PV_2 = 45{,}000$, $r = 10\%$, $D_{portfolio}^{mac} = 3$.

$\Rightarrow$ First: $D_{portfolio}^{MOD} = \frac{D_{portfilio}^{mac}}{1+r} = \frac{3}{1.1}$

Moreover

$$\frac{3}{1.1} = \underbrace{\frac{PV_1}{PV_{portfolio}}}_{\frac{15{,}000}{60{,}000}} \times D_1^{mod} + \underbrace{\frac{PV_2}{PV_{portfolio}}}_{\frac{45{,}000}{60{,}000}} \times \boxed{D_2^{mod}} \leftarrow \text{target}$$

$\Rightarrow D_2^{MOD} \approx 3.036$ Ⓑ

Exam FM Question 150:

Liabilities 1000 at the end of year 1 and X at the end of year 2.

Investments available are:

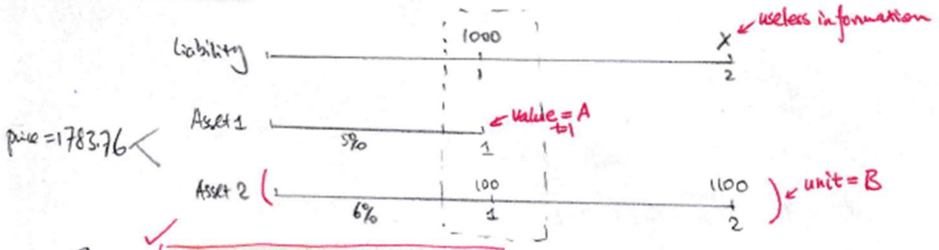
i) 1-year zero-coupon bonds, annual yield of 5%

ii) 2-year bonds with a par value of 1000, 10% annual coupons, annual yield of 6%

Exact cash flow matching. The purchase price of this portfolio is 1783.76.

Question: what is the amount invested in the one-year zero-coupon bond?

Exam FM Question150



Solve:

$$\boxed{\begin{array}{l}\text{Assume: } \text{Vaule}_{\text{Asset 1}, t=1} = A \\ \text{Unit}_{\text{Asset 2}} = B\end{array}}$$

Then (eq1): $A + 100B = 1000$

(eq2): $\frac{A}{1.05} + \left(\frac{100}{1.06} + \frac{1100}{1.06^2}\right) \times B = 1783.76$

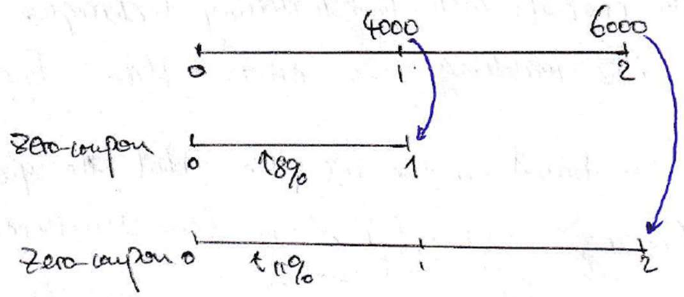
$$\Rightarrow \begin{cases} A = 915 \\ B = 0.85 \end{cases} \Rightarrow \boxed{\text{Amt Invest} = \frac{A}{1.05}} = \frac{915}{1.05} = 871 \text{ Ⓒ}$$

Exam FM Question 151:

Liabilities of 4000 and 6000 at the end of years 1 and 2. Investments available are 1-year zero-coupon bonds with an annual yield of 8% and 2-year zero-coupon bonds with an annual yield of 11%.

Question: what is the amount that must be invested today to have exactly matching?

Exam FM Question 151



Solve: $\text{Investment} = \frac{4000}{1.08} + \frac{6000}{1.11^2} \approxeq 8573.438$ Ⓑ

Exam FM Question 152. (similar to Q146. Just change " Mac → Mod")

Give: ① 20-year bond, $i^{old} = 10\%$, $D_{mac}^{old} = 11$

② $i^{new} = 10.25\%$

Question: bond's Price Percentage Change $\left(\frac{\text{Bond Price}^{new} - \text{Bond Price}^{old}}{\text{Bond Price}^{old}}\right)$, using 1st order **modified** approximation

Solve: Modified Approximation: (有 trick, 没有 power)

$$P^{new} = P^{old}\left(1 + \frac{\overset{10.25\%}{i^{new}} - \overset{10\%}{i^{old}}}{1+\underset{10\%}{i^{old}}} \cdot \underset{11}{D_{mac}^{old}}\right) = 0.975\ P^{old}.$$

$$\Rightarrow \frac{P^{new}}{P^{old}} = 0.975 \Rightarrow \text{Bond's Price Perkentage Change} = \frac{P^{new}}{P^{old}} - 1 = 0.975 - 1 = -2.5\%\ \text{Ⓒ}$$

Exam FM Question 153.

Give: ① n-year Bond, $P^{old} = 1000$ (par), $D_{mac}^{old} = 7.959$, $i^{old} = 7.2\%$.

② $i^{new} = 8.0\%$

Question: P^{new}, using: 1st order **macaulay** approximation

Solve. **Macaulay** Approximation:

$$P^{new} = P^{old}\underbrace{\left(\cancel{1+}\ \frac{1+i^{old}}{1+i^{new}}\right)^{D_{mac}^{old}}}_{1000 \times \left(\cancel{1+}\ \frac{1+7.2\%}{1+8.0\%}\right)^{7.959}} = 942.54\ \text{Ⓑ}$$

Exam FM Question 154

Give: (1) $D_{mod}^{old} = 8$, $P^{old} = 112.955$, $i^{old} = 6.4\%$

(2) E^{MAC}: 1st order "Macaulay" Approximation: $i^{new} = 7\%$

E^{mod}: 1st order "Modified" Approximation: $i^{new} = 7\%$

Solve: **Macaulay** Approximation: $P^{new} = P^{old}\left(\frac{1+i^{old}}{1+i^{new}}\right)^{D_{mac}^{old}}$ where $D_{mac}^{old} = (1+i^{old})D_{mod}^{old} = (1+6.4\%)\cdot D_{mod}^{old}$

$= 1.064 \times 8$.

$$= \underset{112.955}{P^{old}}\left(\frac{1+0.064}{1+0.07}\right)^{(1.064\times 8)} \cong 107.676$$

Modified Approximation:

$$P^{new} = P^{old}\left(1 - \frac{i^{new} - i^{old}}{1+i^{old}} \cdot D_{mac}^{old}\right)$$

$\cdot D_{mod}^{old}$

$$= P^{old}\left[1 - (i^{new} - i^{old}) \cdot D_{mod}^{old}\right]$$

$$= \underset{112.955}{P^{old}}\left[1 - (0.07 - 0.064) \cdot 8\right] \cong 107.533$$

Question: $E_{mac} - E_{mod} = 107.676 - 107.533 = 143$ Ⓔ

Exam FM Question 155

Give: portfolio $\begin{cases} (1)\ \text{Bond 1: } D^{old}_{mac} = 7.28\ ,\quad P^{old} = 35{,}000 \\ \text{Bond 2: } D^{old}_{mac} = 12.74,\quad P^{old} = 65{,}000 \end{cases}$

$$\Rightarrow \begin{cases} D^{old\_portfolio}_{mac} = \frac{35.000}{35{,}000+65{,}000} \cdot 7.28 + \frac{65{,}000}{35{,}000+65{,}000} \cdot 12.74 = 10.829 \\ P^{old} = Value^{portfolio} = 35{,}000 + 65{,}000 = 100{,}000 \end{cases}$$

(2) $i^{old} = 4.32\%$

(3) $P^{new}_{1^{st}\text{ order Macaulay}} = 105{,}000$

Question: i^{new}

Solve:

$$\underset{\overset{\nearrow}{105{,}000}}{P^{new}_{Macaulay}} = \underset{\overset{\uparrow}{100{,}000}}{P^{old}} \left(\frac{1+\overset{\overset{4.32\%}{\downarrow}}{i^{old}}}{1+\underset{\overset{\nwarrow}{\text{Question}}}{i^{new}}}\right)^{\overset{\overset{10.829}{\swarrow}}{D^{old}_{mac}}}$$

$$\Rightarrow \left(\frac{1.0432}{1+i^{new}}\right) = \left(\frac{21}{20}\right)^{\frac{1}{10.829}} \Rightarrow i^{new} = 3.85\%\ Ⓒ \quad //$$

Exam FM Question 156:

An annuity due with PV 123,000. An annual rate of 5%, modified duration is D_mod.

Uses the first-order Macaulay approximation to estimate the present value of the annuity due at an annual rate was 5.4%. The present value to be 121,212.

Question: what is D_mod if the modified duration of the annuity at 5%?

Exam FM Question 156

Solve: give: $i^{old} = 5\%$, $Price^{old} = 123{,}000$, $i^{new} = 5.4\%$, $Price^{new} = 121{,}212$,

Use " First order Macaulay duration" formula: $\underset{121211}{P^{new}} = \underset{123000}{P^{old}} \left(\frac{1+\overset{0.05}{i^{old}}}{1+\underset{0.054}{i^{new}}} \right)^{D^{mac}_{old}} \Rightarrow D^{mac}_{old} \doteq 3.851177$

Again: $\underset{3.851177}{D^{mac}_{old}} = (1+\underset{0.05}{i_{old}}) \boxed{D^{mod}_{old}}$ ←target $\Rightarrow D^{mod}_{old} \cong 3.667787$ Ⓐ//

Exam FM Question 157:

Loan 1000 one year from now and 1000 two years from now.

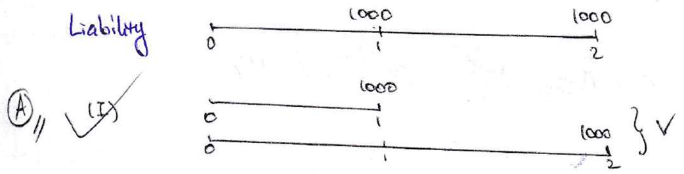
Exact cash-flow matching, which of the following strategy can be used?

I. Purchases a 1-year zero-coupon bond and a 2-year zero-coupon bond. Each with a face amount of 1000.

II. The company deposits 1859.41 into an account that currently earns an annual rate of 5% that is subject to change in one year.

III. The company purchases an asset that has the same duration as the liabilities and a larger convexity.

Exam FM Question 157



(II) Notice: the interest i 5% will change in one year, which does NOT garantee 1000 in t=2.

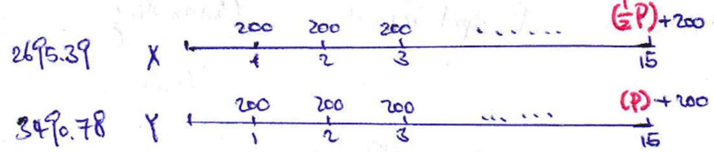
(III) Same duration & larger convexity. is NOT CF matching. more like Redington immunization.

Exam FM Question 158:

X and Y: two 15-year par value bond, each pay an annual coupon of 200 at the end of the year. The face value of X is one-half the face value of Y. At an annual yield of i, the price of X is 2695.39 and the price of Y is 3490.78.

Question: what is the coupon rate for X?

Exam FM Question158 Question asks (1) coupon rate ; (2) Bond X's "Face Vaule" = $\frac{1}{2}P$,



Solve: (eq1): $2695.39 = 200\cdot a_{\overline{15}|i} + (\frac{1}{2}P)v^{15} \xleftrightarrow{\times 2} \Rightarrow Pv^{15} = 2695.39\times 2 - 400\cdot a_{\overline{15}|i}$

(eq2): $3490.78 = 200\cdot a_{\overline{15}|i} + (P)v^{15}$ ← plug into

$\Rightarrow 100 a_{\overline{15}|i} = 1900 \Rightarrow \underline{\underline{a_{\overline{15}|i} = 9.5 \ \& \ i = 6.34\%}}$ plug into (eq1).

We have: $2695.39 = 200\times(9.5) + (\frac{1}{2}P)\times(\frac{1}{1.0634})^{15} \Rightarrow \frac{1}{2}P = 2000$

Thus: $\boxed{r_{\text{coupon rate}} = \frac{200}{\frac{1}{2}P}} = 10\%$ Ⓓ

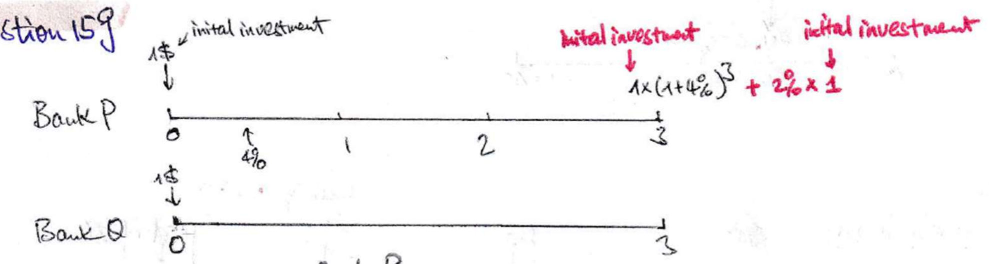
Exam FM Question 159:

Bank P offers a 3-year certificate of deposit with an annual rate of 4%. In addition, a bonus of 2% of the initial investment paid at the end of the third year.

Bank Q offers a 3-year certificate of deposit without any bonus.

Question: what is the annual rate that Bank Q would need to offer to give the same annual yield as the certificate from Bank P?

Exam FM Question 159



Solve: $AV_{t=3} = \overbrace{1\times(1+4\%)^3 + \underbrace{1\times 2\%}_{\text{2\% of the initial investment}}}^{\text{Bank P}} = \overbrace{(1+i)^3}^{\text{Bank Q}} \Rightarrow i \cong 4.61275\%$ Ⓓ

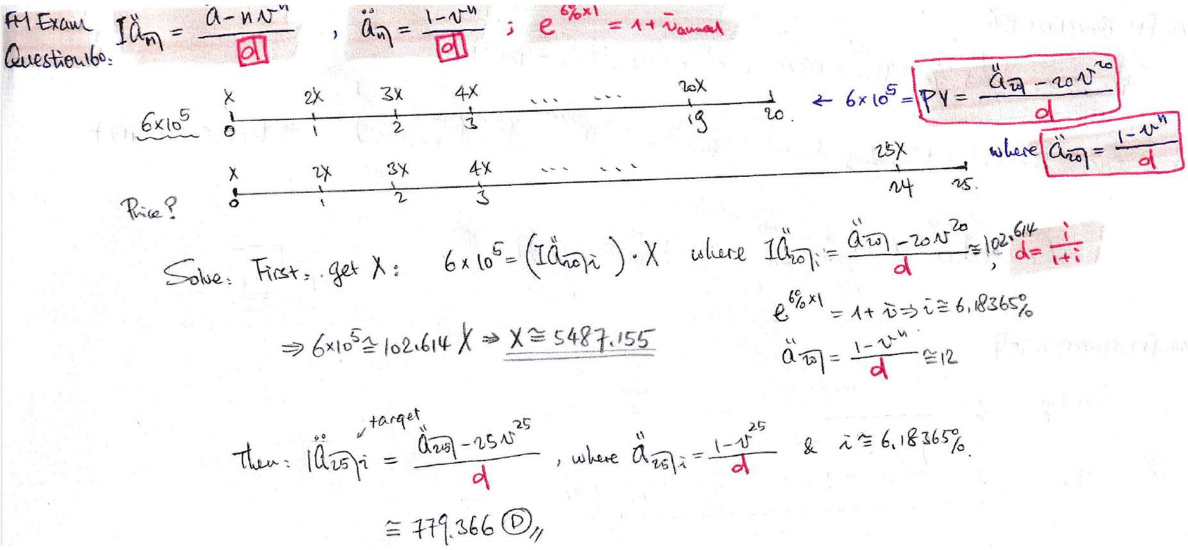
Bonus : (1) "+" NOT "×" : 2% of the "initial investment".

(2) With Bonus : yield will ↑. more than 4%.

Exam FM Question 160:

20-year arithmetically increasing annuity-due, price is 600,000, annual payments, the first payment is X, each payment thereafter is X more than the previous one. 25-year arithmetically increasing annuity-due provides annual payments, the first payment is X, each payment thereafter is X more than the previous one. A continuously compounded annual rate of 6%.

Question: what is the price of the 25-year annuity?



FM Exam Question160: $I\ddot{a}_{\overline{n}|} = \frac{\ddot{a} - nv^n}{\boxed{d}}$, $\ddot{a}_{\overline{n}|} = \frac{1-v^n}{\boxed{d}}$; $e^{6\%\times 1} = 1 + i_{annual}$

$\leftarrow 6\times 10^5 = \boxed{PV = \frac{\ddot{a}_{\overline{20}|} - 20v^{20}}{d}}$

where $\boxed{\ddot{a}_{\overline{20}|} = \frac{1-v^n}{d}}$

Solve: First, get X: $6\times 10^5 = (I\ddot{a}_{\overline{20}|i})\cdot X$ where $I\ddot{a}_{\overline{20}|i} = \frac{\ddot{a}_{\overline{20}|} - 20v^{20}}{d} \approx 102.614$, $d = \frac{i}{1+i}$

$e^{6\%\times 1} = 1 + i \Rightarrow i \cong 6.18365\%$

$\ddot{a}_{\overline{20}|} = \frac{1-v^n}{d} \cong 12$

$\Rightarrow 6\times 10^5 \cong 102.614X \Rightarrow \underline{X \cong 5487.155}$

Then: $\overset{target}{I\ddot{a}_{\overline{25}|i}} = \frac{\ddot{a}_{\overline{25}|} - 25v^{25}}{d}$, where $\ddot{a}_{\overline{25}|i} = \frac{1-v^{25}}{d}$ & $i \cong 6.18365\%$.

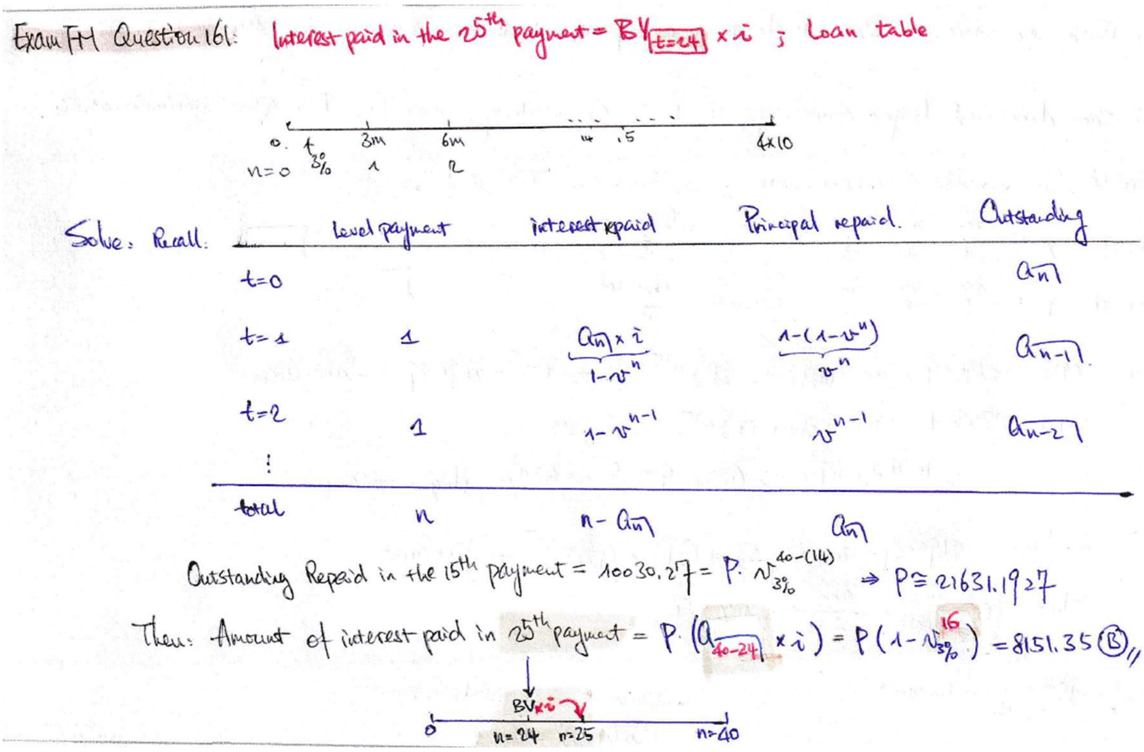
$\cong 779.366$ Ⓓ//

Exam FM Question 161:

A 10-year loan with level end-of-quarter payments. An annual rate of 12% convertible quarterly.

The amount of principal repaid in the 15th payment is 10,030.27.

Question: what is the amount of interest paid in the 25th payment?



Exam FM Question161: Interest paid in the 25th payment = $BV_{\boxed{t=24}} \times i$; Loan table

Solve: Recall.

	Level payment	interest repaid	Principal repaid.	Outstanding
t=0				$a_{\overline{n}|}$
t=1	1	$\underbrace{a_{\overline{n}|}\times i}_{1-v^n}$	$\underbrace{1-(1-v^n)}_{v^n}$	$a_{\overline{n-1}|}$.
t=2	1	$1-v^{n-1}$	v^{n-1}	$a_{\overline{n-2}|}$
⋮				
total	n	$n - a_{\overline{n}|}$	$a_{\overline{n}|}$	

Outstanding Repaid in the 15th payment = $10030.27 = P\cdot v_{3\%}^{40-(14)}$ $\Rightarrow P \cong 21631.1927$

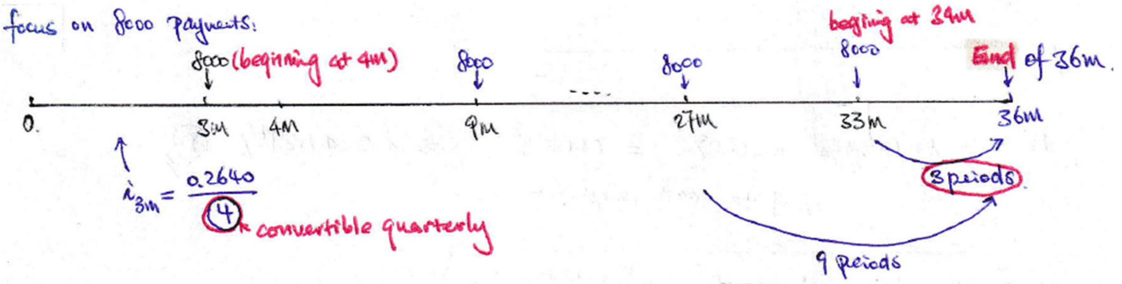
Then: Amount of interest paid in 25th payment $= P\cdot(a_{\overline{40-24}|}\times i) = P(1 - v_{3\%}^{16}) = 8151.35$ Ⓑ//

Exam FM Question 162:

Trish has a loan of 4000 at the beginning of month 1. Every month thereafter, she made a payment of X in the middle of the month. At the beginning of month 4, and every 6 months thereafter, she borrowed an additional 800. Trish's loan balance is 4000 again at the end of month 36. Annual rate for the loan is 26.4%, convertible quarterly.Question: which of the following is an equation of value that can be used to solve for X?

Exam FM Question 162:

Solve. focus on 8000 payments:



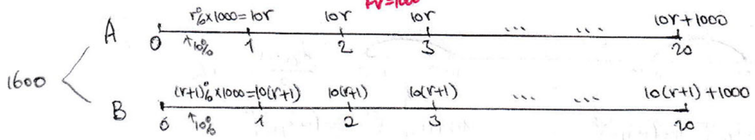
$i_{3m} = \frac{0.2640}{④}$ ← convertible quarterly

Check choices: choose the one which meets $\left\langle \begin{array}{l} (1)\ \frac{0.2640}{④} \leftarrow \\ (2)\ n=1 \leftrightarrow \frac{800}{(1+i_{3m})^{1}} \leftarrow \end{array} \right.$ ⇒ (E)//

Exam FM Question 163:

Two 20-year bonds A and B, each with annual coupons, an annual rate of 10%, and a face amount of 1000. The total combined price of two bonds is 1600. Bond B's annual coupon rate is equal to Bond A's annual coupon rate plus 1%. Question: what is the annual coupon rate of Bond A?

Exam FM. Question 163 Bond: "last period": $\underbrace{1000v^{20}}_{FV=1000}$ [NOT $(10r+1000)v^{20}$]



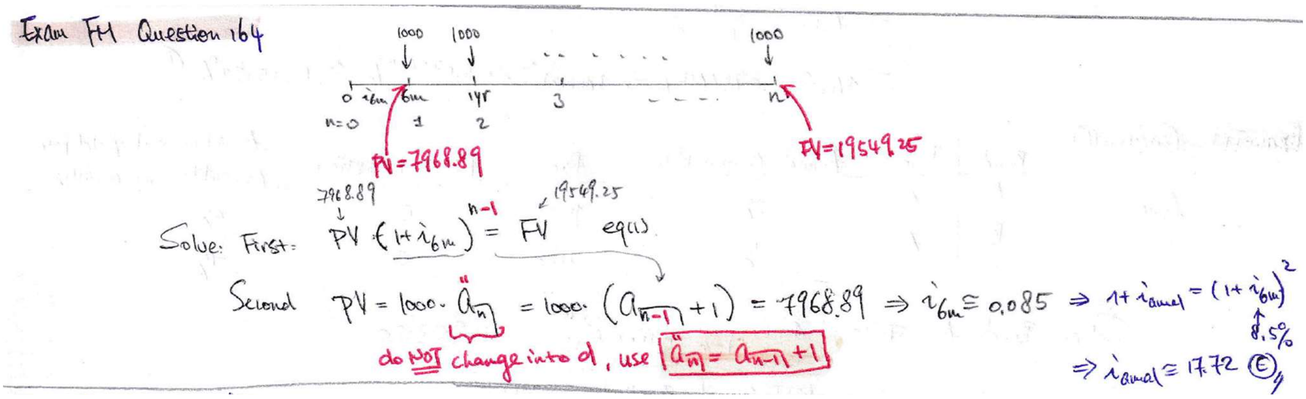
Solve: $1600 = \left(10r \cdot \underbrace{a_{\overline{20}|10\%}}_{8.51356} + 1000\, v_{10\%}^{20} \right) + \left(10(r+1) \cdot a_{\overline{20}|10\%} + 1000\, \underset{\substack{\uparrow \\ 0.148644}}{v_{10\%}^{20}} \right)$

$\Rightarrow r \cong 7.15\%$ Ⓑ//

Exam FM Question 164:

Annuity provides level payments of 1000 every six months. An annual rate of i, Future value at the time of the last payment is 19,549.25 and the present value of at the time of the first payment is 7,968.89.

Question: what is i?



Exam FM Question 164

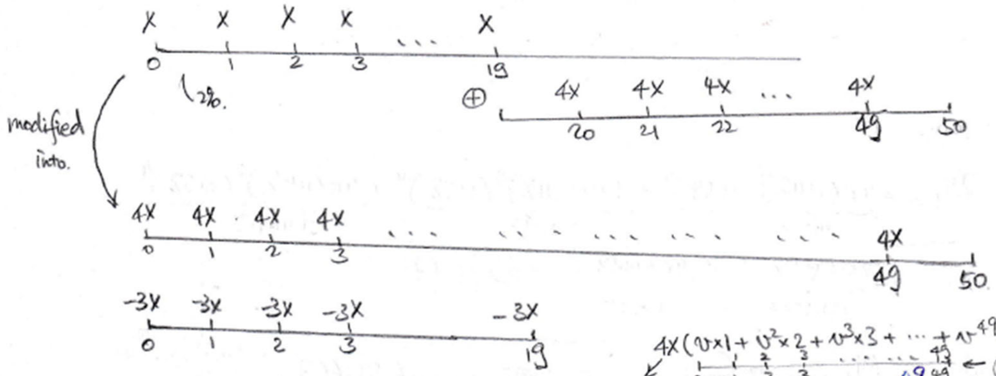
Solve: First: $\overset{7968.89}{PV}\left(1+i_{6m}\right)^{n-1} = \overset{19549.25}{FV}$ eq(1)

Second $PV = 1000\cdot \underbrace{\ddot{a}_{\overline{n}|}} = 1000\cdot\left(a_{\overline{n-1}|}+1\right) = 7968.89 \Rightarrow i_{6m} \cong 0.085 \Rightarrow 1+i_{annual} = \left(1+\underset{8.5\%}{i_{6m}}\right)^2$

do NOT change into d, use $\boxed{\ddot{a}_{\overline{n}|} = a_{\overline{n-1}|}+1}$

$\Rightarrow i_{annual} \cong 17.72$ Ⓔ

Exam FM Question 165



$$4X(v\times1+v^2\times2+v^3\times3+\cdots+v^{49}\times49) \leftarrow \boxed{Ia_{\overline{49}|2\%}}$$

Solve:
$$D^{mac} = \frac{4X\cdot0 + (4X v\times1) + (4X v^2\times2) + (4X v^3\times3) + \cdots\cdots + (4X v^{49}\times 49)}{\underbrace{(4X+4Xv+4Xv^2+\cdots+4Xv^{49})}_{4X\underbrace{(1+v+v^2+\cdots+v^{49})}_{\ddot{a}_{\overline{50}|2\%}}} + \underbrace{(-3X-3Xv-3Xv^2-\cdots-3Xv^{19})}_{-3X\underbrace{(1+v+v^2+\cdots+v^{19})}_{\ddot{a}_{\overline{20}|2\%}}}}$$

$$-3X(v\times1+v^2\times2+\cdots+v^{19}\times19)$$

$$+\ \frac{-3X\cdot0 - (3Xv\times1) - (3Xv^2\times2) - (3Xv^3\times3) - \cdots - (3Xv^{19}\times19) \leftarrow \boxed{Ia_{\overline{19}|2\%}}}{4X\ddot{a}_{\overline{50}|2\%} - 3X\ddot{a}_{\overline{20}|2\%}}$$

where.
$$\left.\begin{aligned} 4X\ddot{a}_{\overline{50}|2\%} &= 4X\cdot\frac{1-v_{2\%}^{50}}{d_{2\%}} = 128.208X \\ -3X\ddot{a}_{\overline{20}|2\%} &= -3X\cdot\frac{1-v_{2\%}^{20}}{d_{2\%}} = -50.035X \end{aligned}\right\} \Rightarrow 4X\ddot{a}_{\overline{50}|2\%} - 3X\ddot{a}_{\overline{20}|2\%} = 78.123X$$

$$\Rightarrow D^{mac} = \frac{4X\cdot\overbrace{Ia_{\overline{49}|2\%}}^{Ia_{\overline{49}|2\%}} - 3X\cdot\overbrace{Ia_{\overline{19}|2\%}}^{Ia_{\overline{19}|2\%}}}{\underbrace{4X\ddot{a}_{\overline{50}|2\%} - 3X\ddot{a}_{\overline{20}|2\%}}_{78.123X}} = \frac{\cancel{4X}\times655.2 - \cancel{3X}\times147.49}{78.123\cancel{X}} = \frac{2620.8 - 442.47}{78.123} \cong 27.88\ \textcircled{B}$$

where:
$$Ia_{\overline{49}|2\%} = \frac{\overset{31.673}{\ddot{a}_{\overline{49}|2\%}} - 49v_{2\%}^{49}}{2\%} = \frac{31.673 - 18.569}{2\%} = 655.2$$

$$Ia_{\overline{19}|2\%} = \frac{\overset{15.992}{\ddot{a}_{\overline{19}|2\%}} - 19v_{2\%}^{19}}{2\%} = \frac{15.992 - 13.042}{2\%} = 147.49$$

$$\ddot{a}_{\overline{49}|2\%} = \frac{1-v_{2\%}^{49}}{d_{2\%}} = 31.673$$

$$\ddot{a}_{\overline{19}|2\%} = \frac{1-v_{2\%}^{19}}{d_{2\%}} = 15.992.$$

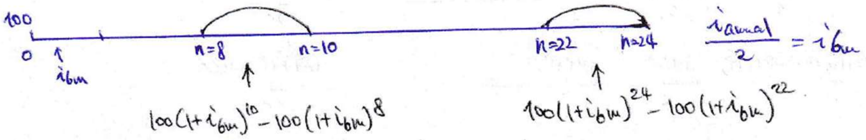
Exam FM Question 166:

Deposits 100 into a bank account at time 0. An annual rate of I compounded semi-annually.

The total amount of interest credited in the 12th year is twice the amount of interest credited in the 5th year

Question: what is i?

Exam FM Question 166



Solve: $100(1+i_{6m})^{24}-100(1+i_{6m})^{22} = 2\times\left[100(1+i_{6m})^{10}-100(1+i_{6m})^{8}\right]$

$\Rightarrow \cancel{100}(1+i_{6m})^{\cancel{22}\,14}\cancel{\left[(1+i_{6m})^2-1\right]} = 2\times\cancel{100}(1+i_{6m})^{\cancel{8}}\times\cancel{\left[(1+i_{6m})^2-1\right]}$

$\Rightarrow (1+i_{6m})^{14}=2$

$\Rightarrow i_{6m}\doteq 5.075664\% \Rightarrow i_{annual}\doteq 2\times 5.075664\% \doteq 10.15132\%$ Ⓐ

Exam FM Question 167:

Exam FM Question 167

Give:

Bond	Price	Annual Coupon Rate	Par	Year to redeption	Annual nominal yield rate convertible semi-anually
A	X	8%	1000	5	6%
B	X	y	1000	5	7%

Solve: Bond A: Price $= \underbrace{40\,a_{\overline{10}|3\%} + 1000\,v_{3\%}^{10}}_{PMT=40;\ I/Y=3;\ n=10;\ FV=1000} = 1085.3020$

$\Rightarrow$ Bond B: Price $= 1085.3020 = \underbrace{P\cdot a_{\overline{10}|3.5\%} + 1000\,v_{3.5\%}^{10}}_{FV=-1085.3020,\ n=10,\ I/Y=3.5,\ FV=1000} \Rightarrow P=45.2568$

Thus: $\boxed{\frac{r}{2}} = \frac{P}{1000} = \frac{45.2568}{1000} = 4.5257\% \Rightarrow r \simeq 9.0514\%$ Ⓓ

Exam FM Question 168:

A 15-year loan at an annual rate of i with payments of 50 at the end of each year. Now pay off the loan early by making extra payments of 30 with each of the 6th through 10th regularly scheduled payments. As a result, the loan will be paid off at the end of 10 years.

Question: which of the following equations is correct?

(A) $50a_{\overline{15}|} = 50a_{\overline{10}|} + 30a_{\overline{5}|}$

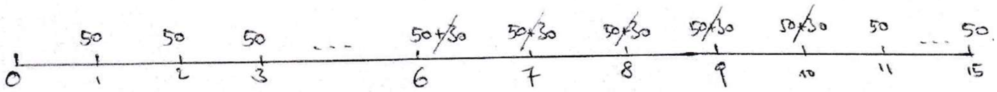
(B) $50s_{\overline{15}|} = 50s_{\overline{10}|} + 30s_{\overline{5}|}$

(C) $50v^5 s_{\overline{15}|} = 50s_{\overline{10}|} + 30s_{\overline{5}|}$

(D) $50s_{\overline{15}|} = 50s_{\overline{10}|} + 30v^5 s_{\overline{5}|}$

(E) $50s_{\overline{15}|} = 50s_{\overline{10}|} + 30(1+i)^5 s_{\overline{5}|}$

Exam FM Question 168



Solve: First: observe all the options: Left hand-side is "Forward". 15-period:

So: $50 s_{\overline{15}|}$: move to t=15

Now: Let's look at the right hand-side: $50 s_{\overline{10}|}$ & $30 s_{\overline{5}|}$: move to t=10

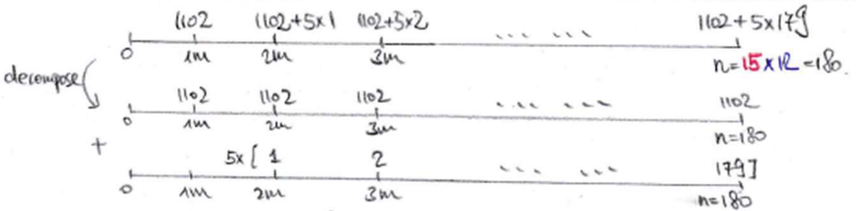
Thus: $\underline{\underline{50 s_{\overline{15}|}}}$ should "$\underset{v^5}{\underline{\text{backward 5 periods}}}$" $\Rightarrow$ Ⓒ //

Exam FM Question 169:

A loan needs to be repaid over 20 years. The first payment is 1102 at the end of the 1st month. Each subsequent monthly payment is 5 more than the previous one.

Question: what is the accumulated value at the end of 15 years using an annual rate of 6.5%?

Exam FM Question 169 $I\ddot{S} = \frac{\ddot{S}-n}{i \to d}$, $\ddot{S} = \frac{(1+i)^n-1}{\boxed{d}}$, in convertible case. Do $\underline{\text{NOT}}$ use PV→ FV



$(1+i_{1m})^{12} = 1.065$

$\Rightarrow i_{1m} = 0.52617\%$

Solve: $\cancel{AV = \underbrace{PV}\times(1+\underset{\underset{0.52617\%}{\uparrow}}{i_m})^{180}}$ "Do $\underline{\underline{\text{NOT}}}$ use "PV $\xrightarrow{\text{then}}$ FV" in the "convertible case"

$$AV = 1102\cdot\underbrace{S_{\overline{180}|0.52617\%}}_{\underbrace{\frac{(1+0.52617\%)^{180}-1}{0.52617\%}}_{298.733}} + 5\cdot\underbrace{I\ddot{S}_{\overline{179}|0.52617\%}}_{\underbrace{\frac{\ddot{S}_{\overline{179}|0.52617\%}-179}{0.52617\%}}_{22565.52845}}$$

where $\ddot{S}_{\overline{179}|0.52617\%} = \frac{(1+0.52617\%)^{179}-1}{\boxed{d}}$

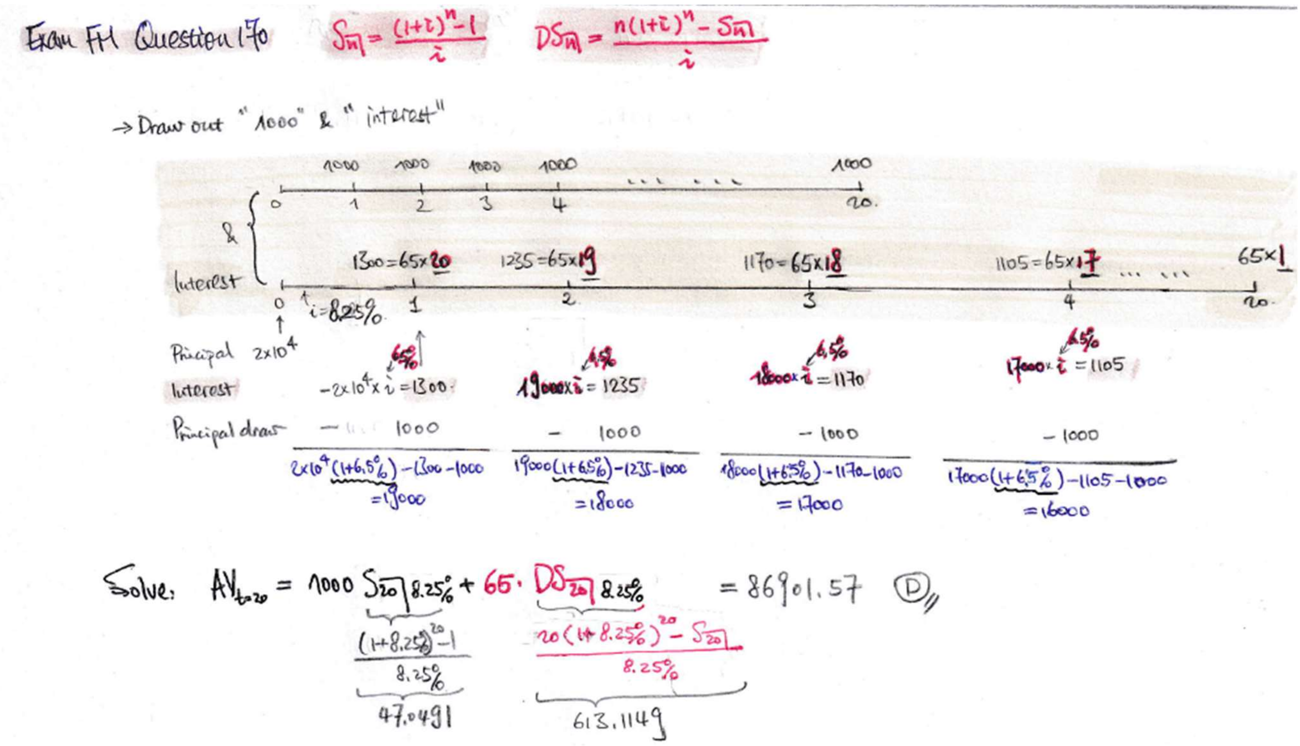
$\cong 297.73304$

$= 329203.8112 + 5\times 22565.52845 \cong 442031.4535$

Exam FM Question 170:

A balance of 20,000, earns an annual rate of 6.5%. At the end of each year, the interest earned and an additional 1000 is withdrawn. The annual withdrawals of interest and principal are deposited into Fund K, which earns an annual rate of 8.25%. At the end of the 20th year, the accumulated value of Fund K is x.

Question: what is x?



Exam FM Question 170 $\quad s_{\overline{n}|} = \frac{(1+i)^n - 1}{i} \quad Ds_{\overline{n}|} = \frac{n(1+i)^n - s_{\overline{n}|}}{i}$

→ Draw out "1000" & "interest"

Principal 2×10^4

Interest $-2\times10^4 \times i = 1300$ · 6.5% · $19000 \times i = 1235$ · 6.5% · $18000 \times i = 1170$ · 6.5% · $17000 \times i = 1105$ · 6.5%

Principal draw -1000 · -1000 · -1000 · -1000

$2\times10^4(1+6.5\%) - 1300 - 1000 = 19000$

$19000(1+6.5\%) - 1235 - 1000 = 18000$

$18000(1+6.5\%) - 1170 - 1000 = 17000$

$17000(1+6.5\%) - 1105 - 1000 = 16000$

Solve: $AV_{t=20} = 1000\, \underbrace{s_{\overline{20}|8.25\%}}_{\underbrace{\frac{(1+8.25\%)^{20}-1}{8.25\%}}_{47.0491}} + 65 \cdot \underbrace{Ds_{\overline{20}|8.25\%}}_{\underbrace{\frac{20(1+8.25\%)^{20} - s_{\overline{20}|}}{8.25\%}}_{613.1149}} = 86901.57$ Ⓓ