SOA and CAS: Exam FM[1]
Written Solutions: 1-84

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This document only provides written solutions to official example problems 1-84. For official sample questions, check out the official websites of Society of Actuaries and the Casualty Actuarial Society.

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Exam FM Question 1

① Bruce: deposits 100 into an account, which has "norminal rate of interest 4%" convertible semi-annually

② Peter: deposits 100 into an account, which has "force of inerest δ"

③ After 7.25 year, AV are the same

Question: δ

Solve.

$$AV_{Bruce} = 100\left[\left(1+\frac{4\%}{2}\right)^2\right]^{7.25} \quad (1.02^2 = 1.0404)$$

$$AV_{Peter} = 100\cdot e^{\int_0^{t=7.25}\delta\, dt} = 100\, e^{7.25\delta}$$

$$\Rightarrow 1.0404 = e^{\delta} \Rightarrow \ln(1.0404) = \delta = 0.0396 \quad \text{(C)}$$

Exam FM Question 2

① Kathryn: deposits 100 into an account, at the begining of each 4-year period, for 40 years

this account credits interest at i

② AV at the end of 40 years is X. which is 5 times the AV at the end of 20 years

Question: X

100	100	100	100	100	100	100	100	100	100	
0	4	8	12	16	20	24	28	32	36	40

Solve.

$$AV_{40} = 100\left[(1+i)^{40} + (1+i)^{36} + (1+i)^{32} + \cdots + (1+i)^4\right] = 100(1+i)^4\left[1+\cdots+(1+i)^{36}\right] = 100(1+i)^4\frac{1-(1+i)^{36}(1+i)^4}{1-(1+i)^4}$$

$$\left(\frac{a_1 - a_n q}{1-q}\right)$$

$$AV_{20} = 100\left[(1+i)^{20} + (1+i)^{16} + (1+i)^{12} + \cdots + (1+i)^4\right] = 100(1+i)^4\left[1+\cdots+(1+i)^{16}\right]$$

$$= 100(1+i)^4\frac{1-(1+i)^{16}(1+i)^4}{1-(1+i)^4}$$

$\because AV_{40} = 5\cdot AV_{20}$

$$\therefore 100(1+i)^4\frac{1-(1+i)^{40}}{1-(1+i)^4} = 5\cdot 100(1+i)^4\frac{1-(1+i)^{20}}{1-(1+i)^4}$$

This gives: $1-(1+i)^{40} = 5 - 5(1+i)^{20}$

Assume: $(1+i)^{20} = X$

we have: $X^2 - 5X + 4 = 0$

$(X-4)(X-1) = 0$

$\Longrightarrow$ $(1+i)^{20} = 4$ (delete 1, since $i>0$)

$\Rightarrow$ $(1+i)^4 = 4^{\frac{1}{5}} = 1.3195$

Then: $\text{Question} = AV_{40} = 100(1+i)^4\cdot\dfrac{1-(1+i)^{40}}{1-(1+i)^4} \cong 6194.8357$ (E)

($1-(1+i)^{40} = -15$, $1-(1+i)^4 = -0.3195$)

Exam FM Question 3

① Eric: deposits 100 into a saving account, at time 0, nominal rate i, compounded semi-annually

② Mike: deposits 200 into a saving account, at time 0, simple rate at i

③ Eric and Mike, earn the same amount of interest, during the last 6 months of the 8th years.

Question: i

Solve:
$$Interest_{Eric} = 100\left[\left(1+\frac{i}{2}\right)^2\right]^8 - 100\left[\left(1+\frac{i}{2}\right)^2\right]^7\left(1+\frac{i}{2}\right) = 100\left(1+\frac{i}{2}\right)^{15}\left(1+\frac{i}{2}-1\right) = 100\left(1+\frac{i}{2}\right)^{15}\cdot\frac{i}{2}$$

$$Interest_{Mike} = 200\,(1+i\cdot \underset{8}{t}) - 200\,(1+i\cdot \underset{7.5}{t}) = 200\cdot 0.5i$$

$\because$ $Interest_{Eric} = Interest_{Mike}$

$$\therefore 100\left(1+\frac{i}{2}\right)^{15}\cdot\frac{i}{2} = 100i \Rightarrow \left(1+\frac{i}{2}\right)^{15} = 2 \Rightarrow i = 2\left(2^{\frac{1}{15}}-1\right) \cong 9.4588\% \quad Ⓒ$$

Exam FM Question 4

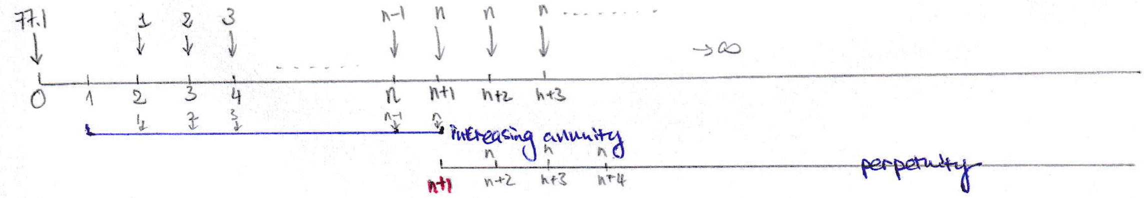
① A perpetuity costs 77.1, makes end-of-year payments

② Perpetuity pays 1 at the end of year 2, 2 at the end of year 3, ... n at the end of year $(n+1)$.

③ After year $(n+1)$, payments remain constant at n

④ annual effective rate 10.5%.

Question: n



Solve: $77.1 = v\cdot I\ddot{a}_{\overline{n}|} + v^{n+1}\cdot n\cdot \frac{1}{i}$ ($\frac{1}{i}$ ← $a_{\overline{\infty}|}$)

$I\ddot{a}_{\overline{n}|} \to \dfrac{\ddot{a}_{\overline{n}|} - nv^n}{i}$

$$= \frac{v\ddot{a}_{\overline{n}|} - nv^{n+1}}{i} + \frac{nv^{n+1}}{i}$$

$$I\ddot{a}_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{i}$$

$$= \frac{v\ddot{a}_{\overline{n}|}}{i} \quad (\text{Recall: } v\ddot{a}_{\overline{n}|} = a_{\overline{n}|})$$

$$= \frac{a_{\overline{n}|}}{i} \quad \left(\text{Recall: } a_{\overline{n}|} = \frac{1-v^n}{i}\right)$$

$$= \frac{1-v^n}{i^2} \quad \left(i = 10.5\%, \Rightarrow v = \frac{1}{1+i} = \frac{1}{1.105}\ ;\ i^2 = 0.105^2\right)$$

$$\Leftrightarrow 0.1499725 = v^n \Leftrightarrow n\cdot \underset{\ln\frac{1}{1.105}}{\ln v} = \ln(0.1499725) \Leftrightarrow n \cong \frac{1.8973}{0.099845} = 19$$

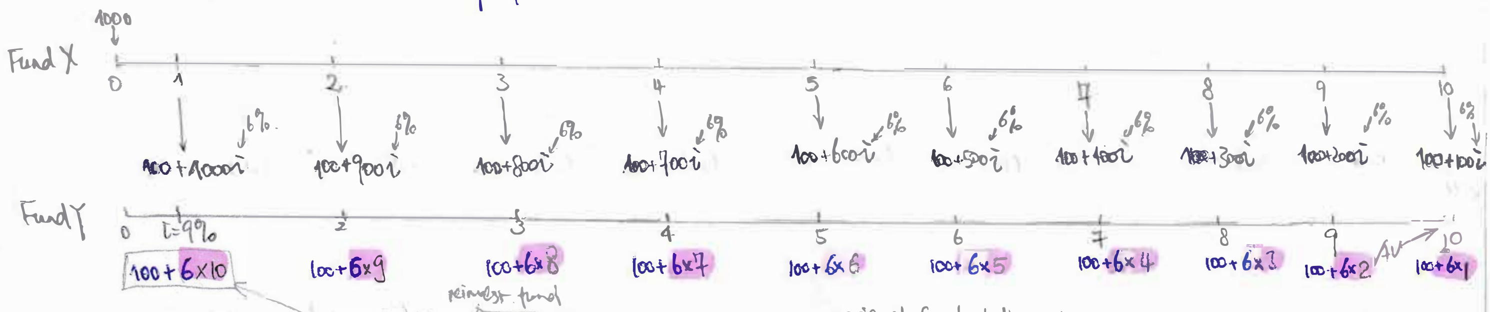
Exam FM Question 5

① 1000 deposited into Fund X, effective rate 6%, at the end of each year, the interest earned plus an additional 100 is withdrawn from the fund.

② At the end of the 10^{th} year, fund is depleted.

③ The annual withdrawals of interest & principal are deposited into Fund Y, earns an rate 9%.

Question: AV of Fund Y at the end of year 10



Solve: $100 \cdot s_{\overline{10}|0.09} + 6 \cdot (Ds)_{\overline{10}|0.09} + 0$

$= 100 \cdot \frac{(1+i)^{n} - 1}{i} + 6 \times \frac{n(1+i)^n - s_{\overline{n}|}}{i}$

(15.192930; $i=0.09$, $n=10$; 15.19293)

$$DS = \frac{n(1+i)^n - s_{\overline{n}|}}{i}$$

$= 1519.2930 + 565.38 \approx 2084.67$ Ⓒ

Exam FM Question 6

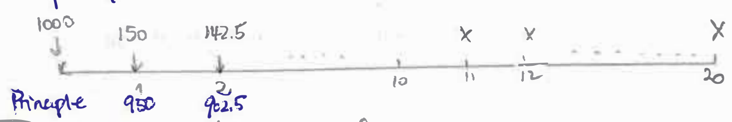
① 20-year loan of 1000 is repaid with payments at the end of each year

② Each of the first ten payments = 150% amount of interest due

③ Each of the last ten payments = X

④ effective rate of 10%.

Question: X



Solve: $t=1$ ∵ interest due $= 1000 \times 10\% = 100$

∴ $150\% \times$ interest due $= 150$

Thus: Payment Amt at $t=1$ is 150, which
- 100: use to pay interest
- 50: use to pay principle; Principle Outstanding $= 1000 - 50 = 950$

$t=2$ ∵ interest due $= 950 \times 10\% = 95$

∴ $150\% \times$ interest due $= 142.5$ (7.5)

Thus: Payment Amt at $t=2$ is 142.5, which
- 95: use to pay interest
- 47.5: use to pay principle; Principle Outstanding $= 950 - 47.5 = 902.5$

$t=3$ ∵ interest due $= 902.5 \times 10\% = 90.25$ (7.125)

∴ $150\% \times$ interest due $= 135.375$

Thus: Payment Amt at $t=3$ is 135.375, which
- 90.25: use to pay interest
- 45.125 use to pay principle; Principle Outstanding $= 902.5 - 45.125 = 857.375$

⋮

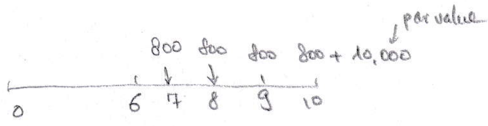
$$PV = (150v + 142.5v^2 + 135.375v^3 + \cdots) + (v^{10} \cdot a_{\overline{10}|10\%})$$

Exam FM Question 7

(Same as Question 6. Principle Amt Outstanding = PV of all Future CFs

Give: A 10,000 par value, 10-year bond, 8% annual coupons, is bought at a premium to yield an annual effective rate of 6%

Question: Calculate the interest portion of the 7th coupon



Solve: "Principle Amount Outstanding" = PV of all future cash flows

∴ $\text{"Principle Amount Outstanding"}_{t=6} = 800\, a_{\overline{4}|6\%} + 10{,}000\, v^4 \quad (n=4,\ i=6\%)$, where $a_{\overline{n}|} = \frac{1-v^n}{i}$

$= 800 \times 3.4651 + 10{,}000 \times 0.79209$

$\cong 10692.98$

Thus: "Interest Portion" of 7th coupon = "Principle Amt Outstanding" × $i_{6\%}$, where "Principle Amt Outstanding" = 10692.98

$\cong 641.58$ Ⓑ //

Exam FM Question 8

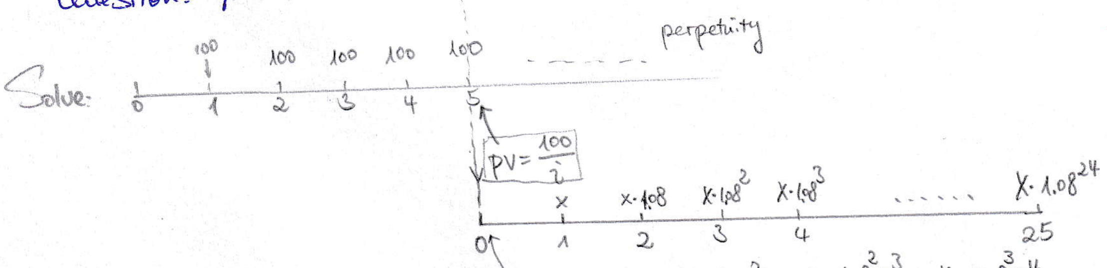
Give: ① A perpetuity-immediate pays 100 per year.

② Immediately after the 5th payment, the perpetuity is exchanged for a 25-year annuity-immediate that pay X at the end of the first year. Each subsequent annual payment will be 8% greater than the preceding payment

③ The annual effective rate of interest is 8%

Question: X ?

Solve:



$PV = X\cdot v + X\cdot 1.08\, v^2 + X\cdot 1.08^2 v^3 + X\cdot 1.08^3 v^4 + \cdots + X\cdot 1.08^{24} v^{25}$

$= X\cdot v\left(1 + 1.08\, v + 1.08^2 v^2 + 1.08^3 v^3 + \cdots + 1.08^{24} v^{24}\right)$

(#1, #2, …, #25 terms; $v = \frac{1}{1.08}$, $v^2 = \left(\frac{1}{1.08}\right)^2$, $v^3 = \left(\frac{1}{1.08}\right)^3$, $v^{24} = \left(\frac{1}{1.08}\right)^{24}$)

$\boxed{PV = X\cdot v\cdot 25}$

Thus: $\frac{100}{i} = X\cdot v\cdot 25$ (perpetuity = PV of CFs) where $v = \frac{1}{1.08}$, $i = 8\%$

$\Rightarrow X = 54$ Ⓐ //

Exam FM Question 9

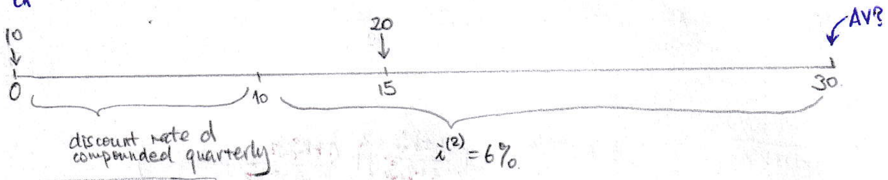
Give: ① Jeff deposits 10 into a fund, and deposits 20 fifteen years later.

② For the first 10 year, give "discount rate d" compounded quarterly; Afterwards, 6% compounded semi-annually

③ AV at 30-year is 100

Question: d

Solve:



Recall. discount rate apply at AV

$$AV \cdot \left[\left(1-\frac{d^{(m)}}{m}\right)^{m}\right]^{n} = PV$$ (1-year)

$$\Leftrightarrow AV = PV \cdot \left[\left(1-\frac{d^{(m)}}{m}\right)^{-m}\right]^{n}$$ (1-year)

$$10 \cdot \left[\left(1-\frac{d^{(4)}}{4}\right)^{-4}\right]^{10} \underbrace{\left[\left(1+\frac{6\%}{2}\right)^{2}\right]^{20}}_{3.2620} + 20\underbrace{\left[\left(1+\frac{6\%}{2}\right)^{2}\right]^{15}}_{48.5452} = 100$$

$$\Leftrightarrow \left[\left(1-\frac{d^{(4)}}{4}\right)^{-4}\right]^{10} = 1.5774$$

$$\Leftrightarrow -40 \cdot \ln\left(1-\frac{d^{(4)}}{4}\right) = \ln(1.5774)$$

$$\Leftrightarrow \ln\left(1-\frac{d^{(4)}}{4}\right) = -0.0114$$

$$\Leftrightarrow 1-\frac{d^{(4)}}{4} = e^{-0.0114} = 0.9887$$

$$\Leftrightarrow d^{(4)} = 0.0453$$ (C)

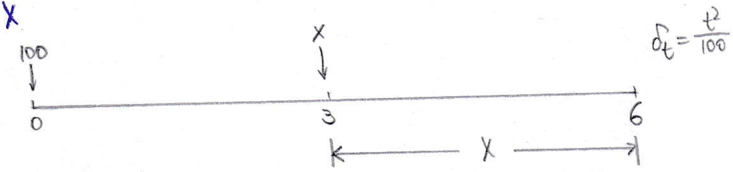
Exam FM Question 10

Give: ① Ernie makes 100 at time 0, X at time 3

② force of interest: $\delta_t = \frac{t^2}{100}$ $(t>0)$

③ The Amount of interest: earned from time 3 to time 6 is X

Question: X



NOTE: Interest earned by 100 $= 100 \cdot e^{\int_0^6 \frac{t^2}{100}dt} - 100 \cdot e^{\int_0^3 \frac{t^2}{100}dt}$

Write "separately": Do NOT write: $\int_3^6 (X)$

Solve: ☆ "Total Value" at t=6 − "Total Value" at t=3

$$100 \cdot e^{\int_0^6 \delta_t dt} + X \cdot e^{\int_3^6 \delta_t dt} \quad - \quad 100 \cdot e^{\int_0^3 \delta_t dt} + X$$

$$\because e^{\int_0^6 \frac{t^2}{100}dt} = e^{\frac{1}{100}\cdot\frac{1}{3}\int_0^6 dt^3} = e^{\frac{1}{300}(6^3-0^3)} = e^{0.72}$$

$$e^{\int_3^6 \frac{t^2}{100}dt} = e^{\frac{1}{100}\cdot\frac{1}{3}\int_3^6 dt^3} = e^{\frac{1}{300}(6^3-3^3)} = e^{0.63}$$

$$e^{\int_0^3 \frac{t^2}{100}dt} = e^{\frac{1}{100}\cdot\frac{1}{3}\int_0^3 dt^3} = e^{\frac{1}{300}(3^3-0)} = e^{0.09}$$

$\therefore$ Question. X: $\boxed{100e^{0.72} + X \cdot e^{0.63} - 100 \cdot e^{0.09} - X = X}$ $\Rightarrow 96.02589 = 0.1223894X \Rightarrow X \doteq 784.59$ (E)

Exam FM Question 11

Give: ① perpetuity-immediate (end of period) with varying annual payments

② First 5 years, payment constant = 10 ; Beginning in year 6, payments start to increase

③ For year 6 and all future years, current year's payment is k% larger than the previous year

④ annual effective rate 9.2% ; PV = 167.5 ; k < 9.2

Question: k

PV = 167.5 (k<9.2)

Timeline: 0, 1, 2, …, 5, 6, 7, 8, … → ∞ ; payments 10, 10, …, 10, $10(1+k\%)$, $10(1+k\%)^2$, $10(1+k\%)^3$, … ; $i = 9.2\%$

$PV_1 = 10\, a_{\overline{5}|} \; i = 9.2\%$.

$PV_2 = v^5 \cdot 10(1+k\%) \dfrac{1}{1-(1+k\%)}$

$10\left(\dfrac{1+k\%}{1+9.2\%} + \left(\dfrac{1+k\%}{1+9.2\%}\right)^2 + \dfrac{(1+k\%)^3}{(1+9.2\%)^3} + \cdots\right)$

$10 \cdot \dfrac{1+k\%}{1+9.2\%} \cdot \dfrac{1}{1-\frac{(1+k\%)}{1+9.2\%}}$

$PV = 167.5 = 10 \cdot a_{\overline{5}|9.2\%} + v^5 \cdot 10 \cdot \dfrac{1+k\%}{9.2\% - k\%}$ where $a_{\overline{5}|9.2\%} = \dfrac{1-v^5}{i} \cong 3.86955$; $v^5 = \left(\dfrac{1}{1+i}\right)^5 = 0.644$

$= 10 \times 3.86955 + 0.644 \cdot 10 \cdot \dfrac{1+k\%}{9.2\% - k\%}$

$\Rightarrow \dfrac{1+k\%}{9.2\% - k\%} = 20 \Rightarrow X = 0.04 \Rightarrow k\% = 4\% \Rightarrow k = 4$ Ⓐ//

Exam FM Question 12

Give: A 10-year loan of 2000 to be repaid with payments at the end of each year.

(i) Equal annual payments at 8.07% ← level payment

(ii) Installment of 200 each year plus interest on the unpaid balance at i ← similar: level of Principal

The "Sum of payment Amount" under (i) = (ii) are the same,

Question: i.

(i) level payment : payment Amt = "level payment" × 10 (10 level payments), where "level payment" $= \dfrac{2000}{a_{\overline{10}|8.07\%}}$

Timeline: 0, 1, 2, …, 10

(ii) Payment Amt = 200 × 10 (pay 200, 10 time) + $(2000i + 1800i + 1600i + \cdots + 200i)$ (interest Amt)

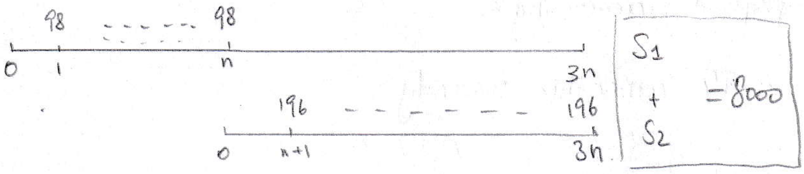
$= i \cdot (200 + \cdots + 1600 + 1800 + 2000)$ ← $\dfrac{n(a_1 + a_n)}{2}$

Solve: $\dfrac{2000}{a_{\overline{10}|8.07\%}} \times 10 = 200 \times 10 + i \cdot \dfrac{10}{2} \cdot (200 + 2000)$, with $a_{\overline{10}|8.07\%} = 6.6889$

$i = 0.09$ Ⓑ//

Exam FM Question 14

Given: (1)



(2) $(1+i)^n = 2$

Question: i

Solve: Method 1 "Annuities with Block Payments"

$98 s_{\overline{3n}|} + 98 s_{\overline{2n}|} = 8000$ where: $s_{\overline{3n}|} = \frac{[\overbrace{(1+i)^n}^{2}]^3 - 1}{i}$

Start from the furthest.

$s_{\overline{2n}|} = \frac{[\overbrace{(1+i)^n}^{2}]^2 - 1}{i}$

$\Rightarrow i = 12.25\%$ (C)

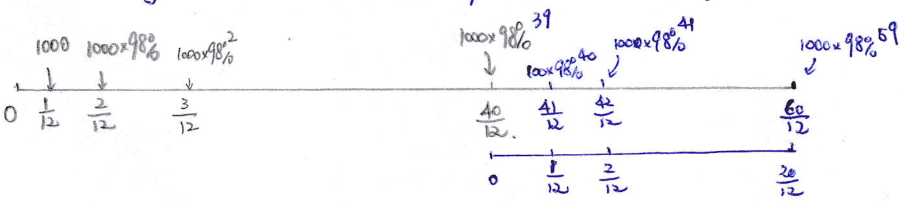
Exam FM Question 13

Given: ① A loan amortized over 5-year, annual interested 9% compounded monthly

② First payment 1000, paid 1 month after the date of loan

③ Each monthly payment will be 2% lower than the prior payment

Question: "Outstanding loan Balance" right after the 40th payment.



Solve: $\text{Outstanding} = 1000 \times 98\%^{40} v + 1000 \times 98\%^{41} v^2 + \cdots + 1000 \times 98\%^{59} v^{20}$

$= 1000 \times 98\%^{40} \cdot v \left(1 + 0.98v + 0.98^2 v^2 + \cdots + 0.98^{19} v^{19}\right)$

$= \underbrace{1000 \times 98\%^{40} \cdot v}_{442.3799} \; \frac{1 - \overbrace{0.98^{19} v^{19} \cdot (0.98v)}^{0.42513}}{\underbrace{1 - 0.98v}_{0.0273}}$ (recall: $\frac{a_1 - a_n q}{1-q}$)

where $v = \frac{1}{1 + \frac{9\%}{12}} \cong 0.99255$

Thus: Question: Outstanding $\cong 6888.97$ (B)

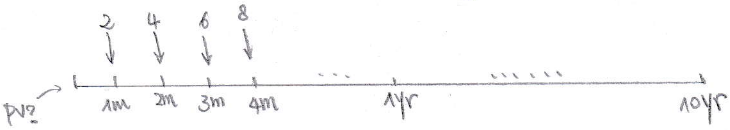
Exam FM Question 15

Give: ① Olga buys a 5-year "increasing annuity" for X ← PV

② Olga will receive 2 at the end of the 1st month, 4 at the end of 2nd month, each month thereafter the payment increases by 2

③ nominal rate is 9% convertible quarterly

Question: X



Solve: First: $\left(1+\frac{9\%}{4}\right)^4 = (1+i_{month})^{12} \Rightarrow i_{month} \cong 0.00744$

Second: $PV = 2 \times (Ia)_{\overline{12\times5}|i_{month}}$

$= 2 \times \frac{\ddot{a}_{\overline{60}|i_{month}} - 60v^{60}_{month}}{i_{month}}$ where $\ddot{a}_{\overline{60}|i_{month}} = (1+i_{month}) \cdot a_{\overline{60}|i_{month}}$

$a_{\overline{60}|i_{month}} = \frac{1-v^{60}}{i_{month}}$

$= 2 \times \frac{48.6135 - 38.448}{0.00744}$

$v = \frac{1}{1+i_{month}}$

$\cong 2732$ (B)

$i_{month} \cong 0.00744$

Exam FM Question 16

Give: ① David can receive: one of the following two payment streams:

(i) 100 at time 0, 200 at time n, 300 at time 2n

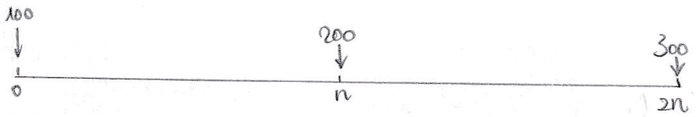
(ii) 600 at time 10

② PVs are the same

③ $v^n = 0.75941$

Question: i

Solve: $100 + 200v^n + 300v^{2n}$



$$100 + 200\underbrace{v^n}_{0.75941} + 300\underbrace{v^{2n}}_{0.75941^2} = 600v^{10}$$

$\Leftrightarrow v = 0.9661 = \frac{1}{1+i} \Rightarrow i = 0.0351$ (A)

Exam FM Question 17

Give: ① **Payments** are made at continuous rate of $(8k+tk)$ $\quad 0 \le t \le 10$

② **Interest** force of interest $\delta_t = \frac{1}{8+t}$

③ After time 10, the account is worth 20,000

Question: Calculate k

Solve: Recall: for AV: $\int_0^n f(t) \cdot e^{+\int_t^n \delta_t \, dt} \, dt$ (always; $f(t) = 8k+tk$)

if PV: $e^{-\int_0^t}$

if AV: $e^{+\int_t^n}$

Thus: Question: $\int_0^{n \to 10} (8k+tk) \cdot \left(e^{+\int_t^{n \to 10} \frac{1}{8+t} dt}\right) dt = 20{,}000$

$e^{\int_t^{n \to 10} d \ln(8+t)} = e^{[\ln(8+10) - \ln(8+t)]} = \frac{18}{8+t}$

$= \int_0^{10} k(8+t) \cdot \frac{18}{8+t} \, dt$

$= 18k(10-0) = \boxed{180k = 20{,}000} \Rightarrow k = 111.11$ (A)

Exam FM Question 18

Give: You have decided to invest in Bond X, n-year bond, semi-annual coupons and the following characteristics:

(1) Par value - 1000

(2) the ratio $\frac{\text{semi-coupon rate}}{\text{semi-annual yield } i_{6m}} = 1.03125$

(3) PV of "redemption value" = 381.50

(4) $(1+i)^{-n} = 0.5889$

Question: Price of Bond X

Solve: Price $= \text{coupon} \times a_{\overline{2n}|i_{6m}} + \left(\text{Redemption} \cdot v^{2n}\right)$ [381.5]

$= 1000 \cdot \frac{\text{coupon rate}}{i_{6m}} (1 - v^{2n})$ [ratio = 1.03125]

$= 1000 \times 1.03125 \times (1 - v^{2n}) + 381.5$ where $v^n = 0.5889$

≈ 1055.11 (D)

Exam FM Question 19

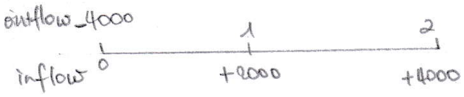
Give: (1) Project P: an investment of 4000 today, investment pays 2000 1-year from today, 4000 2 years from today

(2) Project Q: requires an investment of X two years from today. The investment pays 2000 today, and 4000 1-year from today.

(3) The NPV of 2 projects are equal at an annual effective interest rate of 10%

Question: Caclulate X.

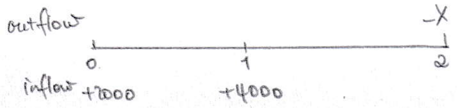
Solve: Project P



$NPV_1 = -4000 + 2000v + 4000v^2$ where $v = \frac{1}{1+10\%}$

$= 1124.0512$

Project Q



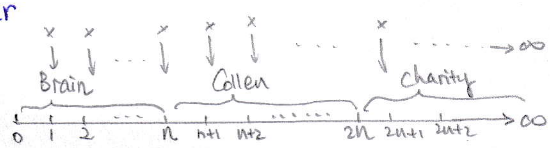
$NPV_2 = -X \cdot v^2 + 2000 + 4000v$ where $v = \frac{1}{1+10\%}$

$\Rightarrow X \cong 5459.83$ (D)

Exam FM Question 20

Give: (1) A perpetuity-immediate pays X per year

(2) Brain: receives the first n payments
Colleen: receives the next n payments
charity: receives the remaining payments



(3) Brain's share of the PV of the orignal perpetuity is 40% $PV_{Brain} = 40\% \cdot \frac{X}{i}$

charity's share is k $\Leftrightarrow PV_{charity} = k \cdot \frac{X}{i}$

Question: Calulate k

Solve: $\because PV_{Brain} = 40\% \cdot \frac{X}{i}$

$\therefore X \cdot a_{\overline{n}|i} = 40\% \cdot \frac{X}{i}$, where $a_{\overline{n}|i} = \frac{1-v^n}{i}$ $\Rightarrow v^n = 60\%$

Again: $\because PV_{charity} = k \cdot \frac{X}{i}$

$\therefore \left(\frac{X}{i}\right) \cdot v^{2n} = k \cdot \frac{X}{i}$

$\Rightarrow k = v^{2n} = (60\%)^2 = 0.36$ (D)

Exam FM Question21

Give: ① Seth, Janice, and Lori, each borrow 5000, for 5-year, nominal rate 12% compounded semi annually

② Seth: pays all interest and principal, at the end of 5-year, as a lump sum.

Janice: interest pays at the end of 6-month period, principal at the end of 5-year

Lori: repay loan, using 10 level payments, at the end of every six-month period

Question: Total interest paid on three loans.

Solve: Seth: $\text{Interest} = \underbrace{\text{Total payment}}_{\text{principal + interest}} - \text{principle} = 5000\Big[\underbrace{\big(1+\tfrac{12\%}{2}\big)^2}_{\text{1-year}}\Big]^5 - \underbrace{5000}_{\text{principal}} = 5000\times(1.6^{10}-1) \cong 3954$ ← 13,1808

Janice: interest 5000i 5000i 5000i — 0, 6m, 1yr, $1\frac{1}{2}$yr, 5yr

$\text{Interest} = (5000i)\times 10$ where $i = \frac{12\%}{2} = 6\%$

$\cong 3000$

Lori: Level Level Level — 0, 6m, 1yr, $1\frac{1}{2}$yr, 5yr

$\text{Level-payment} = \dfrac{5000}{a_{\overline{2\times5}|6\%}} \times 10 - \underbrace{5000}_{\text{principal}} \cong 1793$ ($a_{\overline{2\times5}|6\%}$ ← 7.3601)

Thus: Question $\cong 3954 + 3000 + 1793 \cong 8747$ D //

Exam FM Question22

Give: ① Bruce deposits 100 into bank account; Robbie deposits 50; both at time 0

② Each account earns an annual discount rate d $\overset{\text{same}}{\Longleftrightarrow}$ Both accounts have the same interest rate i

③ Bruce's interest earned during 11 year is X, which is the same as Robbie's interest during 17th yr

Question: X

Solve: $\underbrace{100(1+i)^{10}\cdot i}_{\text{interest during 11 yr}} = 50(1+i)^{16}\cdot i$ ← $100(1+i)^{11} - 100(1+i)^{10}$

$\Leftrightarrow \quad 2 = (1+i)^6$

$\Leftrightarrow \quad (1+i) = 2^{\frac{1}{6}} \Rightarrow i \cong 0.1225$

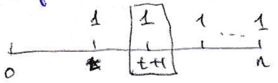
Thus: $X = 100(1+i)^{10}\cdot i \cong 38.9$ E //

Exam FM Question 23

① First: "the Principal Amount Outstanding" at time t, = all Future CFs PV at t $(= X)$

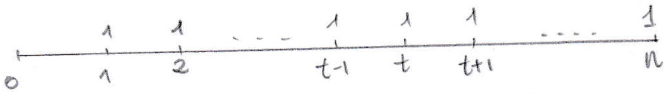
② Second: "the interest paid in year t+1" $= X \cdot i_{annual}$

③ Third: "the Principal Repaid" at time t+1 = "Total Payment at t+1" − "Interest paid in year t+1"



"Total Payment at t+1" = 1; "Interest paid in year t+1" $= X \cdot i_{annual}$

Give: ① Ron: repays a loan, payment 1 at the end of each year for n years



② The annual effective interest rate is i

③ The "Amount of interest paid in year t" + "amount of principal repaid in year t+1" = K

⇔ "Outstanding" at t−1 × i ; "Outstanding" at t−1 $= a_{\overline{n-t+1|}}$

⇔ 1 − "interest paid in year t+1" ; "interest paid in year t+1" = "outstanding" at t × i ; "outstanding" at t $= a_{\overline{n-t|}}$

$$= \frac{1-v^{n-t+1}}{i} \cdot i + \left(1 - \frac{1-v^{n-t}}{i} \times i\right)$$

$$= 1 - v^{n-t+1} + v^{n-t}$$

$$= 1 + v^{n-t}(1-v)$$

↑ check answers all "+"

$$1 - v = \frac{1+i-1}{1+i} = \frac{i}{1+i} = d$$

$$= 1 + v^{n-t} \cdot d \quad (D)$$

Exam FM Question 24.

Give (1) annual effective interest rate $i > 0$

(2) PV of a perpetuity, paying 10, at the end of each 3 year period, with the first payment at the end of year 3, is 32

$$\Leftrightarrow \boxed{PV} = 10v^3 + 10v^6 + 10v^9 + \cdots$$

$$= 10v^3(1 + v^3 + v^6 + \cdots) \quad (n \to \infty)$$

$$= 10v^3 \frac{1}{1-q} \quad \text{where } q = v^3$$

$$= \boxed{\frac{10v^3}{1-v^3} = 32} \Leftrightarrow 10v^3 = 32 - 32v^3 \Rightarrow v^3 = \frac{32}{42}$$

(3) Same effective rate, PV of 1, paying at the end of each 4-month period with first payment at the end of 4 months, is X

$$\Leftrightarrow X = v^{1/3} + v^{2/3} + v^{3/3} + \cdots$$

$$= v^{1/3}(1 + v^{1/3} + v^{2/3} + \cdots)$$

$$= v^{1/3} \cdot \frac{1}{1 - v^{1/3}} = 32.599. \quad \text{(B)}$$

Exam FM Question 25

Give: ① At 2013/12/31, an company is known to have an obligation of 10^6 on 2017/12/31

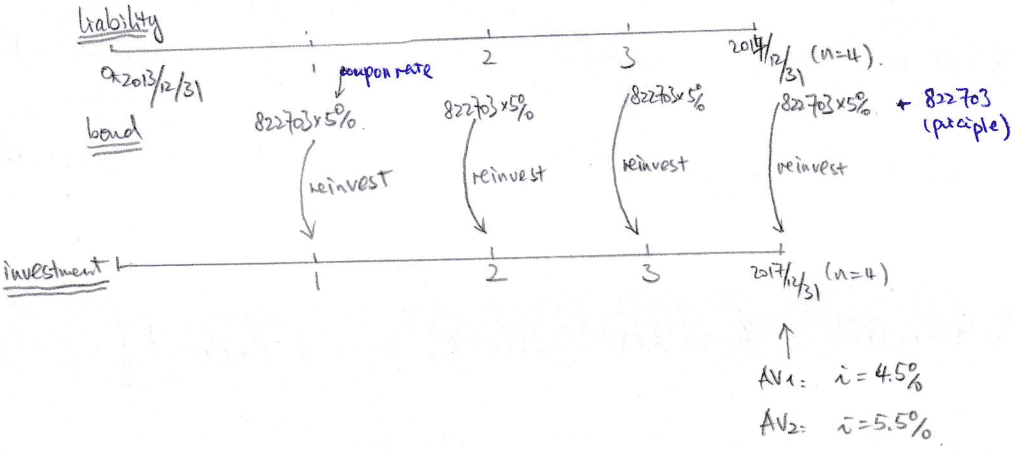
② This company, purchases 4-year, 5% coupon bonds, 822703 par value, redemption value = par value
↳ interest = 5%

③ The coupon of this bond, immediate reinvest at 5% through 2017/12/31,

Now: S1: investment, at 2014/01/01, interest drop 0.5%

S2: investment, at 2014/01/01, interest increase 0.5%

Question: describe company's profit/Loss, as of 2017/12/31, after liability is paid



$$AV_1 = (822703 \times 5\%) \cdot s_{\overline{4}|4.5\%} + 822703 = 998687$$

(coupon rate: 5%; investment return: 4.5%; $(822703 \times 5\%) \cdot s_{\overline{4}|4.5\%}$ = 175984)

$$AV_2 = (822703 \times 5\%) \cdot s_{\overline{4}|5.5\%} + 822703 = 1001324$$

(income, liability)

$$Balance_1 = 998687 - 10^6 = -1313$$

$$Balance_2 = 1001324 - 10^6 = 1324$$

(D)

Exam FM Question 26

Give: ① Company has an obligation to pay the medical costs for a claimat

② Claim costs 5000 today, inflation expected to be 7% per year, Claimant receives 20 payments

③ Claim payment are made at yearly intervals. 1^{st} payment 1-year from now

④ annual effective rate of 5%

Question: Pv of this obligation

	0	1	2	3	...	20
payment		$5000(1.07)$	$5000(1.07)^2$	$5000(1.07)^3$	......	$5000(1.07)^{20}$
PV CFs		$5000(1.07)v_{5\%}$	$5000(1.07)^2 v^2_{5\%}$	$5000(1.07)^3 v^3_{5\%}$	......	$5000(1.07)^{20} v^{20}_{5\%}$

$$PV = 5000(1.07)v_{5\%}\left[1 + 1.07v_{5\%} + (1.07v_{5\%})^2 + \cdots + (1.07v_{5\%})^{19}\right]$$

$$= \underbrace{5000(1.07)v_{5\%}}_{5095.2381} \frac{1-(1.07v_{5\%})^{19}\cdot(1.07v_{5\%})}{1-1.07v_{5\%}} \quad \leftarrow \frac{1-a_n q}{1-q}$$

$\cong 122606.67$ (D) //

Exam FM Question 27

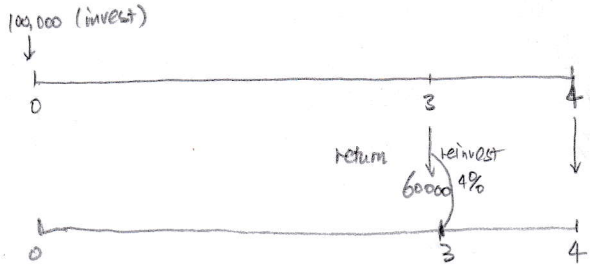
Give: ① investor pays 100,000, for a 4-year investment, returns cash flow 60,000 at year-3, 60,000 at year-4

② Reinvested at 4% per annum effective

③ Annual effective interest rate of 5%

Question: NPV of this investment

Solve:



at $t=4$, $AV_{investment} = 60000(1+4\%) + 60000$

$$\therefore PV_{invest} = v^4_{5\%}\left[60000(1+4\%) + 60000\right] = 100698.7829$$

$$\Rightarrow NPV = -100000 + PV^{income}_{invest} \cong -100000 + 100698.7829 \cong 699 \text{ (C)}$$

Exam FM Question 28.

Give: the following information w.r.t a bond.

(1) par value = 1000

(2) $n = 3$-year

(3) annual coupon rate : 6%

(4) spot rate
- 1-year 7%
- 2-year 8%
- 3-year 9%.

Question: Value of bond.

Solve:

	0	1	2	3
coupon:		1000 × 6% = 60	1000 × 6% = 60	1000 + 1000 × 6% = 60 + 1000
effective rate:		7%	8%	9%

$$PV = \underbrace{\frac{60}{1.07}}_{=56.0747} + \underbrace{\frac{60}{1.08^2}}_{=51.4403} + \underbrace{\frac{1060}{1.09^3}}_{=818.5145} = 926.0295$$ (B)

Exam FM Question 29

Give: the following information w.r.t a bond

i) par value: 1000

ii) term to maturity: 3 years

iii) annual coupon rate: 6% annually

iv) 3, 2, 1-year spot rate are: 9%, 8%, 7%

Bond Price sold at its value

Question: an effective yield for this bond

Solve:

$$\frac{60}{1.07} + \frac{60}{1.08^2} + \frac{1060}{1.09^3} = \boxed{926.0295 = 60v_i + 60v_i^2 + 1060v_i^3} \Rightarrow i \doteq 8.9\% \text{ (E)}$$

use calculator

Alternatively, plug in (A)–(E)'s value

Exam FM Question 30

Give: ① Current Bond Price $P(i) = 100$

② i: yield to maturity 8%

③ $\frac{\partial P(i)}{di} = -700$ ← w.r.t "modified duration"

Question: "Macaulay Duration".

Solve: $D_{mac}(i) = (1+\overset{8\%}{i}) \cdot \underbrace{D_{mod}(i)}_{-\frac{dP(i)/di}{P(i)}} = (1+8\%)\cdot\left(-\frac{-700}{100}\right) = 1.08 \times 7 = 7.56$

$\frac{dP(i)}{di} \leftarrow -700$, $P(i) \leftarrow 100$

Exam FM Question 31

Give: ① A common stock pays a "constant divident" at the end of year, into perpetuity

② An annual effective interest rate 10%

Question: Macaulay Duration: $D_{mac}(i)$

Solve:

	0	1	2	3	......	→ ∞
		D	D	D		

	0	1	2	3	...	$(Ia)_{\overline{\infty}|}$
		1	2	3	...	

$$D_{mac} = \frac{(Dv)\cdot 1 + (Dv^2)\cdot 2 + (Dv^3)\cdot 3 + \cdots}{(Dv) + (Dv^2) + (Dv^3) + \cdots} \leftarrow PV = \frac{D(1\cdot v + 2\cdot v^2 + 3\cdot v^3 + \cdots)}{Dv\underbrace{(1+v+v^2+\cdots)}_{\frac{1}{1-v} \leftarrow \frac{1}{1-q}}} \leftarrow D\cdot (Ia)_{\overline{\infty}|} = \frac{1}{i} + \frac{1}{i^2} \text{ or } \frac{1}{id}$$

$$= \frac{D\left(\frac{1}{i}+\frac{1}{i^2}\right)}{D\left(\frac{v}{1-v}\right)} = \frac{\frac{1}{i}+\frac{1}{i^2}}{\frac{1}{i}} \begin{matrix}\times i^2 \\ \times i^2\end{matrix} = \frac{i+1}{i} = 11 \quad \text{(C)}$$

$$\frac{v}{1-v} = \frac{\frac{1}{1+i}}{1-\frac{1}{1+i}} = \frac{\frac{1}{1+i}}{\frac{1+i-1}{1+i}} = \frac{1}{i}$$

Exam FM Question 32

Give: ① A common stock pay "dividents increasing by 2% per year" into perpetuity.

② Annual effective rate 5%

Question: $D_{mac}(5\%)$

0	1	2	3	4	→ ∞
	$1.02D$	$D1.02^2$	$D1.02^3$	$D1.02^4$	

Solve: $$D_{mac} = \frac{(Dv)\cdot 1.02 + (D1.02^2 v^2)\cdot 2 + (D1.02^3 v^3)\cdot 3 + \cdots}{Dv1.02 + D1.02^2 v^2 + D1.02^3 v^3 + \cdots}$$

$(I\ddot{a})_{\overline{\infty}|i^*}$ where $1.02v_{5\%} = \frac{1}{1+i^*} \Rightarrow i^* \cong 0.0294$

0	1	2	3
1	2	3	4

0	1	2	3
1	1.02	1.02^2	1.02^3

$$= \frac{D\cdot v \cdot 1.02\,[1 + 2\cdot(1.02v) + 3\cdot(1.02v)^2 + \cdots]}{Dv\cdot 1.02\,[1 + (1.02v) + (1.02v)^2 + \cdots]}$$

Sum $= \frac{1}{1-\underbrace{1.02v}_{v^*}} \cong \frac{1}{0.02857} = \frac{1}{d^*}$, $1 - v^* = d^* = \frac{i^*}{1+i^*}$

$$= \frac{\frac{1}{d^{*2}}}{\frac{1}{0.02857}} \quad \text{where } d^* = i^* v^* = 0.0294 \times \frac{1}{1.0294} = 0.02856$$

$$= \frac{\frac{1}{0.02856^2}}{\frac{1}{0.02857}} \cong \frac{1}{0.02856} \cong 35 \leftarrow \frac{1}{d^*}$$

$\frac{1}{d^{*2}}$, $\frac{1}{d^*}$

Exam FM Question 33

Give: Seth ① borrows X for 4-year, annual rate 8%, repay "level-payment" at the end of each year

② "Loan Balance" at the end of 2^{nd} year = 1076.82

③ "Loan Balance" at the end of 3^{rd} year = 559.12

Question: Principal repaid in the first payment

Solve: we know: $\text{factor} = \frac{X}{a_{\overline{4}|8\%}}$

Thus: $\underbrace{\text{factor}}_{\frac{X}{a_{\overline{4}|8\%}}} \cdot a_{\overline{4-2}|8\%} = 1076.82 \Rightarrow X \cong 2000$, $\text{factor} = \frac{X}{a_{\overline{4}|8\%}} = 603.8465$

Then: Question = factor × (Outstanding$_{t=4}$ (period 0) − Outstanding$_{t=3}$ (period 1))

$= 603.8465 \times (a_{\overline{4-0}|} - a_{\overline{4-1}|})$

$\cong 443.83$ (A)

FM Exam, Question 34

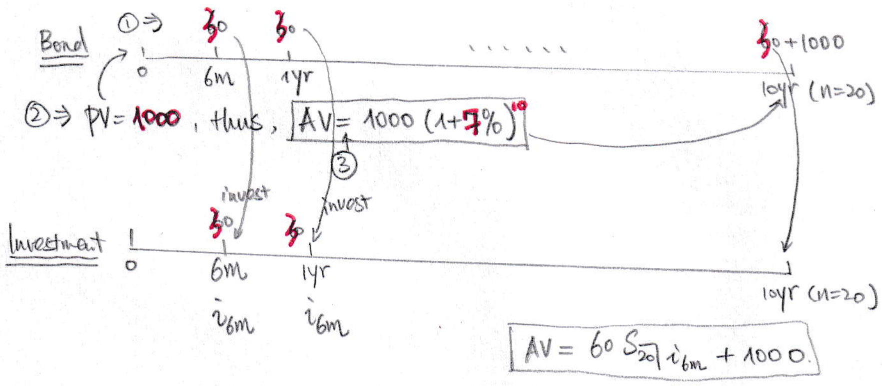
Give: ① Bond: 10-year, 1000 par value, semi-annual coupons at annual rate 6%; annual yield 6% compounded semi-annual

— Bond's Price at t=0 is par value 1000

② Investment: each coupon immediately invest with "annual effective rate i_{annual}"

③ At the end of 10 years, The annual effective yield is 7% on his investment in bond

Question: Calculate i_{annual}



Solve: $1000(1+7\%)^{10} \overset{AV}{=} 30\, s_{\overline{20}|i_{6m}} + 1000 \Rightarrow s_{\overline{20}|i_{6m}} \cong 32.238 \Rightarrow i_{6month} \cong 4.7595$

Thus: $(1+i_{6month})^2 = 1 + i_{annual} \Rightarrow i_{annual} \cong 0.09745$ (B)

Exam FM Question 35:

Give: At age 40, a plan to have a retirement income of 3000 at the beginning of each month starting on 65^{th} birthday. To support the plan, this person makes monthly contribution of X to a fund for 25 years. The annual nominal rate 8% compounded monthly. On the 65^{th} birthday, each 1000 of the fund will provide 9.65 income at the beginning of each month starting immediately till the day he dies

Question: what is X?

Exam FM Question35.

Solve: In order to have 3000: at the beginning of each month on 65^{th} birthday as long as he survives. We assume. Y amount in the fund.

each Y can provide 3000 at the begining of month till die ($\frac{3000}{Y} = \frac{9.65}{1000}$) each 1000 of fund can provide 9.65 income at the beginning till die.

$\Rightarrow Y = 310800.829$

Thus: $X \ddot{s}_{\overline{300}|\frac{8\%}{12}} = 310800.829 \Leftrightarrow X \cdot \underbrace{\frac{(1+i)^{300}-1}{d}}_{957.3689} = 310800.829$. where $d = \frac{i}{i+1} = \frac{\frac{8\%}{12}}{1+\frac{8\%}{12}} \cong 0.006623$

$\Rightarrow X \cong 324.64$ (A)

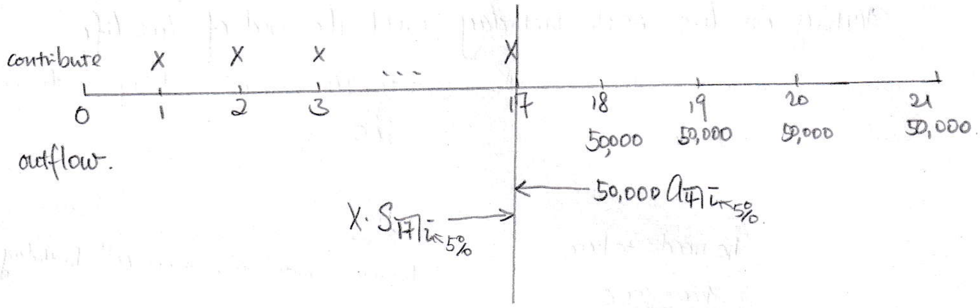
Exam FM Question 36

Give: ① parents: decide at the birth of their daughter that they need to provide 50,000 at each of their daught's 18^{th}, 19^{th}, 20^{th}, 21^{th} birthday to fund her college education.

② plan: contribute X at each of their daughter's 1^{st} till 17^{th} birthday to fund

③ rate: constant 5% annual effective interest rate

Question: Calculate X



Solve: ~~$X \cdot s_{\overline{17}|5\%} = 50,000\, a_{\overline{4}|5\%}$~~

25.840366   3.545951

$$X\left(1+1.05^1+1.05^2+\cdots+1.05^{16}\right) = 50,000\left(v+v^2+v^3+v^4\right) \quad \text{where } v=\frac{1}{1.05}$$

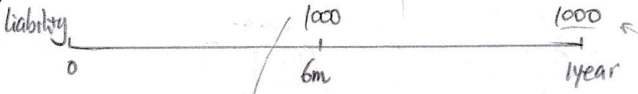
$1.05 \cdot (\quad)$ $1.05 \cdot (\quad)$

$$X\left(1.05+1.05^2+1.05^3+\cdots+1.05^{17}\right) = 50,000\left(1+v^1+v^2+v^3\right)$$

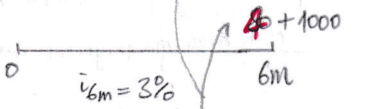
(D)

Exam FM Question 37

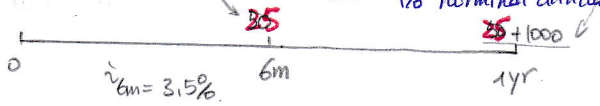
Give: ① Liability, 1000 due 6 month, and 1000 due one year



② Asset 1: face amt 1000, 6 month bond, 8% coupon rate convertible semi-annually and 6% norminal annual rate convertible semi-annually



③ Asset 2: face amt 1000, 1-year bond, 5% coupon rate convertible semi-annually and 7% norminal annual rate convertible semi-annually



Question: Amt of each bond to purchase, to exactly match the liabilites.

Solve: Assume: buy X: Asset 1 ; Y: Asset 2

→ Start with "Asset 2", since it's the only investment has return at 1-yr

[t=1-yr] $1000 = (25 + 1000) \cdot Y \Rightarrow Y = 0.9756$

→ Then:

[t=6m] $1000 = (40 + 1000) X + 25 \cdot Y$

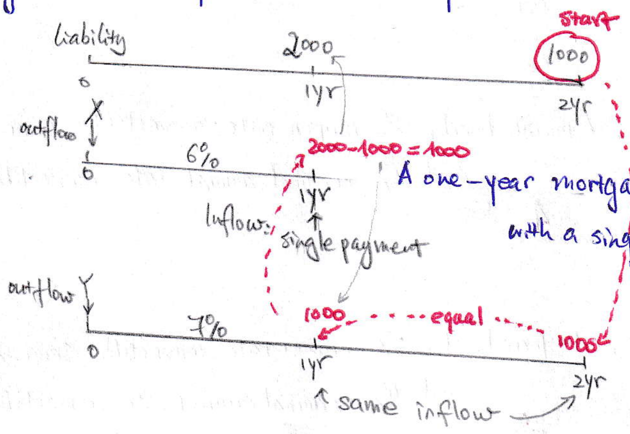
$\Rightarrow X = 0.93809$ (D)

Exam FM Question 38.

Give: ① Joe: liability: 2000 due 1-yr; 1000 due 2-yr.

② Mortgage I:

③ Mortgage II:



A one-year mortgage in which X is lent, which is repaid with a single payment at time one. $i_{annual} = 6\%$.

A two-year mortgage in which Y is lent, repaid with 2 equal annual payment

Question: X+Y

Solve: **First**: Mortgage II's income, at t=2, must = 1000.

Since: Mortgage II's 2 income at t=1 & t=2 are the same.

Thus: at t=1, "$\text{income}_{\text{mortgage } 2}$" = 100

Second: which make the income of "Mortgage 1 at t=1" = 2000 − 1000 = 1000

Thus: $Y = 1000\, a_{\overline{2}|7\%} \simeq 1808.018$

$X = 1000\, v \cong 943.396$ ($v = \frac{1}{1.06}$)

+

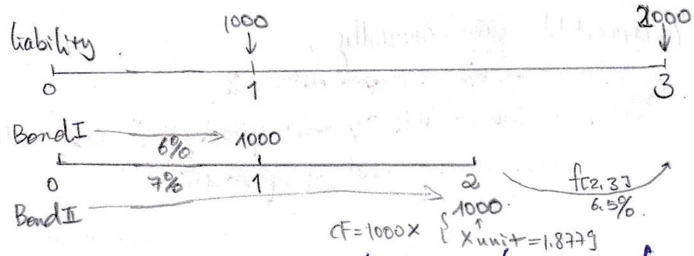
2751.41 Ⓒ

Exam FM Question 39

Give: ① Liability: of 1000 due 1-year, 2000 due 3-year

② Bond: I: 1-year zero-coupon bond, matures for 1,000, 6% yield rate

II: 2-year zero-coupon bond, matures for 1000, 7% yield rate.



③ one-year forward rate, made 2-year from now $f_{[2,3]} = 6.5\%$

Question: Total purchase price (NOT Unit) of Bond I and Bond II to make exactly match

First: **Bond I**: $CF_{t=1}$ match exactly, thus $\boxed{price_{t=0} = PV_{Bond\ I} = 1000\, v_{6\%} \leftarrow \text{Bond I} = 943.396}$

Second: Bond II: first, get unit X

$$1000 \cdot X \cdot (1 + 6.5\%) = 2000 \Rightarrow X_{unit} = 1.8779$$

Then, $\boxed{PV_{Bond\ II} = (1000X) \cdot v^2_{7\%} \leftarrow \text{Bond II} = 1640.26}$ ($X = 1.8779$)

Thus: $PV_{Bond\ I} + PV_{Bond\ II} = 943.396 + 1640.26 \doteq 2583.66$ (A)

Exam FM Question 40.

Give: ① Bond: 20-year par value, annual coupon rate 8%, payable semi-annually, price of 1722.25

② Callable: can be called at par value X, starting at the end of 15-year, after coupon payment

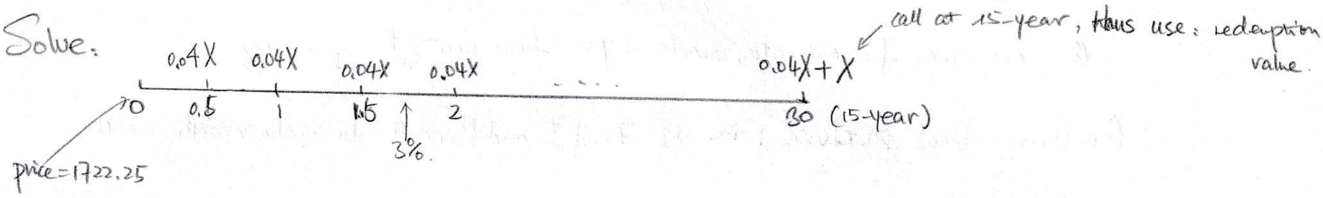
③ The lowest yield rate is 6% convertible semi-annually,

yield rate 6% < coupon rate 8%.

Thus: we know this "lowest yield rate" must achieved by: call this bond at the end of year 15.

Question: X

Solve.



$$1722.25 = 0.04X \cdot a_{\overline{30}|3\%} + X \cdot v^{30}_{3\%}$$

$$\Rightarrow 1722.25 = 0.7840X + 0.41198X = 1.19598X \Rightarrow X = 1440.03$$ ©

Exam FM Question 41

Give: ① A 20-year par value bond, semi-annual coupon of 40, redemption value of 1100

② Call: can be called at 1200 ← par value, any time after end of 15-year, before maturity.

③ A max price, which garantee a 6% semi-annually compounded yield.

par ↓ $g \times 1200 = 40 \Rightarrow g_{6month} = 3.33\%$; $i_{6month} = 3\%$ } ⇒ $g > i$

↑ Smallest price over the various call dates. ⇔

method 1 ⇔ when "coupon rate" > "yield rate", "smallest price" = "highest yield", callable at maturity

method 2 ⇔ calculate: price, when call at maturity; price, when call at t=15; choose the "lower price"

⇒ timeline: 0 ($i = 3\%$), 1 (40, ↑6m), 2 (40, ↑1yr), . . . , 40 (20-year): 40 + Redemption 1100 (∵ call at maturity, ⇔ use "redemption" 1100)

if call before 40, use 40 + 1200 ← call price

$$PV = 40\,a_{\overline{40}|3\%} + 1100\,v^{40}_{3\%} = 1261.80 \quad (B)$$

$a_{\overline{40}|3\%} = \left(\frac{1 - v^{40}}{i_{3\%}}\right)$ → 924.59; $1100\,v^{40}_{3\%}$ → 337.21

Recall:

- if call at 15-year ⇔ price $= 40\,a_{\overline{30}|3\%} + 1200 \cdot v^{30}_{3\%}$
- if call at 20-year (maturity) ⇔ Price $= 40\,a_{\overline{40}|3\%} + 1100 \cdot v^{40}_{3\%}$

Exam FM Question 42.

Give: ① Bond: 10-year par value bond, annual nominal coupon rate 4%, payable semi-annually, price 1021.50

② Call: at par value X, starting at the end of year 5

③ lowest yield rate: annual nominal of 6%

Question: X

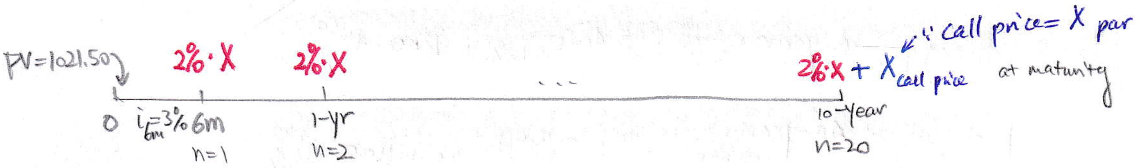
> "Bond Price > Call" 或 "coupon rate > yield rate i": want **lowest** yield, call at "earlist"
>
> "Bond Price < Call price" 或 "Coupon rate < yield rate i": want **lowest** yield, call at "maturity"

Solve:

$\boxed{\text{coupon}_{annual} \text{ rate} = 4\%} < \boxed{\text{yield rate}_{annual}\ 6\%}$

$\Leftrightarrow \boxed{\text{coupon rate}_{6m} = 2\%} < \boxed{\text{yield rate}_{6m} = 3\%}$

Thus: want "lowest yield", must call at "maturity" n=20 / 10-year



$$PV_{price} = \boxed{1021.50 = (2\% \cdot X)\, a_{\overline{20}|3\%} + X \cdot v^{20}_{3\%}} \Rightarrow X = 1200 \quad Ⓔ$$

Exam FM Question 43

Give:
① Bond: 10-year, par value with normiral coupon rate 4%, semi-annual, Bond price 1021.50
② Call: call at "100 over par value of 1100" (call price: 1200), starting at year 5, ending 6m before maturity
③ Minimal yield, convertible semi-annually.

Question: this minimal yield.

"Bond Price (1021.50) < Call Price (1200)" ⟺ "coupon rate < yield rate" ⟺ reach "min yield", must call at the end (end of call period)

∵ end of call period = 6m before maturity

Thus: use [call value] ($1200) + coupon 19 (9 m)

par value (corresponding to: coupon)

Solve: [if call]: (at n=19)

$$PV = 1021.50 = (2\% \times 1100) \cdot a_{\overline{19}|i_{6m}} + (1200 \cancel{+ 2\% \times 1100}) \cdot v_{6m}^{19}$$

22 [PMT] 19 [N] 1021.50 [+/-] PV 1200 [FV] [CPT] [I/Y] ☆

$\Rightarrow i_{6m} \cong 2.86\%$

Thus: $(1 + i_{annual}) = (1 + i_{6m})^2$ (2.86%) $\Rightarrow i_{annual} = 1.058 - 1 = 5.8\%$

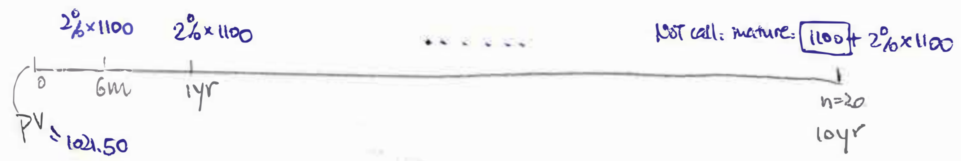
[if NOT call]: (at n=20; maturity at par value.)

$$PV = 1021.50 = (2\% \times 1100) \cdot a_{\overline{20}|i_{6m}} + 1100 + 2\% \times 1100$$

$\Rightarrow i_{6m} \cong 2.46\%$

Thus: $(1 + i_{annual}) = (1 + i_{6m})^2$ (2.46%) $\Rightarrow i_{annual} = 1.04972 - 1 = 4.97\%$ small, thus, Don't call

$\Rightarrow i_{annual} = 4.97\%$ (B)



Exam FM Question 44

Give: ① Liability: 15 annual payments of 35,000.

Timeline: 0, 1 (35,000), 2 (35,000), ..., 15 (35,000)

② Asset: for immunization

Asset 1 (X): 0 → 5, zero coupon 5-year Bond

Asset 2: 0 → 10, zero coupon 10-year Bond

③ $i_{annual} = 6.2\%$

④ Duration ($D_{mac}(v)$): D_{mac} – Liability = D_{mac} – Asset

PV: PV – Liability = PV – Asset

Question: Amount invested in 5-year Bond (X)

Assume: At $t=0$, "\$X" invests in 5-year Bond

Then, \$"$PV_{Liability} - X$" invests in 10-year Bond., since $PV_{Liability} = 35{,}000\, a_{\overline{15}|\, i=6.2\%} = 335530.30$

⇒ "\$335530.30 − X" invests in 10-year Bond

Again; $$D_{mac}(\text{Liability}) = \frac{(35000v)\cdot 1 + (35{,}000v^2)\cdot 2 + \cdots + (35000v^{15})\cdot 15}{35000v + 35000v^2 + \cdots + 35000v^{15}} \leftarrow PV = 335530.30$$

$$= \frac{35000\,(1v + 2\cdot v^2 + 3\cdot v^3 + \cdots + 15\cdot v^{15})}{335530.30} \leftarrow (Ia)_{\overline{15}|\, i=6.2\%} = \frac{\ddot{a}_{\overline{15}|i} - 15v^{15}}{i}$$

($\ddot{a}_{\overline{15}|i} = (1+i)a_{\overline{15}|i}$ = 10.180948; $a_{\overline{15}|}$ = 9.58658; $15v^{15}$ = 6.08448; $= 66.072$)

$= 6.89214$

Which gives: $$\frac{\$X}{PV}\cdot 5 + \frac{\$335530.30 - X}{PV}\cdot 10 = 6.89214 \quad (D_{mac}(\text{Asset}) = D_{mac}(\text{Liability}))$$

(PV = 335530.30; 3.107856; 3.78428)

$X = 208556$ Ⓒ

Exam FM Question 45

Give: Liability: need to be repaid using 16 annual payments:

(1) 1^{st} payment 2000 : due 1 year from now

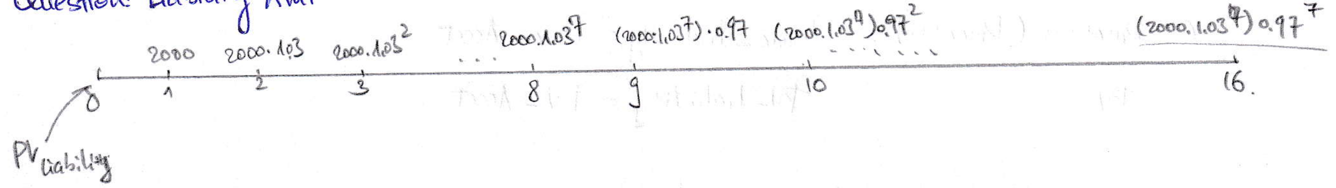
(2) Next 7 payments, each 3% larger than the preceding one $X_t = 1.03 X_{t-1}$

(3) From 9^{th} to 16^{th}, each will be 3% less than the preceding one $X_t = 0.97 X_{t-1}$

(4) $i_{annual} = 7\%$

Question: Liability Amt

base is different, can NOT cancel



Solve: $PV_{liability}$ = part 1: $2000v + (2000\cdot1.03)v^2 + (2000\cdot1.03^2)\cdot v^3 + \cdots + (2000\cdot1.03^7)v^8$

+ part 2: $v^8\left((2000\cdot1.03^7)\cdot0.97\cdot v + (2000\cdot1.03^7)\cdot0.97^2 v^2 + \cdots + (2000\cdot1.03^7)\cdot0.97^8 v^8\right)$

part 1 $\because 2000\cdot1.03v^2 + (2000\cdot1.03^2)v^3 + \cdots + (2000\cdot1.03^7)v^8 = 2000\cdot1.03\cdot v^2\left(1 + 1.03v + (1.03v)^2 + \cdots + (1.03v)^6\right)$

$0.2341\ldots \quad \dfrac{1-(1.03v)^6(1.03v)}{1-1.03v} = \dfrac{a_1 - a_n q}{1-q}$

$1-1.03v = 0.03738$

$\Rightarrow \cong 11268.3877$

$\therefore$ part 1 $\cong 13137.56$

part 2 part 2 $= v^8\cdot2000\cdot1.03^7\cdot v\cdot0.97\left(1 + 0.97v + \cdots + 0.97^7 v^7\right)$

$\dfrac{1 - v^7\cdot v\cdot(0.97)^8}{1 - v\cdot0.97}$

$\cong 7552.22$

$\Rightarrow$ Sum $\cong 20689.78$ Ⓐ

Exam FM Question 46

Give: ① $\delta_t = \dfrac{\frac{t^2}{100}}{3+\frac{t^3}{150}}$

② deposits 500 now

Question: at when, AV = 2000

Solve: $2000 = 500 \cdot e^{\int_0^n \delta_t \, dt}$

$\Leftrightarrow e^{\int_0^n \delta_t \, dt} = 4$

$\Leftrightarrow e^{\int_0^n \frac{\frac{t^2}{100}}{3+\frac{t^3}{150}} dt} = 4$, notice: $d^{\ln}(3+\frac{t^3}{150}) = \frac{3}{150}t^2 = \boxed{\frac{1}{50}t^2}$

Then. $e^{\frac{1}{2}\int_0^n \frac{\boxed{\frac{t^2}{50}}}{3+\frac{t^3}{150}} dt} = e^{\frac{1}{2}\ln(3+\frac{t^3}{150})\Big|_0^n} = e^{\frac{1}{2}[\ln(3+\frac{n^3}{150}) - \ln(3+0)]}$

$\ln(3+\frac{n^3}{150}) - \ln(3+0) = \ln(1+\frac{n^3}{450})$ ← $\ln(a)-\ln(b) = \ln(\frac{a}{b})$

$= \left[e^{\ln(1+\frac{n^3}{450})}\right]^{\frac{1}{2}}$

$= (1+\frac{n^3}{450})^{\frac{1}{2}} = 4$

$\Rightarrow n = 18.8988$

Exam FM Question 47

① 40-year bond. purchase at a discount ⇔ P<C

② Bond pays annual coupons

③ Amount for accumulation of discount in 15th coupon is 194.82
↳ (Principal Repaid: Discount case, P<C, coupon rate < yield rate)
Amount for accumulation of premium in 20th coupon is 306.69
↳ (Principal Repaid: Premium case, P>C, coupon rate > yield rate)

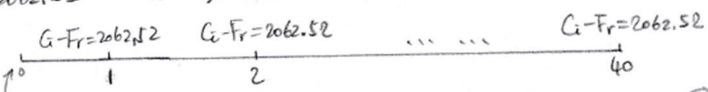
Question: Amount of discount in the purchase price?

Solve: Recall. Loan: Principal Repaid

t=1	v^n
t=2	v^{n-1}
t=3	v^{n-2}
⋮	⋮
t=n	v

$\Rightarrow$ 15th principal repaid $= (C-P)\cdot v^{40-14}$
20th principal repaid $= (C-P)\cdot v^{40-19}$
$\Rightarrow v = 0.91324 \Rightarrow C-P = 2062.53$ (Amount of Discount), $i = 0.095$

$\Rightarrow C-P = 2062.53 = -(Fr-C) = C-Fr$



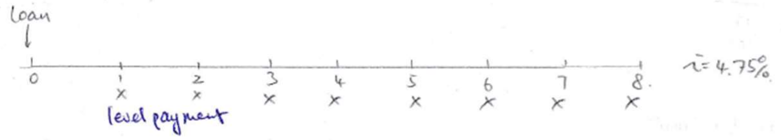
$2062.52\, a_{\overline{40}|i=9.5\%}$ = Amount of Discount in Price = 21.135 (E)

Exam FM Question 48:

Give: Loan today repaid with 8 level payments with the first payment made 1 year from today. The payments are calculated based on an rate of 4.75%. The principal portion of the 5^{th} payment is 699.68.

Question: What is the total amount of interest paid on this loan?

FM Exam Question 48



① Recall: "principal portion" of level payment 1

t=1	v^n
t=2	v^{n-1}
t=3	v^{n-2}
⋮	⋮
t=n	v

$\Rightarrow$ 5th payment principal portion $= X v^{8-4} = X v^4 = 699.68$

$\Rightarrow X = 842.39$

② Recall: level payment 1 case: total amt of interest = Total payment $(1 \times n)$ − Total principal repaied $(1 \times a_{\overline{n}|})$

Thus: in this case: total amt $= X \cdot 8 - X \cdot a_{\overline{8}|4.75\%}$

$= X(8 - a_{\overline{8}|4.75\%})$ (X = 842.39)

$= 1239.125$ Ⓐ

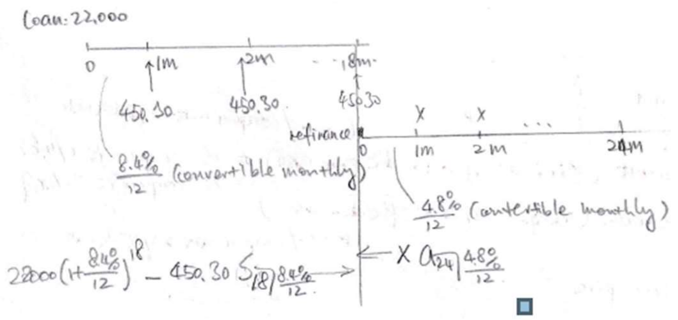
Exam FM Question 49:

Give: A loan of 22,000 with n monthly payments of 450.30 starting in one month. One larger payment in n+1 months to pay off the loan. An annual rate of 8.4% convertible monthly. Immediately after the 18yh payment, the investor refinances the loan to pay off the remaining balance with 24 monthly payments starting one month later. This refinanced loan uses an annual nominal interest rate of 4.8% convertible monthly.

Question: What is the amount of the new monthly payment?

Exam FM Question 49

Give



Solve: $22000\left(1+\frac{8.4\%}{12}\right)^{18} - 450.30 \cdot s_{\overline{18}|\frac{8.4\%}{12}} = X \cdot a_{\overline{24}|\frac{4.8\%}{12}}$

$X = 715.27$ Ⓓ

Exam FM Question 50

Give: Bond: bought at "par value", 7-year, 5000 par value, coupon rate paid 7.6% semi-annual

Question: Macaulay duration $D_{mac}(i)$

Solve: Recall: A bond bought at par ⇔ { ① coupon rate = yield rate. ② $D_{mac} = \ddot{a}_{\overline{n}|\text{yield}}$.

$$\Rightarrow \begin{cases} n = 7\times 2 \\ \text{yield}_{6m} = \frac{7.6\%}{2} = 3.8\% \\ D^{6m}_{mac} = \ddot{a}_{\overline{14}|3.8\%} = (1+i_{6m})\cdot a_{\overline{14}|3.8\%} \end{cases}$$

$\Rightarrow D^{6m}_{mac} = 11.11 \Rightarrow D^{1year}_{mac} = \frac{D^{6m}_{mac}}{2} = \frac{11.11}{2} = 5.55$ Ⓒ

↑ double the periods

Exam FM Question 51

Give: Old: n-year bond, bought at par ($P^{old} = 1000$), $D^{old}_{mac} = 7.959$, $i = 7.2\%$

New: $i = 8\%$

Question: New bond price, using "First-order Modified" Approximation. Ⓐ

Solve: $$P_{new} = P_{old}\cdot\left(1 - \frac{i^{new} - i^{old}}{1+i^{old}}\cdot D^{old}_{mac}\right) = 1000\left(1 - \frac{8\% - 7.2\%}{1+7.2\%}\cdot 7.959\right) = 940.60$$

$$\overset{or}{=} P_{old}\left[1 - (i^{new} - i^{old}) \times D^{old}_{mod}\right]$$

Exam FM Question 52

Give: 3 zero-coupon bonds:

Maturity	Price
2	0.90703
3	0.85892

Question: Calculate: $f_{[2,3]}$

Solve: $i^{2\text{-year}}_{spot}$: $(1+i^{2\text{-year}})^2 \cdot 0.90703 = 1 \Rightarrow i^{2\text{-year}}_{spot} = 0.04999$

$i^{3\text{-year}}_{spot}$: $(1+i^{3\text{-year}})^3 \cdot 0.85892 = 1 \Rightarrow i^{3\text{-year}}_{spot} = 0.052.$

$$\Rightarrow (1+i^{2\text{-year}}_{spot})^2\cdot(1+f_{[2,3]}) = (1+i^{3\text{-year}}_{spot})^3 \Rightarrow f_{[2,3]} = 0.056$$ Ⓔ

Exam FM Question 53:

Give: ① bond sells for 5000, 5000 is par value, 8-year bond, annual coupon rate 5% annually.

② d_1: Macaulay duration, just before the 1st coupon payment

③ d_2: Macaulay duration, just after the 1st coupon payment

Question: $\frac{d_1}{d_2}$

d_0: Macaulay duration, buy at par $\Leftrightarrow D^{mac} = \ddot{a}_{\overline{n}|i} = \ddot{a}_{\overline{8}|5\%} = (1+i)a_{\overline{8}|5\%} \approx 6.7864$

Solve: d_1: before the 1st coupon payment $\Leftrightarrow d_1 = D^{mac} - 1 = 5.7864$

d_2: after the 1st coupon payment $\Leftrightarrow d_2 = \ddot{a}_{\overline{n-1}|} = \ddot{a}_{\overline{7}|5\%} = 6.0757$ ($=(1+i)a_{\overline{7}|5}$)

$\Rightarrow \frac{d_1}{d_2} = \frac{5.7864}{6.0757} = 0.9524$

Exam FM Question 54:

Give: ① Liability: 99 at t=1, 102 at t=2, 100 at t=3

② Investment:

Asset1 A: maturity 1, $i_{yield} = 6\%$, coupon rate = 7%, annual coupon bond, par = 100

Asset2 B: maturity 2, $i_{yield} = 7\%$, coupon rate = 0%, zero coupon bond, par = 100

Asset3 C: maturity 3, $i_{yield} = 9\%$, coupon rate = 5%, annual coupon bond, par = 100

Question: Unit of Bond A

Solve: Start from t=3, $100_{liability} = Unit\_C \times (5+100) \Rightarrow Unit\_C = 0.95238$

at t=2: $102_{liability} = Unit\_B \times (~~0+~~100) + \underbrace{Unit\_C \times (5\% \times par)}_{4.7619} \Rightarrow Unit\_B = 0.9724$

at t=1: $99_{liability} = \underbrace{Unit\_A \times (7+100)}$ + ~~Unit_B × 0~~ $+ \underbrace{Unit\_C \times (5)}_{4.7619}$ (5 = c% × par)

$\Rightarrow Unit\_A = 0.8807$ (A)

Exam FM Question 55: (B) "Redington Immunization" requires "frequent rebalancing".

Exam FM Question 56:

Give: ① Liability: 6000 at t=4

② Asset: A at t=2, B at t=6

③ $i = 5\%$

} "Redington Immunization"

Question: $|A-B|$

Solve: Redington Immunization
- version ① $PV^{Liability} = PV^{Asset}$
- ② $D^{Liability}_{mac} = D^{Asset}_{mac}$
- ③ $Convexity^{Asset} > Convexity^{Liability}$

① NPV = 0
② $\frac{dNPV}{di} = 0$
③ $\frac{d^2NPV}{di^2} > 0$

① $NPV = -600v^4 + Av^2 + Bv^6 = 0$ $\frac{\because \text{Need } \frac{\partial}{\partial i}}{\text{化为}(1+i)\text{形式}}$ $-600(1+i)^2 + A(1+i)^4 + B = 0$

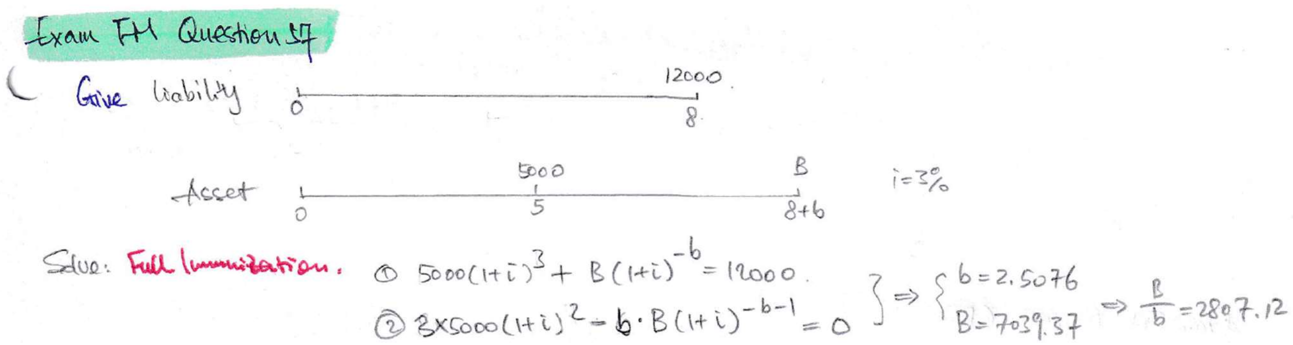
② $\frac{\partial NPV}{\partial i} = [-600(1+i)^2 + A(1+i)^4 + B]' = -1200(1+i) + 4A(1+i)^3 = 0 \Rightarrow$

Exam FM Question 57:

Give: (1) A liability of 12,000 due in 8-year. Liability will be met with payments of 5,000 in 5-year and B in 8+b years

(2) Full immunization with an annual effective interest rate of 3%

Question: What is $\frac{B}{b}$?



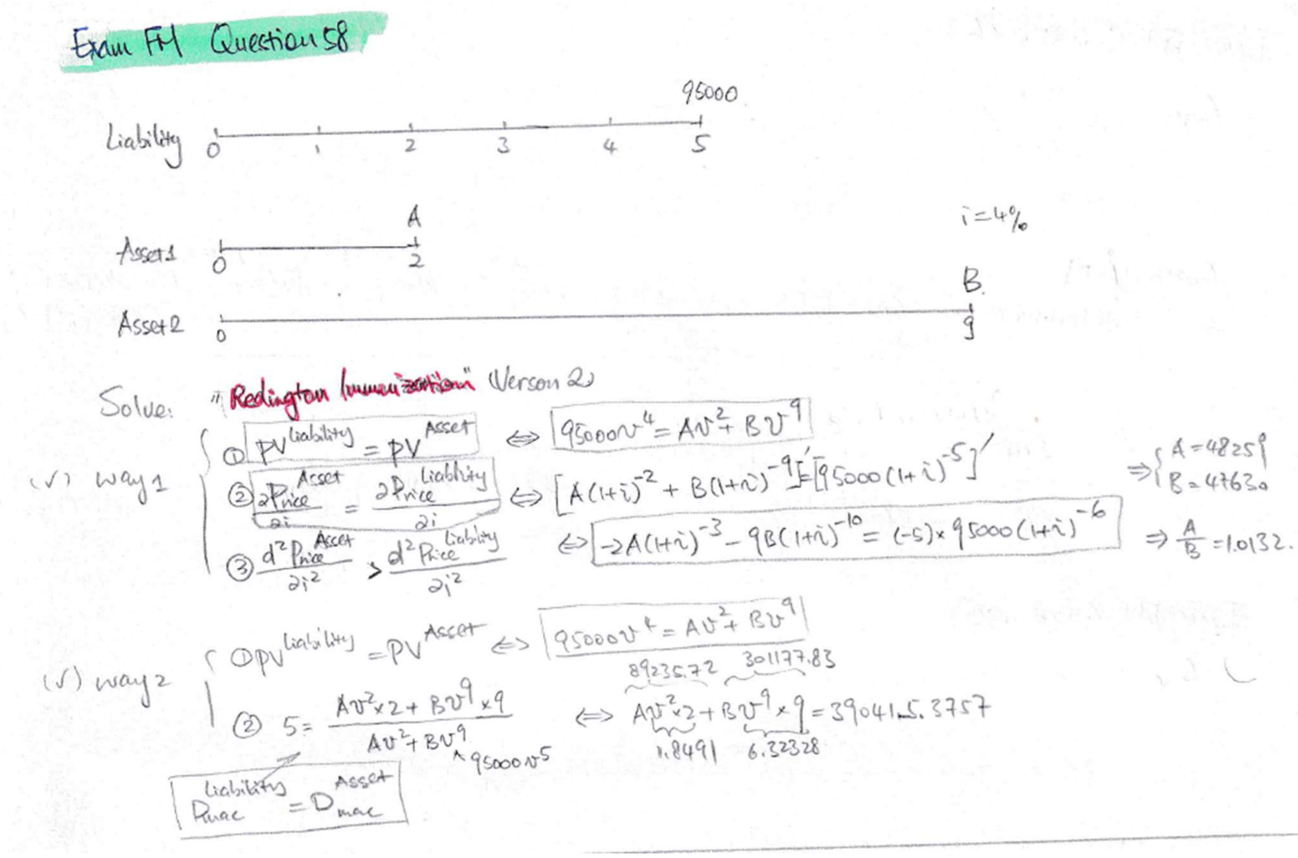
Exam FM Question 58:

Give: (1) Assets at time 2 of A and at time 9 of B.

(2) Liability of 95,000 at time 5.

(2) Redington immunization with an annual effective interest rate of 4%

Question: What is $\frac{A}{B}$?



Exam FM Question 59: annual coupon rate (i+0.04) paid semi-annually⇔ coupon rate_(6m)=(1+0.04)/2

Give: Two bonds: bond A and bond B:

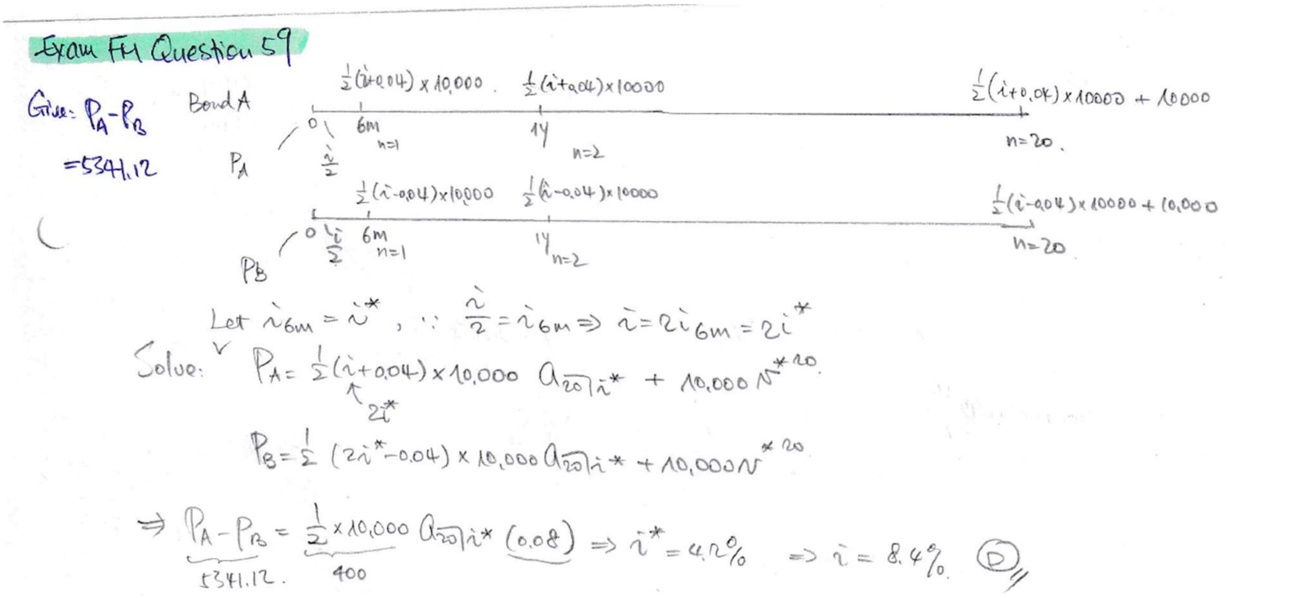
(i) Each bond is a 10-year bond with semi-annual coupons redeemable at 10,000. Par value is 10,000. An annual interest rate of I convertible semi-annually

(ii) Bond A: annual coupon rate (i+0.04) paid semi-annually

(iii) Bond B: annual coupon rate (i-0.04) paid semi-annually

(iv) The price of Bond A is 5,341.12 greater than the price of Bond B.

Question: What is i?

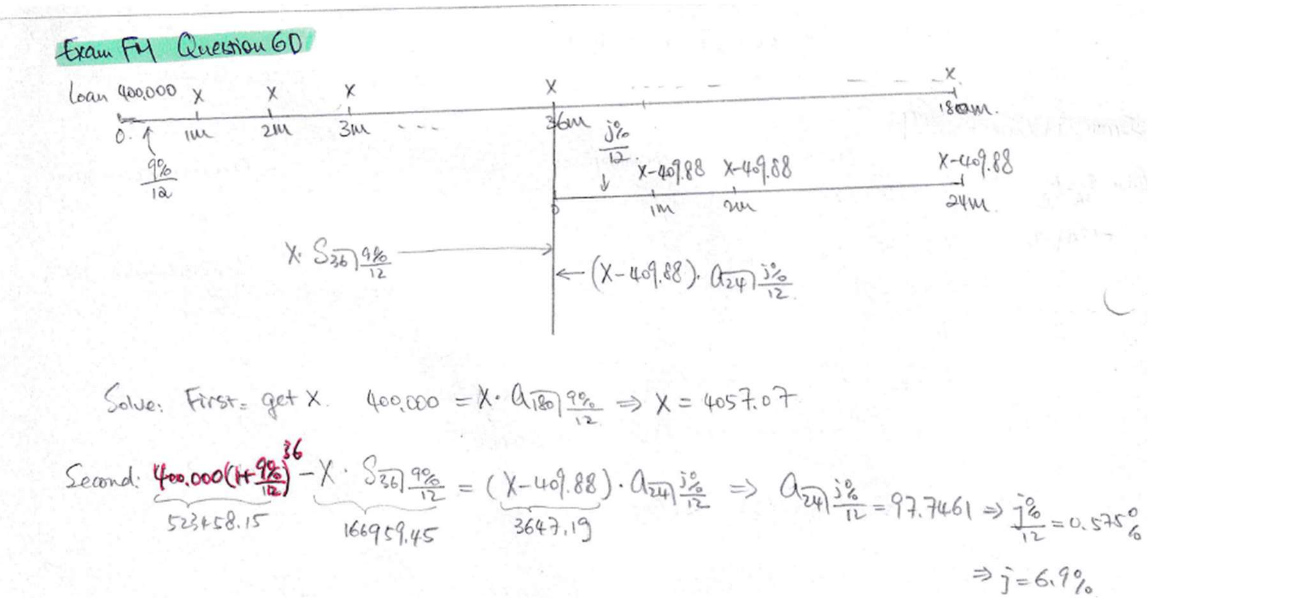


Exam FM Question 60

Give: (1) A 15-year loan for 400,000 with level end-of-month payments at an annual nominal interest rate of 9% convertible monthly. Immediately after the 36th payment, the borrow decides to refinance the loan at an annual interest rate of j convertible monthly. The remaining is kept at 12-year and level payments made at the end of month.

(3) Each payment is 409.88 lower than each payment from the original loan.

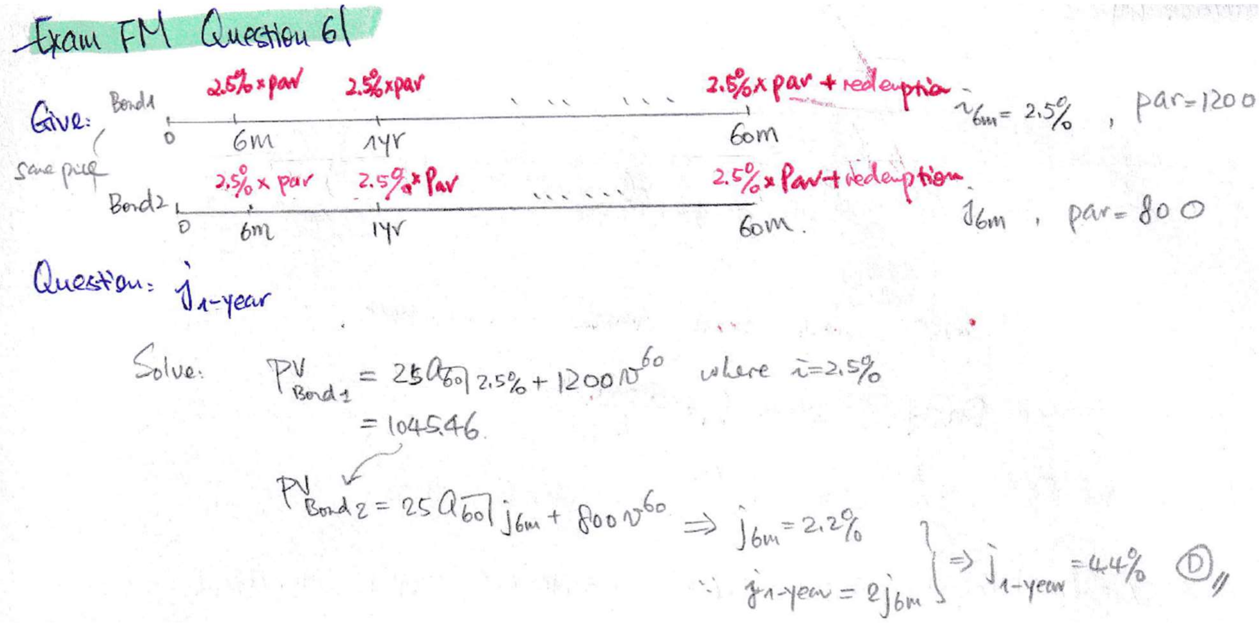
Question: What is j?



Exam FM Question 61:

Give: Two 30-year bonds with same purchase price. Each has an annual coupon rate of 5% paid semi-annually, and par value of 1000. The first bond has an annual yield rate 5% compounded semi-annually and redemption value 1200. The second bond has an annual yield rate j compounded semi-annually and redemption value 800.

What: What is j?

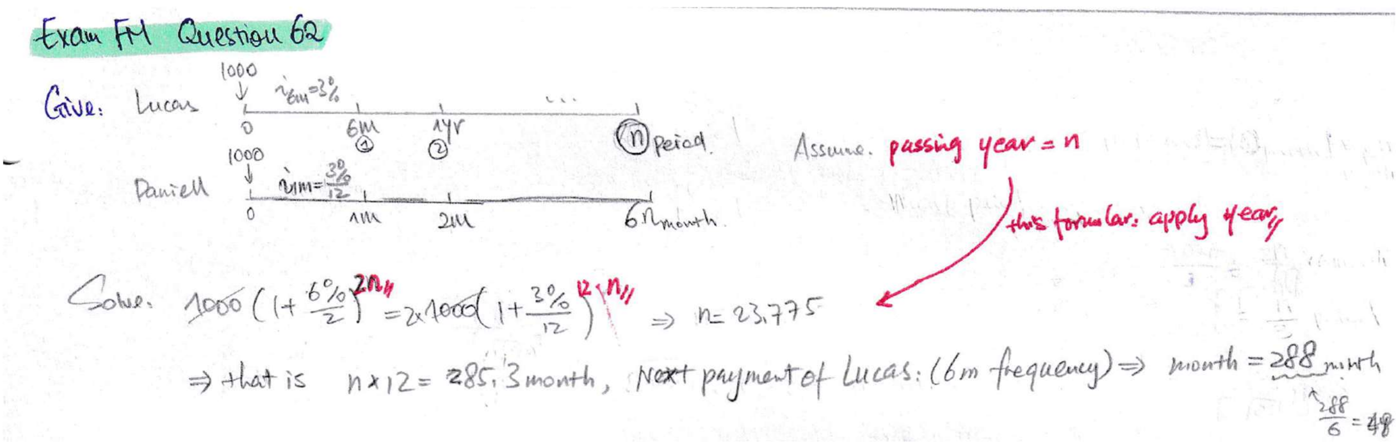


Exam FM Question 62:

Give: (1) Lucas opens a bank account with 1000 and accumulate at an annual nominal rate of 6% convertible semi-annually.

(2) Danielle opens a bank account with 1000 at the same time as Lucas and accumulate at an annual nominal rate of 3% convertible monthly.

What: What is the number of months required for the amount in Lucas's account to be at least double the amount in Danielle's account?



Exam FM Question 63:

Give: Bill and Joe each put 10 into accounts at t=0. Bill's account: constant rate of K/25. Joe's account: force of rate, δ_t = 1/(K+0.25t). At the end of four year, the amount in each account is X

Question: What is X?

Exam FM Question63

Give: Bill: $i_{annual} = \frac{k}{25}$ $(k>0)$

Joe: $i_{annual} = e^{\int_0^1 \frac{1}{k+0.25t} dt}$ (forward)

At the end of 4 year, Amt of Bill and Joe are both X.

Question: X

Solve: $10(1+\frac{k}{25})^4 = 10 \cdot e^{\int_0^4 \frac{1}{k+0.25t} dt}$

both sides: $4\ln(1+\frac{k}{25}) = \int_0^4 \frac{1}{k+0.25} dt = \frac{1}{0.25} \ln(k+0.25t)\Big|_0^4 = 4[\ln(k+1) - \ln(k)] = 4\ln\frac{k+1}{k}$

$\Rightarrow 1+\frac{k}{25} = \frac{k+1}{k} \Rightarrow k=5 \Rightarrow$ Thus, $X = 10\cdot(1+\frac{k}{25})^4 = 20.74$ (A)

Exam FM Question 64:

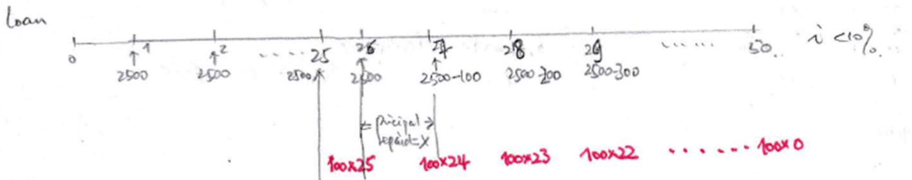
Give: (1) 50-year loan, repaid at the end of each year. The loan payment is 2500 for each of the first 26 years.

(2) The payments decrease by 100 per year thereafter.

(3) Interest on the loan is charged at rate i. The principal repaid in year 26 is X.

Question: What is the amount of interest paid in the first year?

Exam FM Question 64



Solve: $\text{Outstanding}_{t=25} = 100 \cdot Da_{\overline{25}|i}$ where $Da = \frac{n - a_{\overline{n}|}}{i}$

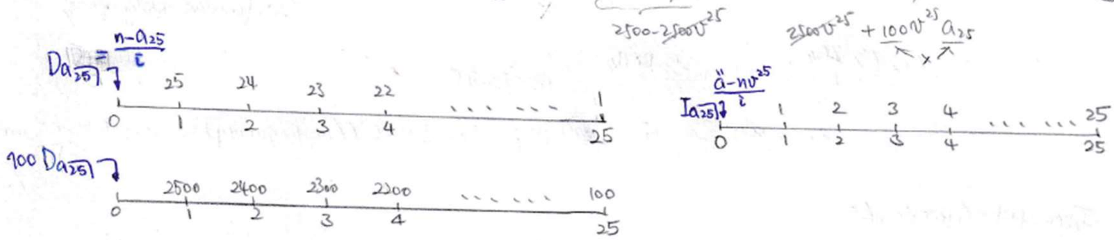
Then $\text{interest}_{26} = 100 \cdot Da_{\overline{25}|i} \cdot i = 100 \cdot \frac{n - a_{\overline{n}|}}{i} \cdot i = 100(25 - a_{\overline{25}|i})$

$\Rightarrow$ Principal Repaid $\boxed{X}$ = Level Payment$_{26}$ − interest$_{26}$ = $2500 - 100(25 - a_{\overline{25}|i}) = \boxed{100\, a_{\overline{25}|i}}$

$\Rightarrow \text{Outstanding}_{t=0} = 2500\, a_{\overline{25}|i} + v^{25}(100 \cdot Da_{\overline{25}|i})$, where $Da_{\overline{25}|i} = \frac{n - a_{\overline{n}|}}{i}$

$\Rightarrow \text{interest}_{t=1} = i \cdot 2500\, a_{\overline{25}|i} + 100v^{25}(25 - a_n) = i \cdot 2500 \frac{1-v^{25}}{i} + 100v^{25}(25 - a_{\overline{25}|}) = 2500 - X \cdot v^{25}$ (D)

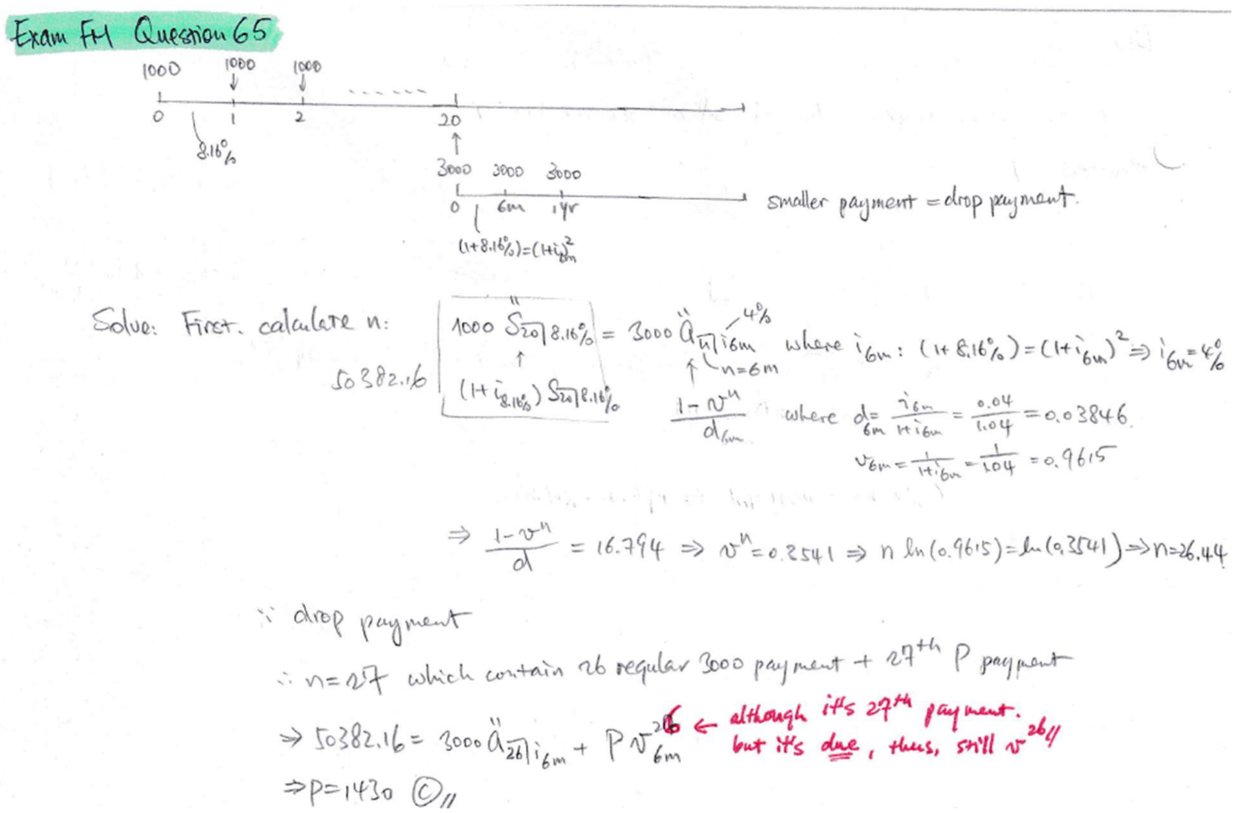
$2500 - 2500v^{25}$; $2500v^{25} + 100v^{25} a_{25}$ (X)



Exam FM Question 65:

Give: A deposit of 1000 into a fund at the beginning of each year for 20 years. At the end of 20 years, starting making semi-annual withdraw of 3000 at the beginning of each 6 months. A smaller final withdrawal to exhaust the fund. Annual rate 8.16%

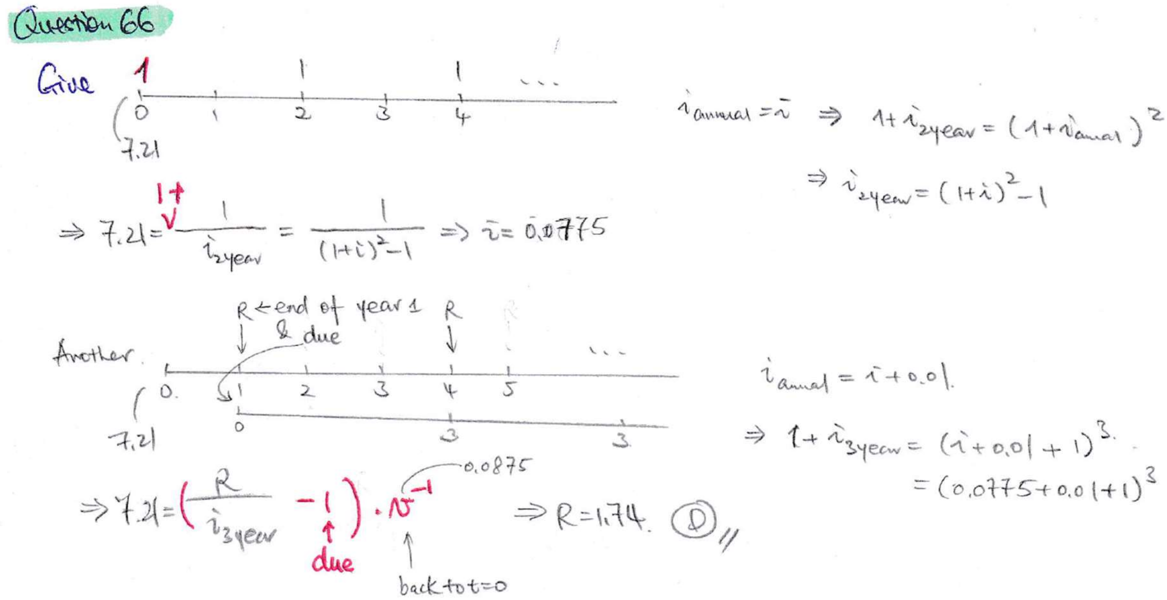
Question: What is the amount of final withdraw?



Exam FM Question 66:

Give: PV of a perpetuity paying 1 every two years with first payment due immediately is 7.21 at an annual rate of i. Another perpetuity paying R every 2 years with the first payment due at the beginning of year 2 has the same PV with an annual rate i+0.01

Question: What is R?



Exam FM Question 67:

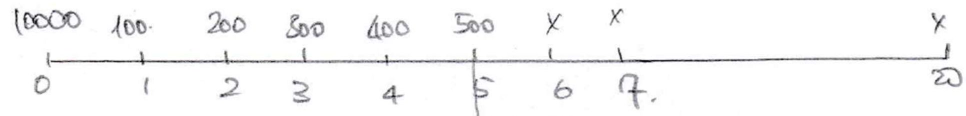
Give: (1) A loan of 10,000 is repaid with a payment made at the end of each year for 20 years. The payments are 100, 200, 300, 400, and 500 in years 1 and 5.

(2) In the subsequent 15 years, equal annual payments of X

(3) Annual rate is 5%

Question: What is X?

Question 67



$\Leftarrow 100 \cdot (Ia)_{\overline{5}|5\%} = \dfrac{\ddot{a} - n v^n}{i}$ where $\ddot{a}_{\overline{n}|} = (1+i)\, a_{\overline{n}|}$

$\Leftarrow X \cdot a_{\overline{20}|5\%}$

$$\Rightarrow 10000 = 100 \cdot \underbrace{\frac{(1+5\%)\, a_{\overline{n}|} - 5v^5}{i}}_{\frac{4.546 - 3.9176}{0.05} = 1256.8} + \underbrace{v^5 \cdot X \cdot a_{\overline{20}|5\%}}_{8.13273X} \quad \left(v = \frac{1}{1.05}\right)$$

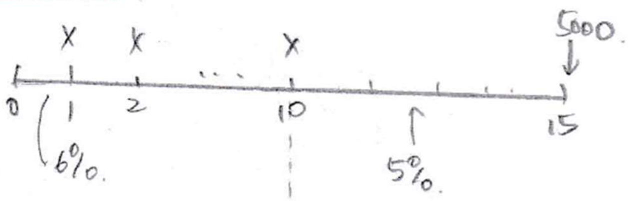
$\Rightarrow X = 1075$ (E)

Exam FM Question 68:

Give: Need to accumulate 5,000 in a fund at end of 15 years. Make equal deposits of X at then end of each year for the first 10 years. The fund earns an annual rate of 6% during the first 10 years and 5% for the next 5 years.

Question: What is X?

Question 68.



Solve: $X \cdot \underbrace{s_{\overline{10}|6\%}}_{\frac{(1+6\%)^{10} - 1}{i}} \cdot (1+5\%)^5 = 5000$

$16.822X = 5000 \Rightarrow X = 297.22$ (C)

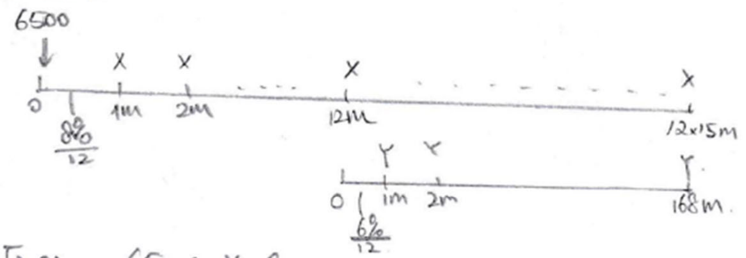
Exam FM Question 69:

Give: (1) A 15-year loan for 65,000 with end-of-month level payments. The annual rate of the loan is 8% convertible monthly.

(2) Immediately after the 12th payment, the remaining loan balance is re-amortized. Same maturity date, but the annual rate is changed to 6% convertible monthly.

Question: What is the new end-of-month payment?

Question 69



Solve. First: $65000 = X \cdot a_{\overline{180}|\frac{8\%}{12}} \Rightarrow X = 621.17$

Second $\underbrace{+65000\left(1+\frac{8\%}{12}\right)^{12}}_{70394.968} - \underbrace{621.17\, s_{\overline{12}|\frac{8\%}{12}}}_{7733.52} = Y \cdot \underbrace{a_{\overline{168}|\frac{6\%}{12}}}_{113.48} \Rightarrow Y = 552.18$

Exam FM Question 70:

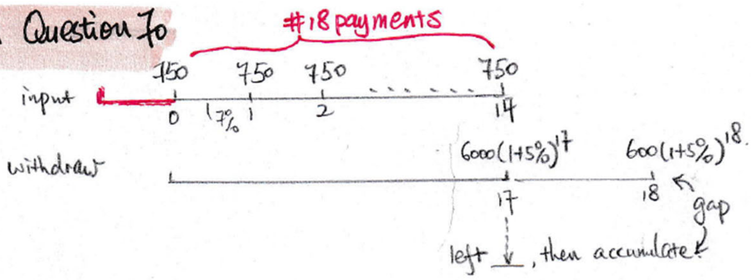
Give: (1) College tuition is 6,000 for the current school year, payable in full at the beginning of the school year. Tuition grows at an annual rate of 5%.

(2) A parent sets up a saving fund which earns an annual rate of 7%. The parent deposits 750 at the beginning of each school year for 18 years, with the first deposit made at the beginning of the current school years. Immediately following the 18th deposit, the parent pays tuition for the 18th school year from the fund.

(3) The amount of money needed, in addition to the balance in the fund, to pay tuition at the beginning of the 19th school year is X

Question: What is X?

Exam FM Question 70



Solve: $AV^{input}_{t=17} = 750\, s_{\overline{18}|7\%} = 750 \times \frac{(1+7\%)^{18}-1}{7\%} = 25499.27$ (#payments)

Then: After $6000(1+5\%)^{17}$ withdraw: left $= 25499.27 - 6000(1+5\%)^{17} = 11747.16$

Then: t=18 accumulate to $11747.16(1+7\%) = 12569.4613$

t=18 Withdraw Amt: $6000(1+5\%)^{18} = 14439.7154$ ⇒

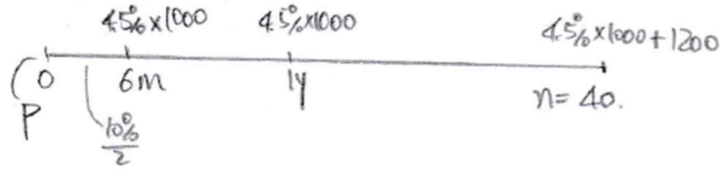
GAP $= 14439.7154 - 12569.4613 \cong 1870.254$ (E)

Exam FM Question 71:

Give: A 1,000 par value 20-year bond sells for P and yields a nominal interest rate of 10% convertible semi-annually. The bond has 9% coupons payable semi-annually and a redemption value of 1,200.

Question: What is P?

Exam FM Question 71



Solve: $P = 45\,a_{\overline{40}|5\%} + 1200\,v^{40} = 942.61$

$45\,a_{\overline{40}|5\%} = 772.1589$, $1200\,v^{40} = 170.4548$

Exam FM Question 72:

Give: (1) 10-year callable bond with face amount of 1,000 for price P. The bond has an annual nominal coupon rate of 10% paid semi-annually.

(2) The bond can be called at par on every other coupon payment date, beginning with second coupon payment date.

(3) Investor earns at least an annual nominal yield of 12% compounded semi-annually regardless of when the bond is redeemed.

Question: What is the largest possible value of P?

Exam FM Question 72.

Give: ① bond, 10-year, Face value = 1000, $r_{annual} = 10\%$ semi-annually

② yield: 12% compounded semi-annually

③ Call: 2^{nd} coupon date

Question: largest P

Solve: Recall: largest P ⟺ lowest yield ⟺ {Discount Case: To achieve "lowest yield", "latest Call" n=20, notice, lastest call = no call

[Coupon rate < Yield rate]

Thus: Redemption Value ≠ call value

Should = par value (∵ no say in this question, thus, redemption = par)

$P = 50\,a_{\overline{20}|6\%} + 1000\,v^{20} = 885.30$ Ⓐ

$50\,a_{\overline{20}|6\%} = 573.50$, $1000\,v^{20} = 311.80$

Exam FM Question 73:

Give: the term structure of interest rates:

Length of investment in years	Spot rate
4	9.00%
5	9.50%

Question: What is the one-year forward rate, deferred four years, implied by this term structure?

Solve $(1+9.0\%)^4 \cdot (1+f_{[4,5]}) = (1+9.5\%)^5 \Rightarrow 11.5\%$ (C)

Exam FM Question 74:

Give: Two retirements benefit options:

(1) First option: a lump sum of 374,500 at retirement

(2) Second option: receive monthly payments for 25 years starting 1 month after retirement. For the first year, the amount of each monthly payment is 2,000. For each subsequent year, the monthly payments are 2% more than the monthly payments from the previous year. Using an annual nominal rate of 6% compounded monthly, the PV of the second option is P

Question: which of the following is true?

(A) P is 323, 440 more than the lump sum option amount

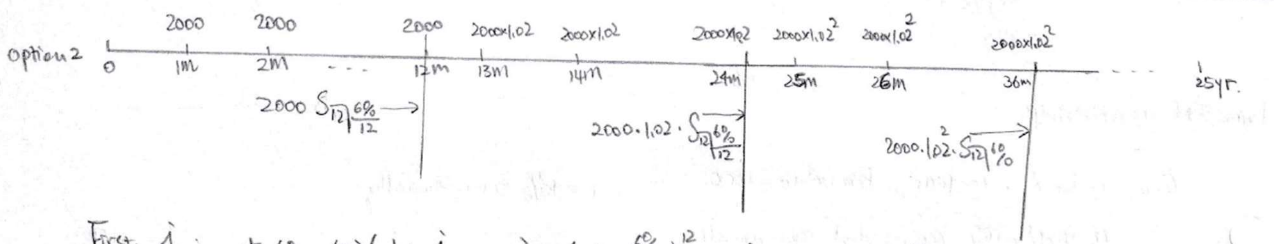
(B) P is 107,170 more than the lump sum option amount

(C) The lump sum option amount is equal to P.

(D) The lump sum option amount is 60 more than P.

(E) The lump sum option amount is 64,090 more than P.

Exam FM Question 74



First: $i_{1\text{-year}} \neq 6\% \Leftrightarrow (1+i_{1\text{-year}}) = (1+\frac{6\%}{12})^{12} \Rightarrow i_{1\text{-year}} = 6.1678\%$

Second: $PV = 2000\, s_{\overline{12}|\frac{6\%}{12}} \cdot v_{1\text{-year}} + 2000 \cdot 1.02\, s_{\overline{12}|\frac{6\%}{12}} \cdot v^2_{1\text{-year}} + \cdots + 2000 \cdot 1.02^{24} \cdot s_{\overline{12}|\frac{6\%}{12}} \cdot v^{25}_{1\text{-year}}$

$= \underbrace{2000\, s_{\overline{12}|\frac{6\%}{12}}}_{24671.12} \cdot v_{1\text{-year}} \left(1 + v_{1\text{-year}} \cdot 1.02 + \cdots + v^{24}_{1\text{-year}} \cdot 1.02^{24}\right)$

$\frac{1-(v_{1\text{-year}} \cdot 1.02)^{25}}{1-v_{1\text{-year}} \cdot 1.02}$ $\quad v_{1\text{-year}} \approx 0.9419$

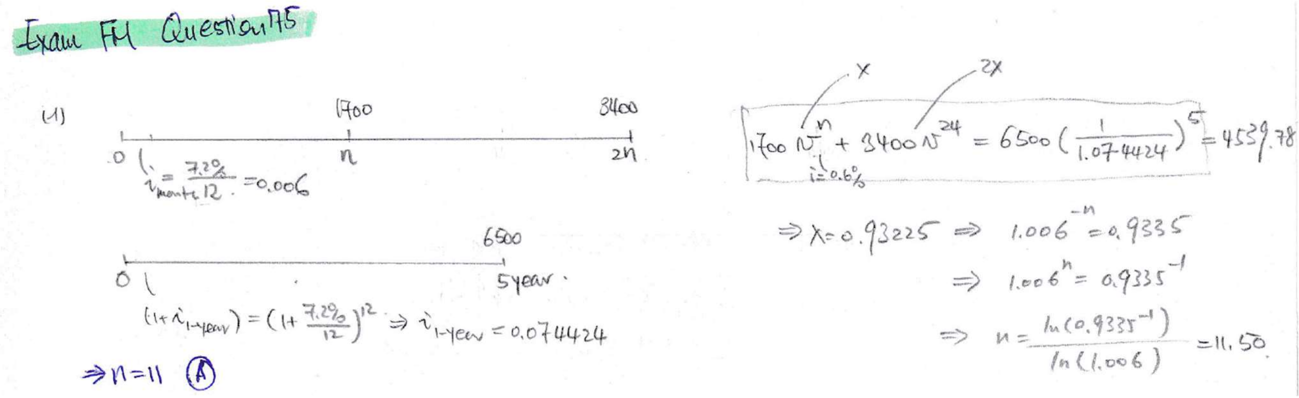
$= \frac{0.632562}{0.0392567} = 16.11346895$

(D≈57) $= 374442.46$ (D)

Exam FM Question 75:

Give: Save money for the first college tuition. Deposit 1,700 n months from today and another 3,400 2n months from today. All deposits earn a nominal rate of 7.2% compounded monthly.

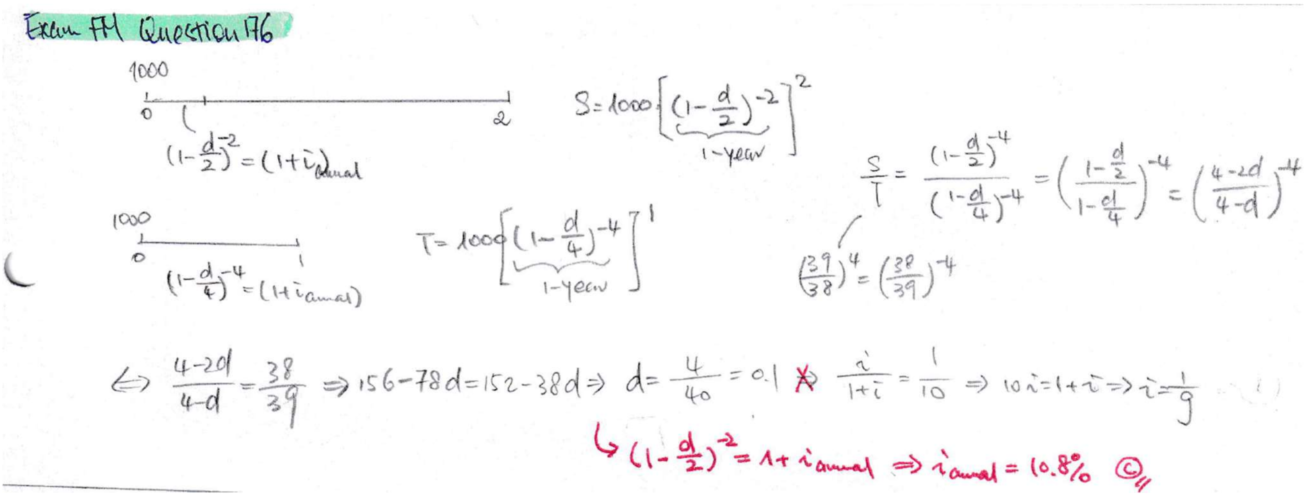
Question: What is the maximum integral value of n such that the parents will have accumulated at least 6,500 5-year from today?



Exam FM Question 76:

Give: S be the accumulated value of 1,000 invested for 2-year at a nominal annual rate of discount d convertible semi-annually, which is equivalent to an annual effective interest rate of i. T be the accumulated value of 1,000 invested for 1-year at a nominal annual rate of discount s convertible quarterly. S/T=(39/38)^4

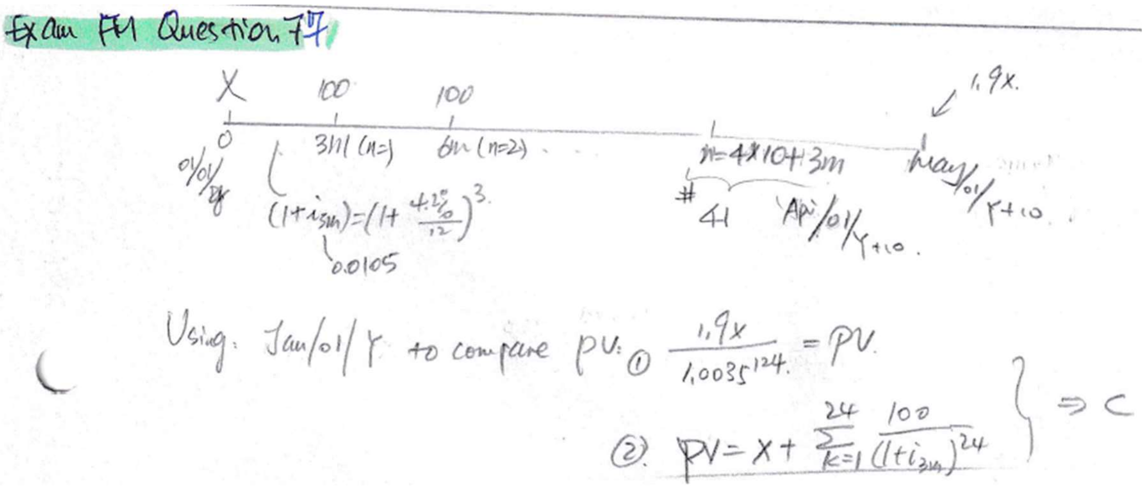
Question: What is i?



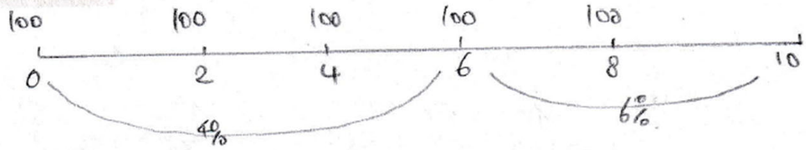
Exam FM Question 77:

Give: A retirement account pays an annual nominal rate of 4.2% convertible monthly. On 1st January of year y, the investor's balance was X. The investor then deposited 100 at the end of every quarter. On 1st May of year (y+10), the account balance was 1.9X.

Question: Which of the following is an equation of value that can be used to solve for X?



Exam FM Question 78



Solve: $AV_{t=10} = \underbrace{100(1+4\%)^6(1+5\%)^4}_{153.8} + \underbrace{100(1+4\%)^4(1+5\%)^4}_{142.197} + \underbrace{100(1+4\%)^2(1+5\%)^4}_{131.46913}$

$+ \underbrace{100(1+5\%)^4}_{121.5506} + \underbrace{100(1+5\%)^2}_{110.25} = 659.2667$

Meanwhile: $AV_{t=10} = 100\,\ddot{s}_{\overline{5}|i_{2\text{-year}}} \Rightarrow \ddot{s}_{\overline{5}|i_{2\text{-year}}} = 6.592667 = \dfrac{(1+i_{2\text{-year}})^5 - 1}{d_{2\text{-year}}}$

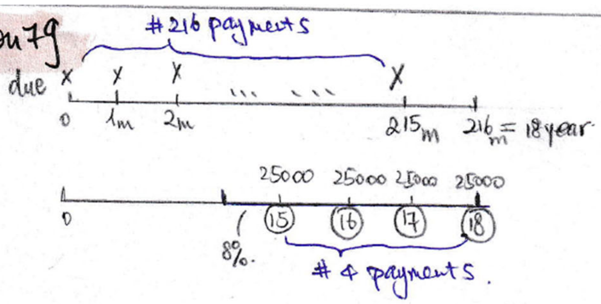
Annual
(A) 4.18%
(B) 4.4%
(C) 4.50% $\Rightarrow i_{2year} = 9.2025\% \rightarrow 6.56$
(D) 4.58% $(1+4.58\%)^2 = 1 + i_{2\text{-year}} \Rightarrow i_{2\text{-year}} = 9.3698\%$ (✓) $\rightarrow 6.59$
(E) 4.78% $\Rightarrow i_{2year} = 9.7885\% \rightarrow 6.67$

Exam FM Question 79:

Give: To support a college education plan, John makes deposits into a fund earning interest at an annual rate of 8%. He deposits XX at the beginning of each month for 18 years. In the 16th to 19th years, John makes a withdraw of 25000 at the beginning of each year. The fund balance becomes 0 after the final withdraw.

Question: what is X?

Exam FM Question 79



where $1.08=(1+i_{1m})^{12} \Rightarrow i_{1m} \cong 0.6434\%$

Solve: $AV_{t=18year} = X\ddot{s}_{\overline{216}|i_{1m}}^{0.6434\%} = s_{\overline{4}|8\%} \times 25000.$

$\ddot{s}_{\overline{216}|} = \frac{(1+0.6434\%)^{216}-1}{d_{0.6434\%}} = 468.646$

$s_{\overline{4}|8\%} = \frac{(1+8\%)^4-1}{8\%} = 4.50612$

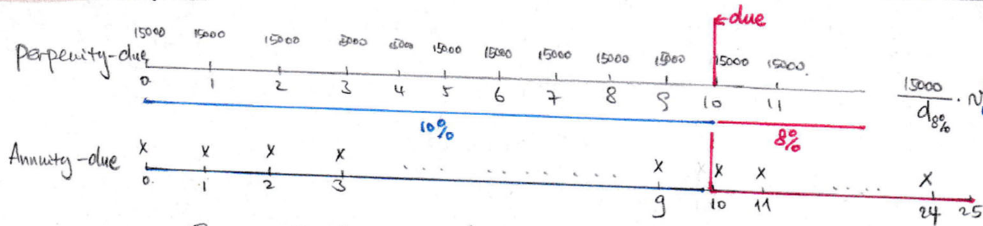
$\Rightarrow 468.646X = 2500 \times 4.50612 = 112652.8 \Rightarrow X \cong 240.38$ (C)

Exam FM Question 80:

Give: A perpetuity with annual payments of 15000. This perpetuity immediately exchanged the perpetuity for a 25-year annual-due having the same present value. The annuity-due has annual payments of X. All the PVs are based on an annual effective rate of 10% for the first 10 years and 8% thereafter.

Question: what is X?

Exam FM Question 80



$\frac{15000}{d_{8\%}}\cdot v_{10\%}^{10} + 15000\,\ddot{a}_{\overline{10}|10\%}$

$X\ddot{a}_{\overline{10}|10\%} + X\ddot{a}_{\overline{15}|8\%}\cdot v_{10\%}^{10}$

Solve: PV (perpetuity-due) = PV (Annuity-due)

$\frac{15000}{d_{8\%}}\cdot v_{10\%}^{10} + 15000\,\ddot{a}_{\overline{10}|10\%} = X\ddot{a}_{\overline{10}|10\%} + X\cdot\ddot{a}_{\overline{15}|8\%}\cdot v_{10\%}^{10}$ where $d_{8\%} = \frac{8\%}{1.08}$

$\frac{15000}{d_{8\%}}\cdot v_{10\%}^{10} = 78072.51611$

$15000\,\ddot{a}_{\overline{10}|10\%} = 15000 \times \frac{1-v^{10}}{d} = 101385.3572$

Left side: 179457.8733

$\ddot{a}_{\overline{10}|10\%} = 6.759$; $\ddot{a}_{\overline{15}|8\%} = 9.244237$; $\ddot{a}_{\overline{15}|8\%}\cdot v_{10\%}^{10} = 3.564$

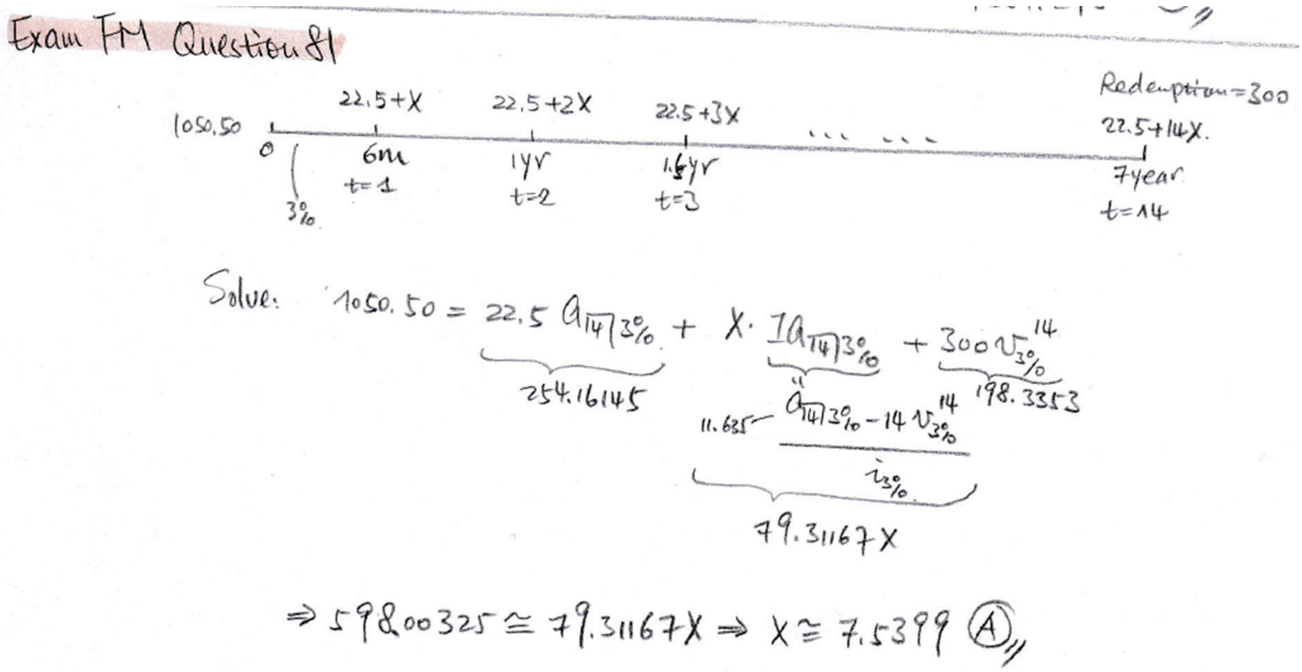
Right side: $10.323X$

$\Rightarrow 10.323X = 179457.8733 \Rightarrow X \doteq 17384.275$ (B)

Exam FM Question 81:

Give: A bond is redeemable for 300 in 7 years. The investor has just received a coupon of 22.50 and each subsequent semi-annual coupon will be X more than the preceding coupon. The PV of this bond immediately after the payment of coupon is 1050.50. Annual nominal yield rate of 6% convertible semi-annually

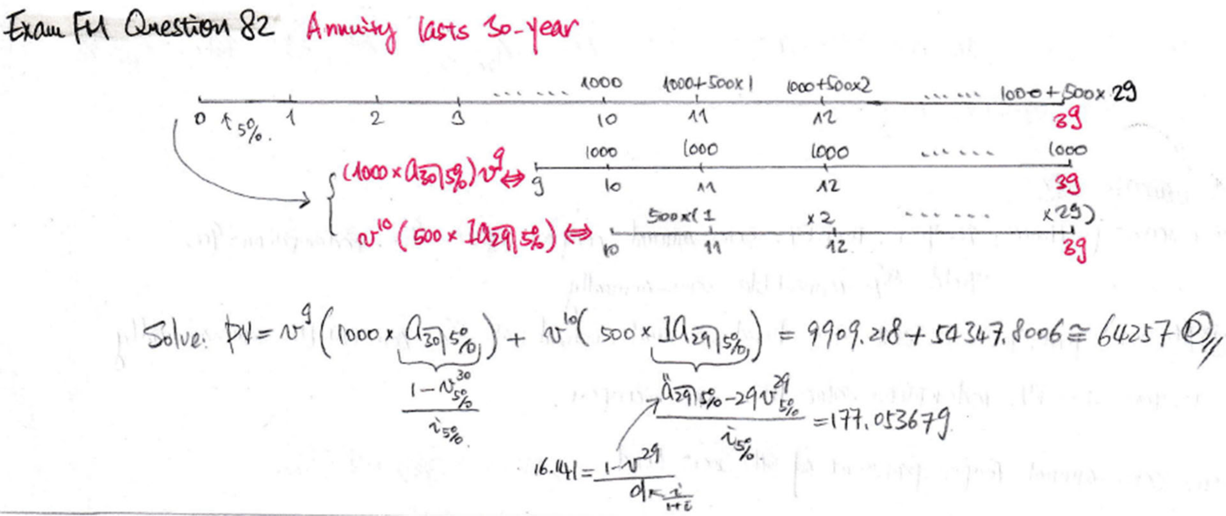
Question: what is X?



Exam FM Question 82:

Give: 30-year annuity is used to pay off a loan taken out today at 5% annual rate. The first payment of the annuity is due in 10 years in the amount of 1000. The subsequent payments increase by 500 each year

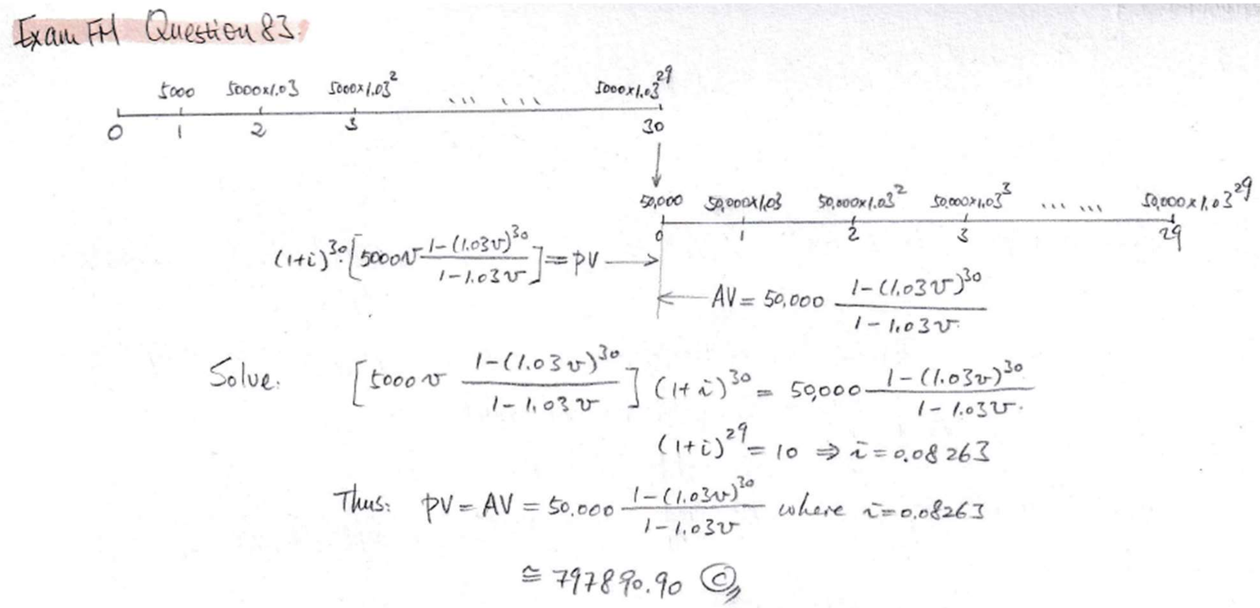
Question: what is "the amount of the loan"?



Exam FM Question 83:

Give: A women worked for 30 years before retiring. At the end of the 1st year of her employment, she deposited 5000, each subsequent year, she deposited 3% more than the prior year. A total of 30 deposits are made. The women will withdraw 50000 at the beginning of the 1st year of her first retirement year. Annual withdrawal at the beginning of each subsequent year for a total 30 withdrawals. Each withdraw is 3% more than the prior year. The account earns a constant annual effective interest rate,

Question: what is the account balance after the final deposit and before the first withdrawal?



Exam FM Question 84:

Give: Perpetuity-due with quarterly geometric payment series. A price of 100,000 based on an annual effective rate of i. The first and second quarterly payments are 2,000 and 2,010

Question: what is i?