

Duration Mismatch¹

Yi Li ²

January 16, 2024

In theory, durations of assets and liabilities should match. If a company's duration gap is zero³, then the company is not affected by (is immunized against) interest rate risk. For insurance companies, the duration of liabilities is normally provided by actuaries, while duration of assets is handled by the investment department. Duration mismatch, as known as liquidity mismatched, is one of the major Asset-Liability Mismatches besides currency mismatch and interest rate mismatch in Asset and Liability Management (ALM).

The impacts of duration mismatch:

(1) Duration of assets is larger than duration of liabilities⁴: when interest rates rise, values of assets and liabilities both fall, but the value of the asset side falls more due to its longer duration, further reducing the value of equity. This was the main reason for the collapse of SVB Bank.

(2) Duration of liabilities is larger than duration of assets⁵: when interest rates fall, values of assets and liabilities both raise, but the value of the asset side increases less. It is a decrease in the value of the asset compared to the liability side, which leads to a decrease in firm's equity.

We know that liquidity mismatch caused SVB bank to fall. In 2019, the average liability duration of life insurance companies operating in China is more than 12 years, while the average asset duration is around 6 years. Wang, Wang, and Chen (2021) reveal that Chinese listed companies often use short-term debt to finance long-term investments.

Macaulay Duration is a measure of the sensitivity of bond prices to changes in interest rates, which is related to, but is *not*, the slope of the price yield curve⁶. We know that by applying Taylor's theorem to $\Delta P = P(r + \Delta r) - P(r)$, we can obtain

$$\begin{aligned}\Delta P &\approx \left(-\frac{D^{mac}}{1+r} * P\right) * \Delta r + \frac{C}{2} P(\Delta r)^2 \\ &\approx (-D^{mod} * P) * \Delta r + \frac{C}{2} P(\Delta r)^2\end{aligned}$$

where D^{mac} is the Macaulay Duration, D^{mod} is the Modified Duration, and C is the convexity.

The tangent line underestimates prices rise when yields fall and overestimates the prices fall when yields increase. The last part of the formula is simply a convexity adjustment used for error correction. Line duration prediction produces an error, which is the difference between tangent line and curve. See figure below



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²Email: liyifinhub@outlook.com. Please email me if you find any errors.

³The duration gap is the difference in duration between assets and liabilities. It measures the timing matching of asset inflows and liability outflows.

⁴Positive duration gap.

⁵Negative duration gap.

⁶The figure of bond price against the yield to maturity.

This means that the slope is $-\frac{D^{mac}}{1+r} * P = -D^{mod} * P$, known as the dollar duration. For example, give $D^{mac} = 1$, 5% simple rate, 100 par, then a 100bps increase in rate corresponds to a $(-\frac{1}{1+5\%} * 100) * 0.01 = 0.95$ decrease in price.

Convexity, also known as modified convexity, is related to the second derivative of the price of bond w.r.t. yield defined as

$$C = \frac{\frac{d^2 P(i)}{di^2}}{P}$$

Recall, Modified Duration is related to the first derivative of the price of bond w.r.t. yield, defined as

$$D^{MOD} = -\frac{\frac{dP(i)}{di}}{P} = \frac{D^{mac}}{1+r}$$

In SOA and CAS FM exam, you will be asked to calculate the macaulay duration and the modified duration. The macaulay duration is further used in the first-order macaulay approximation and the first-order modified approximation.

$$Macaulay\ Duration = \frac{PV_1 * t_1 + PV_2 * t_2 + \dots + PV_n * t_n}{PV_1 + PV_2 + \dots + PV_n = P}$$

$$Modified\ Duration = \frac{Macaulay\ Duration}{1+i} = -\frac{\frac{\partial P}{\partial i}}{P}$$

In FM exam, there are two types of immunization: Redington Immunization and Full Immunization. Redington Immunization: discount the CFs to time 0. Full Immunization: discount the CFs to the time of single liability CF. The rest is the same, take the first and second derivate w.r.t interest rate respectively. Then, you will find the answer. Please check my website [SOA and CAS Exam FM](#) for more information.

Reference:

Yizhong Wang, Ting Wang, Lifang Chen (2021). Maturity mismatches of Chinese listed firms. Pacific-Basin Finance Journal. Volume 70.