# Crude Oil Price Co-movements: A Revisit

Yi Li*

January 11, 2024

**Abstract**

We investigate the dependence structure among WTI, Brend, and NEX using copula models. Model is implemented using ARMA(p,q)-EGARCH(1,1)-t for the marginal distributions and five time-invariant copula models for the joint distribution. We find evidence of symmetric tail dependence. Moreover, copula results show the tail dependence between crude oil returns is stronger than the tail dependence between the returns of crude oil and new energy global innovation index. Our findings are useful for portfolio diversification and risk management.

JEL Classification: C22; C46; C51; F41; G32
*Keywords*: Crude oil prices; Renewable energy prices; Copulas; Tail dependence coefficient; Co-movement



## 1 Introduction

We visit the dependence structure among crude oil benchmark prices and renewable energy stock prices in this paper. We try to answer the following questions: Is the tail dependence independent? Is it symmetic? Moreover, is the tail dependence between crude oil returns stronger than the tail dependence between crude oil prices and wilderhill new energy global innovation index returns? The analysis of tail dependence allow us to study the likelihood of getting joint

*Please send correspondence to Yi Li. Telephone: (+86) 134 7289 1956. E-mail: liyifinhub@outlook.com. Personal website: https://yilifinhub.com/

extreme co-movements of rude oil benchmark prices as well as the oil and renewable energy stock prices. That is, we know whether/how those markets boom or carsh together.

Our paper is closely related to Reboredo(2011) and Ghorbel(2018). Reboredo(2011) studies the dependence structure between crude oil benchmark prices using copula models from January 1997 to June 2010. They find evidence of symmetic tail dependence and this dependence is higher between WTI and Brent than other pairs, which suggest globalization is more likely to occur in high liquid markets. Ghorbel(2018) studies the dependence structure between WTI and three renewable energy indices from November 2003 to March 2016 using TAGRCH-based-Vine copula. Their results show positive dependence between oil and renewable energy markets. That is, those markets tend to crash and boom together.

There is an extensive literature studying the dependence structures across international financial markets using copula models. Reboredo(2012) studies the co-movements between oil price and exchange rate. Zhu et.al(2014) studies the dynamic dynamic dependence between oil prices and Asia stock market returns. Fei et al.(2017) uses markov-switching bivariate copula to study the dynamic dependence between CDS spreads and equity prices. LI (2019) studies the time-varying dependence between China sovereign CDS and bank CDS spreads. Chollete et al.(2011) uses copulas to study the international diversification of national stock market indices. Patton (2006) studies the asymmetric dependence between Deutsche mark and yen. Atil et al. (2016) uses dynamic Gaussian and Student's t copula to study the conditional dependence of U.S. and E.U sovereign CDS.

Some clear results emerges from our study. First, we find symmetic tail dependence between oil prices as well as oil prices and renewable energy stock market prices. To be percise, all the return pairs choose student's t copula. We also use AIC adjusted for small-sample bias as the alternative model seclection criteria, results again show student's t copula out-perfroms other copula models. Second, the tail dependence is stronger between oil prices than between oil and renewable energy stock market prices. The tail dependence coefficient between WTI and Brent is 0.495, while the tail dependence coefficient between WTI(Brent) and NEX is 0.032(0.020).

The remainder of the paper is organised as follows. Section 2 provides the modelling framework. Section 3 describes the data. Section 4 summarises results. Section 5 describes the conclusion and provides policy implications.

# 2 Modelling Framework

## 2.1 Marginal Distribution

The daily logarithmic changes of prices, denoted as $r_t$ , are modelled by ARMA(p,q)-EGARCH(1,1)-t model. The model is specificed as

$$r_t = a_0 + \sum_{i=1}^{p} a_i r_{t-i} + \varepsilon_t + \sum_{j=1}^{q} b_j \varepsilon_{t-j}$$
$$\log(\sigma_t^2) = \omega + \beta \log(\sigma_{t-1}^2) + \alpha |\frac{\varepsilon_{t-1}}{\sigma_{t-1}}| + \gamma(\frac{\varepsilon_{t-1}}{\sigma_{t-1}})$$

where $a_0$ and $\omega$ are constant, $a_p$ and $b_q$ stand for the parameters of AR lag p and MA lag q, $\beta$ and $\alpha$ are the parameters of GARCH and ARCH components. EGARCH(1,1) model is used to capture the asymmetric movement of volatility in daily returns. The leverage effect is captured by $\gamma$. $\gamma < 0$ suggests negative shocks(bad news) have more impact on the volatility of returns than positive shocks(good news). In another word, negative returns lead to higher volatility. The filtered returns $x_t = \frac{\varepsilon_t}{\sigma_t} \overset{i.i.d}{\sim} t_v$. That is, we use the Student's t distribution for the distribution of the standardized residuals. The optimal (p,q) combination with $p \in [0, 5]$ and $q \in [0, 5]$ is selected using Akaike Information Criterion.

Table 1: Copula Models

| Copula | Distribution | $\tau^U$ | $\tau^L$ |
|---|---|---|---|
| Gaussian | $C(u, \upsilon; \rho) = \Phi_\rho(\Phi^{-1}(u), \Phi^{-1}(\upsilon))$ | $0$ | $0$ |
| Student's t | $C(u, \upsilon; \rho, \nu) = T_{\rho,\nu}(t_\nu^{-1}(u), t_\nu^{-1}(\upsilon))$ | $g_T(\rho, \nu)$ | $g_T(\rho, \nu)$ |
| Clayton | $C(u, \upsilon; \theta) = (u^{-\theta} + \upsilon^{-\theta} - 1)^{-1/\theta}$ | $0$ | $2^{-1/\theta}$ |
| Gumbel | $C(u, \upsilon; \delta) = \exp(-((-\log u)^\delta + (-\log \upsilon)^\delta)^{1/\delta})$ | $2 - 2^{1/\delta}$ | $0$ |
| SJC | $C(u, \upsilon; \tau^U, \tau^L) = 1 - (1 - [1 - (1 - u)^k]^{-\gamma} + [1 - (1- \upsilon)^k]^{-\gamma} - 1^{-1/\gamma})^{1/k}$ <br> $where\ k = 1/log_2(2 - \tau^U), \gamma = -1/log_2(\tau^L)$ | $\tau^U$ | $\tau^L$ |

This table shows the distribution, upper and lower tail dependence coefficients of copula models. $\Phi_\rho$ is the bivariate standard normal cumulative distribution function with correlation $\rho$. $T_{\rho,\nu}$ is the bivariate Student-t cdf with d.o.f $\nu$ and correlation $\rho$. The lower and upper tail dependence coefficient of Student's t copula is $g_T(\rho, \nu) = 2 * F(-\sqrt{(\nu + 1)\frac{\rho-1}{\rho+1}}, \nu + 1)$.

## 2.2 Copula Function

Gaussian, Student's t, Clayton Gumbel and SJC copulas are used to study the dependence structure. Those five copula models have different tail dependence characteristics. Gaussian copula has zero tail dependence. Gumbel and Clayton copulas have asymmetric tail dependence. To be specific, Gumbel copula has upper tail dependence and zero lower tail dependence. Clayton copula has lower tail dependence and zero higher tail dependence. The Student's t and SJC copulas consider both upper and lower tail dependence. The detailed description of copula functions and their corresponding tail dependence coefficients are shown in Table 1. We use AIC to select the best copula model following Fei et al.(2017), Atil et.al(2016)), Reboredo(2011), and Reboredo(2013).

## 2.3 Estimation and Goodness-of-Fit

Parameters are estimated using a two stages process following Fei et al.(2017), Patton(2013) and Reboredo(2011). First, we estimate the parameters of the marginal distributions. Second, we estimate the parameters of copula models conditional on the margins. To be specific, we denote the parameters of margins and copula as $(\phi_1, \phi_2)\prime$ and $\theta$. The log-likelihood function can be expressed as

$$
\begin{aligned}
L(\theta, \phi) &= \sum_{n=1}^{2} L_n(\phi_n) + L_C(\phi_1, \phi_2; \theta) \\
&= \sum_{n=1}^{2}\sum_{t=1}^{T} \log f_{n,t}(x_{n,t}; \phi_n) + \sum_{t=1}^{T} \log c(u_{1,t}, u_{2,t}; \theta)
\end{aligned}
$$

The two-steps ML estimator of copula parameters is asymptotically normal and consistent. Although it's not efficient, the simulation results from Joe (2005) shows that the efficiency loss is generally small in practice.

We use different methods to evaluate the goodness-of-fit of the marginal distribution. Ljung-Box test is used to test whether the persistence in returns has been accounted. P-value of LB(10) test higher than 0.05 means the null hypothesis that the residuals from ARMA(p,q) are not autocorrelated at lag 10 can not be rejected at 5% significant level. Engle(1982) Lagrange multiplier test is employed to test whether volatility clustering has been accounted. P-value of LM(10) test higher than 0.05 means the null hypothesis that there is no ARCH effect at lag 10 can not be rejected at 5% significant level. Diebold et al.(1998) suggests that û should be i.i.d uniform(0,1) distributed if marginal distribution is correctly specified. This is tested in two steps: First, we use LB statistic to exam the serial correlation of the first fourmoments of $(\hat{u} - \bar{u})^j, j \in \{1, 2, 3, 4\}$. See for example Fei et al.(2017). Second, we use the Kolmogorov–Smirnov (K–S), Cramer–von Mises (C–vM) and Anderson–Darling (A–D) tests to evaluate whether the transforms are Uniform(0,1) distributed. We accept i.i.d and uniform assumptions if LB test and KS tests can not be rejected at 5% significant level.

Table 2: Data Description of Daily Returns

| | Mean | Medium | Max | Min | Jarque-Bera | LB(10) | ARCH-LM(10) |
|---|---|---|---|---|---|---|---|
| WTI | 0.000 | 0.001 | 0.116 | -0.091 | 0.001 | 0.035 | 0.000 |
| Brent | 0.000 | 0.000 | 0.098 | -0.084 | 0.001 | 0.003 | 0.000 |
| NEX | -0.000 | 0.000 | 0.121 | -0.089 | 0.001 | 0.006 | 0.000 |

Daily data for the period from 15 July 2014 to 31 October 2018. Jarque-Bera is the $\chi^2$ statistic for the test of normality. LB(10) is the Ljung-Box statistics for serial correlation in the returns using 10 lags. ARCH-LM(10) is the Engel's LM test for the heteroskedasiticity using 10 lags.

# 3 Data Description

We limited our study to the dependence structure of major(highly liquid) oil and renewable energy benchmarks. (1) WTI, the reference benchmark for light crude oil in United States. (2) Brent, the reference benchmark for North Sea crude oil with similar characteristics similar to WTI. (3) NEX, the wilderhill new energy global innovation index, which is composed of companies worldwide whose innovative technologies focus on clean energy, low CO2, renewables, conservation and efficiency. The daily data is obtain via Bloomberg terminal. Sample period spans from 7/15/2014 to 10/31/2018. All prices are denominated in U.S. dollars[1]. Returns are computed on a continuous compunding basis as $r_t = \ln(P_t/P_{t-1}),$ where $P_t$ and $P_{t-1}$ are current and one-day lagged daily prices.

Table 1 provides statistical description of the return series for the period from 15 July 2014 to 31 October 2018. The linear serial dependence is tested by Ljung-Box test with 10 lags. All three return series have linear serial dependence. We use the Engle (1982) lagrange multiplier test for detecting ARCH effects. Results suggest the null hypothesis of no ARCH effect can be rejected at 5% significant level for all three series. Results on Jarque-Bera test show the null hypothesis of normally distributed return is rejected at 1% significant level for all series.

[1]Footnote: Crude oil prices are denominated in U.S. dollar per barrel.

# 4 Results

Table 3 reports the ML estimates for the marginal distributions. The d.o.f parameter $\upsilon$ has a smaller value for NEX returns than for crude oil returns. This suggests NEX returns have fatter tails than crude oil returns. The leverage parameters for all three return series are all negatively significant at 5% significant level. The negative leverage parameter suggests negative shocks have more impact on the volatility of returns than positive shocks for all return series.

Table 4 shows the goodness-of-fit test of marginal distribution models. The p-values of LB(10) and LM(10) tests show neither autocorrelation nor ARCH effects remain in the filtered standardised residuals. The p-values of Ljung-Box test on the first four moments of the estimated probability integral transformations $(\overset{\wedge}{u} - \bar{u})^j, j \in \{1, 2, 3, 4\}$ at lag 10. P-values are all above 0.05, which suggests we can not reject the i.i.d assumption at 5% significant level for all series. P-values of KS tests are all above 0.05, which indicates the null hypothesis that the transforms are Uniform(0,1) distributed cannot be rejected at 5% significant level. Overall, the results of goodness-of-fit suggest our marginal distribution models are not mis-specified and copula models can correctly capture the co-movement between sovereign CDS returns.

Table 5 reports the results of AIC and tail dependence coefficients for static Normal, Gumbel, Clayton, Stduent's t and SJC copulas. There are several conclusions we can draw from the estimation results: First, Student's t copula out-performs other copula models, which suggests symmetric tail dependence coefficients for all pairs. Second, WTI, crude oil returns, have a higher tail dependence with Brent, other oil returns, than with new energy global innovation index returns. To be specific, table 5 shows the tail dependence coefficient for WTI and Brent pair is 0.492, while the average tail dependence coefficient for crude oil and new energy global innovation index pair is 0.026. Those results are consistent with Reboredo(2011), which uses crude oil prices from January 1997 to June 2010. Reboredo(2011) shows symmetric upper and

Table 3: Marginal Distribution and Goodness-of-Fit

| | WTI | Brent | NEX |
|---|---|---|---|
| Panel A: Marginal Distribution | | | |
| Mean Equation | | | |
| $a_0$ | $0.000(0.001)$ | $-0.000\,(0.000)$ | $0.000(0.000)$ |
| Variance Equation | | | |
| $\omega$ | $-0.072(0.040)$ | $-0.090(0.052)$ | $-0.467(0.126)^*$ |
| $\beta$ | $0.990(0.005)^*$ | $0.988(0.006)^*$ | $0.952(0.012)^*$ |
| $\alpha$ | $0.100(0.025)^*$ | $0.136(0.030)^*$ | $0.129(0.038)^*$ |
| $\gamma$ | $-0.068(0.016)^*$ | $-0.066(0.019)^*$ | $-0.153(0.027)^*$ |
| d.o.f | $12.383(4.943)^*$ | $10.913(3.860)^*$ | $7.369(1.563)^*$ |
| Log-likelihood | 2426.5 | 2510.3 | 3430.7 |
| Panel B: Goodness-of-Fit | | | |
| LB(10) | 0.829 | 0.920 | 0.806 |
| LM(10) | 0.482 | 0.282 | 0.062 |
| First Moment | 0.853 | 0.859 | 0.978 |
| Second Moment | 0.975 | 0.660 | 0.564 |
| Third Moment | 0.762 | 0.772 | 0.401 |
| Fourth Moment | 0.858 | 0.827 | 0.434 |
| Kolmogorov–Smirnov Test | 0.203 | 0.158 | 0.061 |

This table shows the test for the margial distribution models. Standardised residuals are checked for autocorrelation and ARCH effect using Ljung-Box test and Engel's LM test at lag 10. The first fourmoments of $(\hat{u} - \bar{u})^j, j \in \{1, 2, 3, 4\}$ are used to check for i.i.d assumption. First, second, third, fourth moment stand for the p-values of Ljung-Box test on the first four moments of $(\hat{u} - \bar{u})^j$ at lag 10. P-values are also shown from the Kolmogorov–Smirnov(KS), Cramer–von Mises (CvM) tests, Anderson–Darling (AD) tests to evaluate whether the transforms are Uniform(0,1) distributed.

Table 4: Results of Copula Models

| Panel A: AIC | | | | | |
|---|---|---|---|---|---|
| | Normal | Gumbel | Clayton | Student's t | SJC |
| WTI vs Brent | -1277.700 | -1247.270 | -1055.290 | **-1322.500** | -1275.850 |
| WTI vs NEX | -63.980 | -63.236 | -58.663 | **-71.816** | -66.552 |
| Brent vs NEX | -66.966 | -57.077 | -69.370 | **-73.804** | -71.179 |
| Panel B: Tail Dependence Coefficient [lower, upper] | | | | | |
| | Normal | Gumbel | Clayton | Student's t | SJC |
| WTI vs Brent | [0,0] | [0,0.734] | [0.779,0] | **[0.492,0.492]** | [0.720,0.722] |
| WTI vs NEX | [0,0] | [0,0.255] | [0.151,0] | **[0.032,0.032]** | [0.143,0.145] |
| Brent vs NEX | [0,0] | [0,0.246] | [0.183,0] | **[0.020,0.020]** | [0.083,0.199] |

This table panel A shows the AIC of Normal, Gumbel, Clayton, Student's t and SJC copula models. For each pair, bold italic font denotes the selected copula formulation overall. Panel B shows the tail dependence coefficients of the above five copula models. The bold italic font denotes the tail dependence coefficient of the selected best copula formulation.

lower tail dependence between crude oil prices and the implied tail dependence coefficient is 0.57 for WTI and Brent pair. We re-evalute the performance of copula models using AIC adjusted for small-sample bias as in Breymann et al(2003) and Rodriguez(2007). Table 6 shows the AIC adjusted for small-sample bias and all three pairs still choose Student's t copula as in Table 5. This shows our results of copula model selection and tail dependence coefficients are robust.

Table 5: AIC Adjusted for Small-Sample Bias

| | Normal | Gumbel | Clayton | Student's t | SJC |
|---|---|---|---|---|---|
| WTI vs Brent | -1277.700 | -1247.270 | -1055.290 | **-1322.490** | -1275.830 |
| WTI vs NEX | -63.976 | -63.232 | -58.659 | **-71.805** | -66.540 |
| Brent vs NEX | -66.962 | -57.073 | -69.366 | **-73.792** | -71.167 |

This table shows the results of AIC adjusted for small-sample bias(See Reboredo (2011), Breymann et al. (2003) and Rodriguez (2007)) for all three pairs: WTI vs Brent, WTI vs NEX and Brent vs NEX. This is used as a robustness check for the copula model selection results as shown in table 4.

## 5 Conclusion

We exaim the dependence structure of oil market and renewable energy market by study the co-movement of the prices. By using different copula models with different tail dependence structures, we find Student's t copula out-perform other copula formations. This implies symmetric tail dependence for all three pairs: WTI-Brent, WTI-NEX and Brent-NEX. Moreover, the tail dependence coefficient between WTI and Brent are significantly higher than the tail dependence coefficient between WTI(Brent) and NEX.

Those results suggest: First, the globalization of crude oil market. This is consistent with Reboredo(2011). Second, a less integrated crude oil and renewable energy markets. Our results suggest a high likelihood of getting extreme co-movements in crude oil markets. In another word, the high tail dependence implies a high downside (upside) risk in crude oil market investments. The dependence coefficient between oil returns and renewable energy stock market returns is close to zero, which is close to no-tail dependence.

## References


Atil, A., Bradford, M., Elmarzougui, A., Lahiani, A., 2016. Conditional dependence of US and EU sovereign CDS: A time-varying copula-based estimation. Finance Research Letters 19, 42–53.

Breymann, W., Dias, A., Embrechts, P., 2003. Dependence structures for multivariate highfrequency data in finance. Quantitative Finance, 3 (1), 1–16.

Chollete, C., Pena, V.D.L., Lu, C.C., 2011. International diversification: A copula approach. Journal of Banking and Finance 35 (2), 403–417.

Diebold, F.X., Gunther, T.A., Tay, A.S., 1998. Evaluating density forcatses with applications to financial risk management. International Economic Review 39 (4), 863–883.

Engle, R., 1982. Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. Econometrica 50 (4), 987–1007.

Fei, F., Fuertes, A.M., Kalotychou, E., 2017. Dependence in credit default swap and equity markets: Dynamic copula with markov-switching. International Journal of Forecasting 33 (3), 662–678.

Joe, H., 2005. Asymptotic efficiency of the two-stage estimation method for copula-based models. Journal of Multivariate Analysis 94 (2), 401–419.

LI, Y., 2019. Understanding China sovreign CDS. Working Paper.

Mejdoub, H., Ghorbel, A., 2018. Conditional dependence between oil price and stock prices of renewable energy: a vine copula approach. Economic and Political Studies 6 (2), 948-955.

Patton, A.J., 2006. Modelling asymmetric exchange rate dependence. International Economic Review 47 (2), 527–556.

Patton, A.J., 2006. Copula methods for forecasting multivariate time series. Handbook of Economic Forecasting 14 (3), 899–960.

Reboredo, J.,C., 2011. How crude oil prices co-movements? A copula approach. Energy Economics 33 (5), 948-955.

Reboredo, J.,C., 2012. Modelling oil price and exchange rate co-movements. Journal of Policy Modeling 34 (3), 419–440.

Reboredo, J.,C., 2013. Is gold a safe haven or hedge for the US dollar? Implication for risk management. Journal of Banking and Finance 37, 2665–2676.

Rodriguez, J. C., 2007. DMeasuring financial contagion: A copula approach. Journal of Empirical Finance, 14 (3), 403–423.

Zhu, H.M. Li, R., Li, S., 2014. Modelling dynamic dependence between crude oil prices and Asia-Pacific stock market retruns. International Review of Economics and Finance (29), 208–223.